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Fundamental Length Hypothesis in a Gauge Theory Context†

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ABSTRACT

The new gauge formulation of the electromagnetic interaction theory, containing the "fundamental length" ℓ as a universal scale like $\hbar$ and c, is worked out. A key part belongs to the 4-dimensional de Sitter p-space, with the curvature radius $\hbar/\ell c$. In the new approach the electromagnetic potential becomes a 5-vector associated with de Sitter group O(4.1). The extra fifth component, called the τ-photon, similar to scalar and longitudinal photons, does not correspond to an independent dynamical degree of freedom. Respectively, the new local gauge group is larger than the ordinary one and depends intrinsically on the fundamental length ℓ.

The gauge invariant equations of motion, replacing the Dirac-Maxwell equations, are set up. The new formulation is minimal with respect to the 5-potential but is not so in terms of the usual 4-potential. As a result, the underlying physics looks much richer than the ordinary electromagnetic phenomena. The new scheme predicts the existence of the electric dipole moments for charged particles, leading to a direct violation of P- and CP-symmetries, and the new universal correction to the (g-2)-anomaly. Further, some new group of internal symmetry, $SU_\tau(2)$, arises that can be used to describe the μe-symmetry of the electromagnetic interactions. It turns out that $SU_\tau(2)$-symmetry is violated by the 4-fermion type interaction, induced by τ-photons, with associated coupling constant $\sim\alpha\ell^2$. This novel interaction might give rise to the μe-mass difference and processes like $\mu \to 3e$, $\mu \to e\gamma$, etc.

In the limit $\ell \to 0$, the new field equations turn into the Dirac-Maxwell equations for the electron, muon and electromagnetic fields. So, one may consider this approach as a generalization in a profound way of the standard theory of electromagnetic interactions at small distances $\lesssim \ell$ (high energies $\gtrsim 1/\ell$).

The upper bound for the fundamental length ℓ is discussed taking into account the various experimental data.

1. FUNDAMENTAL LENGTH AND DE SITTER MOMENTUM SPACE

In the present talk we shall discuss a new gauge formulation of the electromagnetic interaction theory which is based on a concept of fundamental length. This new hypothetical constant we denote as ℓ. Together with $\hbar$ and c it is expected to regulate all microscopic phenomena. The quantity

$$M = \hbar/\ell c \tag{1.1}$$

is called the fundamental mass.

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The idea of an existence of a new universal length, and therefore mass, that would fix a scale in 4-dimensional space-time, and therefore 4-dimensional momentum-energy space, was discussed in literature of the last four decades in different contexts.[1-14]

Most of the people who tried to introduce a fundamental length into field theory, pursued a quite clear and practical goal: to cure the theory from the ultraviolet divergences. But it turned out that a theory can survive with this chronic disease, and work as a quantitative scheme, if it possesses genetically a renormalizability property. Nowadays, the principle of renormalizability has imperceptibly become one of the corner stones of the quantum field theory. As a result, interest in a fundamental length has almost died out (see, however, refs. 15-18).

The greatest triumph of the renormalization approach to the formulation of the quantum field theory is certainly quantum electrodynamics (QED). The predictions of QED agree with a number of highly precise experiments. The upper bound for the magnitude of the fundamental length, established in experiments in the test of QED at high energies, now is given by

$$\ell \lesssim 10^{-15}\ \mathrm{cm} \qquad . \qquad (1.2)$$

The harmony and elegance of QED make an impression which cannot be darkened even by the obviously algorithmic character of the renormalization procedure. It should be clear that the fundamental length hypothesis is first of all a challenge to contemporary QED. In other words, this hypothesis can survive only if it will lead naturally to modifying QED in a profound way.

The crucial advantage of QED is that the form of the interaction in this theory is dictated by gauge symmetry arguments. It is called the minimal interaction principle and is symbolized by the following substitution law ($\hbar = c = 1$)

$$p_\mu \to p_\mu - e_0 A_\mu(x) \qquad . \qquad (1.3)$$

This substitution leads one to the inhomogeneous Dirac-Maxwell equations for "bare" fields

$$(i\not\partial - e_0 \not A(x) - m_0)\psi(x) = 0 \qquad (1.4a)$$

$$\frac{\partial F^{\mu\nu}(x)}{\partial x^\nu} = e_0 \bar\psi(x)\gamma^\mu \psi(x) \qquad (1.4b)$$

where the field strengths are defined as $F^{\mu\nu}(x) = \frac{\partial A^\mu(x)}{\partial x_\nu} - \frac{\partial A^\nu(x)}{\partial x_\mu}$.

Let us mention, to be complete, that local gauge transformations of the fields $\psi(x)$, $\bar\psi(x)$ and $A(x)$, leaving Eqs. (1.4a)-(1.4b) invariant, are given by*:

$$\psi(x) \to e^{ie_0\lambda(x)}\psi(x) \quad , \quad \bar\psi(x) \to e^{-ie_0\lambda(x)}\bar\psi(x) \qquad (1.5a)$$

$$A_\mu(x) \to A_\mu(x) - \frac{\partial\lambda(x)}{\partial x^\mu} \quad ; \quad \lambda^\dagger(x) = \lambda(x) \qquad . \qquad (1.5b)$$

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*In p-representation the hermiticity condition of λ-function, evidently, becomes

$$\lambda^\dagger(p) = \lambda(-p) \qquad . \qquad (1.6)$$

The rule (1.3) does not contain any scale like ℓ or M and for this reason is universally applied to all space-time intervals and to all values of 4-momenta. Therefore, if one adopts the fundamental length hypothesis it means that the substitution law (1.3) and its consequences are probably invalid or incomplete in the domains

$$|x| \lesssim \ell \tag{1.7a}$$

$$|p| \gtrsim M \quad . \tag{1.7b}$$

Let us consider just one consequence of (1.3). Choosing $e_0 A_\mu = \text{const} = k_\mu$ we obtain obviously, zero field strengths: $F_{\mu\nu}(x) = 0$. But corresponding substitution (1.3) is not yet an identity transformation, namely

$$p_\mu \to p_\mu - k_\mu \quad . \tag{1.8}$$

This is a pseudoeuclidean parallel shift transformation of the 4-dimensional p-space, testifying that a geometry of this space is a Minkowskian one.

So we may conclude that our fundamental length hypothesis challenges the Minkowskian structure of the momentum 4-space in the region (1.7b). But if the momentum 4-space is not everywhere pseudoeuclidean then what is a reasonable alternative? According to a general geometrical classification, the (pseudo)euclidean spaces are those with zero curvature. Their closest neighbors are spaces with non-zero constant curvature. In the present 4-dimensional case, these curved spaces are so-called "de Sitter spaces."

Let us try to impose on 4-momentum space the de Sitter geometry realized on the one-sheeted 5-hyperboloid

$$p_0^2 - p_1^2 - p_2^2 - p_3^2 - M^2 p_4^2 = -M^2 \quad . \tag{1.9}$$

The curvature radius M we identify with the fundamental mass (1.1), assuming that this quantity is large enough (cf. (1.2)). Note that Eq. (1.9) places no constraint on timelike 4-momenta, and it is therefore not in conflict with the construction of Fock space and Poincaré invariance of the S-matrix. Besides (1.9), there exists only one more de Sitter space satisfying the correspondence principle at $M \to \infty$, namely

$$p_0^2 - p_1^2 - p_2^2 - p_3^2 + M^2 p_4^2 = M^2 \quad .$$

But in this geometry we are faced with the universal upper bound for time-like momenta $p_0^2 - \vec{p}^2 \leq M^2$, which is inconsistent with the implementation of a unitary representation of the Poincaré group on Fock space.

Since the mass shell hyperboloids $p^2 = m_1^2$, $p^2 = m_2^2$,... can be equally well embedded into de Sitter p-space (1.9) or in flat Minkowskian p-space free physical particles cannot distinguish between these two geometries. Actually, only virtual (interacting) particles can probe the geometrical structure of 4-momentum space.

In the "flat limit," i.e. in the region of small virtual momenta

$$|p| << M$$

one can neglect the curvature of de Sitter p-space, and therefore the new formalism reduces to the ordinary theory.

For virtual momenta belonging to the region (1.7b), the curvature of de Sitter p-space becomes a crucial factor. It means that the old (Minkowskian) and new (de Sitterian) formalisms should lead to quite different descriptions of particle interactions at small space-time intervals.

A general approach to the construction of quantum field theory on a base of de Sitter p-space has been put forward and investigated in Refs. 19-29.* The concept of local gauge transformations and gauge vector field was transferred to the new geometrical arena in Ref. 30.

2. NEW CONCEPT OF LOCAL GAUGE TRANSFORMATION AND GAUGE VECTOR FIELD

It is clear, independently of arguments connected with p-space geometry, that in a theory based on the fundamental length hypothesis, the notion of a local gauge group should be revised or generalized in some nontrivial manner. However, geometrical or group theoretical arguments allow it to be done in an essentially unique way.

Indeed, one can realize that in de Sitter p-space (1.9), λ-functions parametrizing the gauge transformation in question may be written as follows:

$$\lambda(p_0, \vec{p}, p_4) = \delta(p_0^2 - \vec{p}^2 - M^2 p_4^2 + M^2)\tilde{\lambda}(p_0, \vec{p}, p_4) \tag{2.1}$$

with the hermiticity condition inherited from (1.7)

$$\lambda^\dagger(p, p_4) = \lambda(-p, p_4) \quad . \tag{2.2}$$

The next step is connected with the following observation: if

$$\lambda(p, p_4) = \int e^{-ipx-ip_4\tau} \lambda(x, \tau) d^4x d\tau \tag{2.3}$$

then

$$(\Box - M^2 \frac{\partial^2}{\partial\tau^2} - M^2)\lambda(x, \tau) = 0$$

$$\lambda(x, \tau)^\dagger = \lambda(x, -\tau) \quad . \tag{2.4}$$

So in the new scheme, the gauge functions λ may be treated as local functions of five variables (x^μ, τ), with the obligatory constraints (2.4). The extra space-like variable τ can be

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*The use of such a momentum space in a field theory was pioneered in Refs. 4 and 8. The list of other papers on this subject can be found in Ref. 20. In Ref. 10, the field theory with the momentum space of variable curvature was discussed. We should point out that in all previous attempts to employ a non-euclidean p-space, Poincaré invariance of the theory was not maintained. The concept of nonlocal electromagnetic field based on ideas which were close to a non-euclidean momentum space hypothesis, holding Poincaré invariance, was developed many years ago in Ref. 3.

interpreted, due to its commutativity with the Poincaré group generators, as some internal parameter of the theory. All λ-functions which parametrize the conventional gauge transformations (1.6a)-(1.6b) can be found among functions $\lambda(x, 0)$.

Since the localization of the new gauge group happened to be connected with the configurational 5-space, the relevant gauge vector field, i.e. the electromagnetic potential, has to be a 5-vector. Let us denote it as

$$A_M(x, \tau) = \left(A_\mu(x, \tau), A_4(x, \tau)\right) \quad \mu = 0, 1, 2, 3 \quad . \tag{2.5}$$

The extra component $A_4(x, \tau)$ we call the τ-photon. The neutrality of the electromagnetic field gives rise to the relation

$$A_M(x, -\tau) = \left(A_\mu^\dagger(x, \tau), -A_4^\dagger(x, \tau)\right) \quad . \tag{2.6}$$

The equations of motion for all five components (2.5) are of the form (we put $\hbar=c=\ell=M=1$)[30]

$$\begin{cases} A_\mu(x, \tau) + i \dfrac{\partial A_\mu(x, \tau)}{\partial \tau} - i \dfrac{\partial A_4(x, \tau)}{\partial x^\mu} = 0 \\ A_4(x, \tau) - i \dfrac{\partial A_4(x, \tau)}{\partial \tau} + i \dfrac{\partial A_\nu(x, \tau)}{\partial x_\nu} = 0 \\ \left(\Box - \dfrac{\partial^2}{\partial \tau^2} - 1\right) A_\mu(x, \tau) = 0 \end{cases} \quad . \tag{2.7}$$

It is readily verified that Eqs. (2.7) are invariant under the following gauge transformation:

$$\begin{aligned} A_\mu(x, \tau) &\to A_\mu(x, \tau) - \frac{\partial \lambda(x, \tau)}{\partial x^\mu} \\ A_4(x, \tau) &\to A_4(x, \tau) + i\, \lambda(x, \tau) - \frac{\partial \lambda(x, \tau)}{\partial \tau} \end{aligned} \tag{2.8}$$

where $\lambda(x, \tau)$ satisfies the relations (2.4). Further, due to (2.6), Eqs. (2.7) remain unaltered after the transformation $\tau \to -\tau$ combined with hermitian conjugation. We shall refer to this property as the τ-invariance.

As was shown in Ref. 30 the τ-photon does not correspond to an independent dynamical degree of freedom and can be excluded by an appropriate gauge transformation (2.8). But, similar to scalar and longitudinal photons, it plays the important mediating role in an interaction.[31]

Putting

$$B_M(x, \tau) = e^{-i\tau} A_M(x, \tau) \tag{2.9}$$

one can rewrite (2.8) as follows

$$B_M(x, \tau) \to B_M(x, \tau) - \frac{\partial}{\partial x^M}\left(e^{-i\tau}\lambda(x, \tau)\right) \quad M = 0, 1, 2, 3, 4 \quad . \tag{2.10}$$

The 5-potential $B_M(x, \tau)$ has a clear geometrical meaning in the fiber bundle theory context.*
Introducing the 5-dimensional field strengths

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*A generalization of (2.10) to the non-abelian case is straightforward.[30]

$$F_{MN}(x,\tau) = \frac{\partial B_M(x,\tau)}{\partial x^N} - \frac{\partial B_N(x,\tau)}{\partial x^M} \tag{2.11}$$

we can conclude from this definition and Eqs. (2.7) that

$$F_{\mu 4}(x,\tau) = 0 \quad , \quad \frac{\partial F^{\mu\nu}(x,\tau)}{\partial \tau} = 0 \quad ; \tag{2.12}$$

$$\frac{\partial F^{\mu\nu}(x,0)}{\partial x^\nu} = 0 \quad . \tag{2.13}$$

Eq. (2.13) coincides with the standard Maxwell equation. This prompts that $B_\mu(x, 0)$ can be identified with the ordinary electromagnetic 4-potential $A_\mu(x)$:

$$B_\mu(x,0) \equiv A_\mu(x) \quad . \tag{2.14}$$

A formulation of the free Dirac theory in the 5-dimensional (x, τ)-space is described in Ref. 31. The important feature of that scheme is a use of the 8-component de Sitter spinors

$$\Psi(x,\tau) = \begin{pmatrix} \psi_1(x,\tau) \\ \psi_2(x,\tau) \end{pmatrix} \tag{2.15}$$

where $\psi_a(x, \tau)$ $(a = 1, 2)$ are the conventional 4-component spinors. This is connected with the requirement of the τ-invariance.

3. GENERALIZED PRINCIPLE OF MINIMAL ELECTROMAGNETIC INTERACTION AND NEW FIELD EQUATIONS

The next natural step is to assume that the new theory of electromagnetic interaction has to be invariant under the following group of the local gauge transformations:

$$\Psi(x,\tau) \to e^{ie_o e^{-i\tau}\lambda(x,\tau)}\,\Psi(x,\tau)$$

$$B_M(x,\tau) \to B_M(x,\tau) - \frac{\partial}{\partial x^M}\left(e^{-i\tau}\lambda(x,\tau)\right) \qquad M = 0,1,2,3,4 \tag{3.1}$$

where $\lambda(x, \tau)$ satisfies, as before, the relations (2.4) and e_o is the electric charge. The appropriate generalization of the minimal principle of electromagnetic interaction can be given by the substitution law

$$i\frac{\partial}{\partial x^M} \to i\frac{\partial}{\partial x^M} - e_o B_M(x,\tau) \qquad M = 0,1,2,3,4 \quad . \tag{3.2}$$

In the new approach we obtain the following field equations, substituting the Dirac-Maxwell equations (1.4a)-(1.4b) (the units are $\hbar = c = 1$)[31]

$$(i\partial\!\!\!/ - e_o A\!\!\!/ - m_o)\psi_1(x) = \frac{ie_o \ell\cos\theta_o}{4}\gamma^5\sigma^{\mu\nu}F_{\mu\nu}(x)\psi_1(x) -$$

$$- \frac{e_o\ell\sin\theta_o}{4}\sigma^{\mu\nu}F_{\mu\nu}(x)\psi_1(x) - \frac{e_o^2\ell^2}{8}\left(\bar\psi_1(x)\gamma_\mu\psi_2(x) + \bar\psi_2(x)\gamma_\mu\psi_1(x)\right)\gamma^\mu\psi_1(x) \quad , \tag{3.3a}$$

$$(i\not\partial - e_o\not A - m_o)\psi_2(x) = \frac{ie_o\ell\cos\theta_o}{4}\gamma^5\sigma^{\mu\nu}F_{\mu\nu}(x)\psi_2(x) -$$

$$-\frac{e_o\ell\sin\theta_o}{4}\sigma^{\mu\nu}F_{\mu\nu}(x)\psi_2(x) + \frac{e_o^2\ell^2}{8}(\bar\psi_1(x)\gamma_\mu\psi_2(x) + \bar\psi_2(x)\gamma_\mu\psi_1(x))\gamma^\mu\psi_2(x) \quad , \quad (3.3b)$$

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = e_o\sum_{a=1,2}\Big[\bar\psi_a(x)\gamma^\mu\psi_a(x) - \frac{i\ell}{2}\cos\theta_o\frac{\partial}{\partial x^\nu}(\bar\psi_a(x)\gamma^5\sigma^{\mu\nu}\psi_a(x)) +$$

$$+\frac{\ell}{2}\sin\theta_o\frac{\partial}{\partial x^\nu}(\bar\psi_a(x)\sigma^{\mu\nu}\psi_a(x))\Big] \quad , \quad (3.3c)$$

where $\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$ and

$$\sin\theta_o = \frac{\sqrt{1+m_o^2\ell^2}-1}{m_o\ell} \quad , \quad \cos\theta_o = \sqrt{\frac{2}{1+\sqrt{1+m_o^2\ell^2}}} \quad . \quad (3.4)$$

Eqs. (3.3) contain as the conventional minimal electromagnetic interaction as the non-minimal terms:

i) electric dipole moment (EDM) interaction $\sim \ell\cos\theta_o$;

ii) magnetic dipole moment (MDM) interaction $\sim \ell\sin\theta_o$, called also the Pauli interaction;

iii) τ-photon interaction $\sim e_o^2\ell^2$. Such a name is chosen because this interaction survives even if there are no "ordinary" photons ($A_\mu(x) = 0$) and, therefore, it could be induced only by the τ-photons.

Neglecting the τ-photon interaction in Eqs. (3.3a)-(3.3b), we obtain that the resulting set of equations, together with Eq. (3.3c), becomes invariant under some SU(2)-group operating in (ψ_1, ψ_2)-space. This group will be denoted as $SU_\tau(2)$.

We would like to stress that all interaction terms in Eqs. (3.3),minimal and non-minimal, are originated by the minimal interaction in terms of the 5-potential $B_M(x,\tau)$.

4. INTERPRETATION AND DISCUSSION

We suggest to consider the $SU_\tau(2)$-symmetry, deeply ingrained in our scheme, as a manifestation of the μe-symmetry of the electromagnetic interactions. So, for instance,*

$$\psi_1 = e$$
$$\psi_2 = \mu \quad . \quad (4.1)$$

The τ-photon interaction, violating $SU_\tau(2)$-symmetry, presumably gives rise to the muon-electron mass difference and the processes $\mu \to 3e$, $\mu \to e\gamma$, etc. Analyzing the experimental data, concerning these decays, one obtains the following upper bound for the fundamental length ℓ:

$$\ell \leq 3\cdot 10^{-18}\ \mathrm{cm} \quad . \quad (4.2)$$

Next, let us consider Eqs. (3.3a)-(3.3b) in the Pauli approximation. Assuming that

$$A^o(x) = \phi(r) = \text{arbitrary} \quad , \quad \vec A(x) = \tfrac{1}{2}[\vec H \times \vec r] \quad ; \quad \vec H = \text{const.} \quad , \quad (4.3)$$

and expanding Eqs. (3.3a)-(3.3b) in powers of $1/c$, one finds the following generalized Pauli equation:

$$i\hbar\frac{\partial}{\partial t}\,\phi_a(\vec{r},t) = \left[\frac{\vec{p}^2}{2m_o} - e_o\phi(r) - e_o\frac{(\vec{L}+g\vec{S})\cdot\vec{H}}{2m_oc} + \frac{e_o\cos\theta_o}{Mc}(\vec{S}\cdot\vec{E})\right]\phi_a(\vec{r},t) \quad (a = 1,2) \tag{4.4}$$

where

$$\vec{p} = -i\hbar\frac{\partial}{\partial\vec{r}}\ ;\ \vec{L} = [\vec{r}\times\vec{p}\,]\ ;\ \vec{S} = \frac{\hbar\vec{\sigma}}{2}\ ;$$
$$\vec{E} = -\frac{\partial}{\partial\vec{r}}\,\phi(r)\ ;\ g = 2\sqrt{1+\frac{m_o^2}{M^2}}\quad .$$

The quantity

$$\frac{g-2}{2} = \sqrt{1+\frac{m_o^2}{M^2}} - 1 \tag{4.5}$$

defines the intrinsic anomalous magnetic moment of the Dirac particle in our approach. Writing the EDM-interaction in the canonical form $U_{EDM} = (\vec{d}\cdot\vec{E})$ one obtains the expression for the intrinsic electric dipole moment of the same particle

$$\vec{d} = -e_o\frac{\cos\theta_o}{Mc}\vec{S} = -e_o\ell\frac{\cos\theta_o}{2}\vec{\sigma}\quad . \tag{4.6}$$

Thus, one can conclude:

i) the Dirac particle in this scheme is an extended object from the very beginning;

ii) the theory developed predicts P- and CP-violations in the electromagnetic interaction, since charged particles possess the EDM.

Eq. (4.4) holds in the general case of arbitrary ratio between the particle mass m_o and the fundamental mass M. For leptons, evidently, $m_o/M << 1$ and therefore

$$\left(\frac{g-2}{2}\right)_{\text{lepton}} \simeq \frac{m_o^2}{2M^2} \tag{4.7}$$

$$|\vec{d}|_{\text{lepton}} \approx \frac{e_o\,\ell}{2}\quad . \tag{4.8}$$

Comparing (4.7) with the current theoretical and experimental uncertainties in this quantity we obtain one more upper bound for the fundamental length*:

$$\ell \lesssim 2.6\cdot 10^{-17}\ \text{cm} \tag{4.9}$$

that is not very far from (4.2).

Using (4.2), (4.8) and (4.9) we can conclude that the upper bound for leptonic EDM, in order of magnitude, is at least

$$|\vec{d}|_{\text{lepton}} \lesssim e_o\,(10^{-17} \div 10^{-18})\ \text{cm} \tag{4.10}$$

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*To our knowledge, it is the first time a highly precise experiment, designed for a test of a validity of QED, is used to estimate the fundamental length.

This is consistent with the old experimental data on a direct measurement of the electron and muon EDM,[32] with observed shifts of atomic levels,[32] with parity violation effects in atoms,[33] with the recent search for the parity violation in the polarized electron scattering.[34]

On the other hand, a number of experiments were performed on indirect estimation of the electron EDM through the measurements of EDM that it induces in atoms. From the result obtained[35–38] and (4.8) one might conclude that

$$\ell \lesssim 10^{-24} \div 10^{-23} \text{ cm} \quad . \tag{4.11}$$

Let us emphasize once more that the existence of non-zero EDM for elementary particles should lead to the violation of the CP-symmetry.[39] Thus, according to our approach, the mechanism of the CP-violation may be purely electromagnetic if the theory of the electromagnetic interactions is based on the fundamental length hypothesis.

It does not need any comment that the experimental discovery of the particle EDM would be of great importance for the present theory. Concerning leptons, which have not undergone strong interactions, one should realize that, due to (4.8), the measurement of their EDM is the straight measurement of the fundamental length.

Coming back to the particle MDM we can rewrite this quantity as follows

$$\vec{\mu} = \frac{e_0 g}{2m_0 c}\vec{S} = \frac{e_0}{Mc}\sqrt{\frac{m^2 + M^2}{m^2}}\,\vec{S} \quad . \tag{4.12}$$

So $e_0\hbar/2MC = e_0\ell/2$ plays the role of a minimal magneton attainable only when $m_0 \gg M$. Hence, superheavy Dirac particles, if such objects somewhere exist,* should serve not only as sources of the static Coulomb field but also as sources of the static magnetic field, produced by the magnetic dipole moment $(e_0\hbar\vec{\sigma})/(2Mc)$.

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*Cf. the "maximon" considered by Markov.[52]

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