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Questions of Quark Confinement and Ambiguities in Coulomb Gauge of Yang-Mills Fields

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ABSTRACT

We discuss the ambiguities considered by V. N. Gribov in the formulation of Coulomb gauge in non-Abelian gauge theories. We review the division of gauge field space into a sector with a unique transverse gauge, a sector with a two fold ambiguity in transverse gauge, etc. We argue in a semi-classical fashion that transitions between these sectors readily occur and discuss the connection with ideas of quark confinement in Coulomb gauge. Because of these transitions it appears that the functional integral formulation of Coulomb gauge will be rather more complicated than expected in the past.

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I. INTRODUCTION

The formulation of non-Abelian gauge theories in Coulomb gauge was first carried out many years ago by Schwinger.[1] He subsequently showed that the canonical commutation relations at equal times lead to a Lorentz covariant description of the theory. On that foundation the Coulomb gauge has proved now[2] and then[3] to be an appealing physical framework into which to cast various questions about Yang-Mills fields because it appears to have two degrees of freedom for the field for each gauge group and space-time label.

In a lecture at the 12th Winter School of the Leningrad Nuclear Physics Institute, Gribov[4] has called attention to the fact that in non-Abelian gauge theories there is a non-trivial residual gauge freedom in, at least, Coulomb and Landau gauges. This means that in such gauges, the connection between physical states and gauge field configuration is not unique. Gribov suggested a mechanism[5] whereby this ambiguity might be connected with the confinement of color in Yang-Mills theories via the Coulomb energy term in the Hamiltonian.

We propose in this note to first review and slightly enlarge on Gribov's arguments. Then we will examine the question of "large fields" and the connection with color confinement. In the latter discussion we will consider different sectors of the eigenvalue space of the operator

$$D_j(A)_{ab}\nabla_j = \nabla^2\delta_{ab} + f_{acb}A_j^c(\vec{x})\nabla_j \tag{1}$$

whose determinant gives the usual Faddeev-Papov ghost term[6] in Coulomb gauge. This same operator enters the Coulomb energy in the gauge field Hamiltonian. We will demonstrate, in a semi-classical argument, that the zero eigenvalues of $D_j \nabla_j$, which give infinite values to the Coulomb energy, do not present an impenetrable barrier to the gauge field and are unlikely, we argue, to have a connection with color confinement.

II. RESUMÉ OF THE COULOMB GAUGE

The field equations for the Yang-Mills field $A_\mu^a(x)$ (where a is a group index running $a = 1, \ldots N^2 - 1$ for SU(N) which we shall have in mind here) derived from the familiar Lagrangian

$$\mathscr{L} = -\frac{1}{4g^2} F_{\mu\nu}^{\ \ a} F_a^{\ \mu\nu} \quad , \tag{2}$$

$$F_{\mu\nu}^{\ \ a} = \partial_\mu A_\nu^{\ a} - \partial_\nu A_\mu^{\ a} + f_{abc} A_\mu^{\ b} A_\nu^{\ c} \quad , \tag{3}$$

do not determine all components of A_μ^a. If we impose the Coulomb gauge condition

$$\nabla_j A_j^{\ a} = 0 \quad , \tag{4}$$

and eliminate the dependent time component $A_0^{\ a}$, the Hamiltonian density for the field becomes

$$\mathscr{H} = \frac{1}{2} E_a^{\ j} E_a^{\ j} + \frac{1}{2} B_a^{\ j} B_a^{\ j} - \rho_a \left(\frac{1}{D_j \nabla_j} \right)_{ab} \nabla^2 \left(\frac{1}{D_j \nabla_j} \right)_{bc} \rho_c \quad , \tag{5}$$

where

$$B_a^{\ j} = \frac{1}{2} \epsilon_{jki} F_{ki}^{\ a} \quad , \tag{6}$$

$$D_j(A)_{ab} = \delta_{ab} \nabla_j + f_{acb} A_j^{\ c} \quad , \tag{7}$$

$$\rho_a = + f_{abc} A_j^{\ b} E_j^{\ c} \quad , \tag{8}$$

and $E_j^{\ a}$ is the transverse part of the electric field $F_a^{\ j0}$

$$\nabla_j E_j^{\ a} = 0 \quad . \tag{9}$$

All this is quite familiar from References 1 and 6.

Now in Reference 4 Gribov poses the question whether there may be another vector potential $A'^{\ a}_j$ which is connected to our original $A_j^{\ a}$ by a gauge transformation g and still satisfies $\nabla_j A_j^{\ a'} = 0$. Writing the vector potential in matrix form

$$A_j = - i \sum_a T^a A_j^{\ a} \tag{10}$$

with

$$[T^a, T^b] = i f^{abc} T^c \quad , \tag{11}$$

this is then the question of the existence of non-trivial g in the gauge group so that

$$A'_j = g^{-1}A_j g + g^{-1}\nabla_j g \tag{12}$$

and

$$\nabla_j A_j' + \nabla_j A_j = 0 \quad . \tag{13}$$

This is satisfied for all g such that

$$D_j(A)\left((\nabla_j g)g^{-1}\right) = 0 \quad , \tag{14}$$

where $D_j(A)\phi = \nabla_j\phi + [A_j, \phi]$ in matrix language.

If we were dealing with an abelian theory, we could of course ask the same question, namely whether with any non-trivial $g = e^{-i\omega(x)}$ two transverse fields could be connected. If it were so, then

$$\nabla^2\omega = 0 \tag{15}$$

must be satisfied. Usually we allow only $\omega = 0$ as a solution for this equation by an appropriate choice of boundary conditions. In any case any permissible ω is independent of the vector potential A and leads to physically uninteresting consequences.

Another way of putting this last point is to examine the generator of gauge transformations in the abelian theory

$$G_\Lambda = \int d^3x E_j \nabla_j \Lambda(x) \quad . \tag{16}$$

For E_j which fall off fast enough, we can perform a partial integration and formally set $G_\Lambda = 0$ since $\nabla_j E_j = 0$. That is in Coulomb gauge we have no operator with which to perform a gauge transformation, so the gauge specification is complete.

In the non-Abelian theory we have a more complicated nonlinear equation to solve. Gribov has shown for an SU(2) gauge theory that even when $A_j = 0$, the ambiguity equation (14)

$$\nabla_j(\nabla_j g\, g^{-1}) = 0 \qquad , \tag{17}$$

has non-trivial solutions which for large $|\vec{x}|$ give

$$(\nabla_j g)g^{-1} \underset{|\vec{x}| \to \infty}{\sim} i(\vec{x} \times \vec{\tau})_j / |\vec{x}|^2 \qquad . \tag{18}$$

The τ_j are Pauli matrices. This has been verified and discussed by Jackiw, Muzinich and Rebbi.[7] Furthermore even for $A_j = 0$, there is a four fold infinity of solutions depending on a "location" $\vec{x}_0$ and a scale. The case $A_j \neq 0$ has been examined by Gribov using the ansatz

$$A_j^a = \epsilon_{jak} \frac{x_k}{|\vec{x}^2|} f(|\vec{x}^2|) \qquad , \tag{19}$$

and he again concludes that non-trivial g's exist which move one around in the space of transverse A_j's.

The generator of gauge transformations in the non-Abelian theory

$$G_\Lambda = \int d^3x\, E_j^{\,a} D_j(A)_{ab} \Lambda^b(x) \tag{20}$$

$$= \int d^3x (E_j^{\,a} \nabla_j \Lambda^a(x) + f_{acb} E_j^{\,a} A_j^{\,c} \Lambda^b) \quad , \tag{21}$$

is non-zero even when we may integrate by parts on the first term and use $\nabla_j E_j^{\,a} = 0$. So the origin of the gauge ambiguity lies in the fact that the gauge bosons carry color, charge, and in a non-Abelian theory we can hope to implement gauge transformations within the space of transverse fields.

The connection between these observations on the ambiguity in specifying the gauge when $\nabla_j A_j = 0$ and the Coulomb energy in (5) goes as follows: we imagine that for various sectors of A_j space the ambiguity equation (14) has $N(A) = 1, 2, \ldots$ solutions. What divides $N(A) = 1, 2$, etc.? To establish this we note that in the sector $N(A) = 1$ the unique solution is $g_1 = \underset{\sim}{1}$ (the unit matrix is always a solution). Inside the sector $N(A) = 2$, we will have the solutions $g_1 = \underset{\sim}{1}$ and $g_2 \neq \underset{\sim}{1}$. We suppose, however, that at the <u>boundary</u> between the $N(A) = 1$ and $N(A) = 2$ sectors $g_2 \to \underset{\sim}{1}$. So very close to the boundary

$$g_2 \approx \underset{\sim}{1} + \phi_2 \quad , \tag{22}$$

and the ambiguity equation becomes

$$D_j(A) \nabla_j \phi_2 = 0 \quad . \tag{23}$$

When this equation has a non-zero solution for ϕ_2 we are "just inside" the sector N(A) = 2. Just at the boundary then, the operator $D_j(A)\nabla_j$ develops a zero eigenvalue and the coulomb energy

$$\mathscr{H}_{coul} = \rho \frac{1}{D\cdot\nabla} \nabla^2 \frac{1}{D\cdot\nabla} \rho \tag{24}$$

becomes infinite. Similarly, at the boundary between N(A) = 2 and N(A) = 3 we might suppose that a solution g_3 to the non-linear ambiguity equation develops out of the unit matrix $g_3 \approx \underset{\sim}{1} + \phi_3$ and, of course, $D_j(A)\nabla_j \phi_3 = 0$. So the zero eigenvalues of the operator $D_j(A)\nabla_j$ seem to set the boundaries between sectors between solutions of the gauge ambiguity equation (14).

Furthermore, it is just at these boundaries that $\mathscr{H}_{coul}$ becomes unbounded. So one might argue, a la Gribov,[5] that this provides an origin for color confinement. Imagine two color charges close together and then moving apart. When they are close, the effective interaction is small because of asymptotic freedom and the gauge potentials are small, so

$$D_j(A)\nabla_j \approx \nabla^2 \tag{25}$$

and $\mathscr{H}_{coul}$ is "normal" and harmless. When the charges move apart the effective interaction grows until $D_j(A)\nabla_j$ develops a zero eigenvalue and the Coulomb energy absorbs the kinetic energy of the charges and they stop. By examining the operator $D_j(A)\ _j$ for potentials like (19), Bender, Eguchi and Pagels[8] have argued that indeed

$$H_{coul} = \int d^3x\, \mathscr{H}_{coul} \tag{26}$$

grows (as $|\vec{x}|^3$) as the charges separate. In the next section we will conclude that this does not demonstrate confinement because by choosing other field configurations the vector potential can easily pass, at finite energy, through the barrier raised in $\mathscr{H}_{coul}$ when $D_j(A)\nabla_j$ has a zero eigenvalue.

Before that we wish to discuss one more matter connected with the ambiguity equation. In the usual Faddeev-Popov prescription for the functional integral formulation of the non-Abelian gauge theory,[6] one introduces the gauge condition $\nabla_j A_j = 0$ into the integral over all field configurations A_j by inserting

$$1 = \Delta(A) \int \prod_x dg(x) \prod_x \delta\left[\nabla_j (g^{-1} A_j g + g^{-1} \nabla_j g)\right] \tag{27}$$

into the integral. Then using the gauge invariance of the Yang-Mills action and of $\Delta(A)$, one changes variables from A_j to $A_j' = g^{-1} A_j g + g^{-1} \nabla_j g$ and extracts an infinite factor $\pi_x \int dg(x)$ from the integral. The procedure to follow is still the same in the presence of ambiguities, but the determination of $\Delta(A)$ is remarkably more conplicated.[9] We need to determine all g_n $n = 1, 2, \ldots N(A)$ for which the argument of the δ

functional in (27) vanishes. These g_n are determined precisely by the gauge ambiguity equation (14). Then $\Delta(A)$ is given by

$$\Delta(A)^{-1} = \sum_{n=1}^{N(A)} \left(\det M^{(n)}\right)^{-1} \tag{28}$$

where

$$M^{(n)}_{ab} = \nabla^2 \delta_{ab} + f_{acb} A_j^{(n)c} \nabla_j \quad , \tag{29}$$

with

$$A_j^{(n)} = g_n^{-1} A_j g_n + g_n^{-1} \nabla_j g_n \quad , \tag{30}$$

and as usual the determinant is in group indices as well as space-time. If the unique solution to the ambiguity equation were $g_1 = \underset{\sim}{1}$, then

$$\Delta(A) = \det (\nabla^2 \delta_{ba} + f_{acb} A_j^c \nabla_j) \quad ,$$

which is the familiar Faddeev-Popov result. In general, however, we expect $N(A) \neq 1$ and $g_n \neq \underset{\sim}{1}$ which rather severely complicates the functional integral formulation of Coulomb gauge. Recall that g_n satisfies (14) and is a functional of A_j.

Much of this apparent additional structure would disappear if the "barrier" established by $\mathscr{H}_{coul}$ when $D_j(A)\nabla_j$ has a zero eigenvalue were not penetrable. Then we would naturally assume ourselves to be in the N(A) = 1 sector where $g_1 = \underset{\sim}{1}$ and know we would never leave it. The next section concludes that to be untrue, so the complications appear to persist.

III. TRANSITIONS THROUGH THE COULOMB BARRIER

We want to consider now the possibility that if we prepare the gauge field $A_j^{(1)}(\vec{x})$ at some time t = 0 in the sector N(A) = 1, say, then at some later time T, it will have tunneled through the zero eigenvalue of $D_j(A)\nabla_j$, which causes $\mathscr{H}_{coul}$ to be unbounded, to another field configuration $A_j^{(2)}(\vec{x})$ which lies in the N(A) = 2 sector. That is, we want to know if the transformation function

$$<A_j^{(2)}(\vec{x}),\ T\,|\,A_j^{(1)}(\vec{x}),\ 0>$$

is non-zero. Formally this transition amplitude has the expression

$$\int dA_j^{\,a}(x)\int dE_j^{\,a}(x)\,\delta(\nabla_j A_j^{\,a})\,\delta(\nabla_j E_j^{\,a})\ \times$$

$$\times\ \exp i\int_0^T dt\int d^3x\left[E_j^{\,a}A_j^{\,a}-\mathscr{H}(A_j^{\,a},\ E_j^{\,a})\right]\quad, \tag{32}$$

with $A(\vec{x},\ 0) = A^{(1)}$ and $A(\vec{x},\ T) = A^{(2)}$. We are unable to evaluate this integral exactly, so we will proceed with the following strategy.[10] Replace the time dependence of $A_j^{\,a}(\vec{x},\ t)$ by a function $\kappa(t)$ which at time t = 0 indicates we are in sector N(A) = 1 and at t = T tells us we are in $N(A) = 2$. $\{\kappa(t)$ will turn out to be related to an eigenvalue of an integral operator connected to $D_j(A)\nabla_j$.$\}$ Replace the integral over fields $A_j^a(\vec{x}, t)$ by the path integral over $\kappa(t)$. Evaluate this integral in semi-classical or WKB approximation using the stationary phase approximation to the one-dimensional quantum problem in κ-space. Effectively we are noting that the transition of interest takes place in the parameter space and the other (infinite) degrees of freedom play an inessential role.

To begin, let us look at the condition that the operator $D_j\nabla_j$ have a zero eigenvalue

$$(\nabla^2\delta_{ab} + f_{acb}A_j^c\nabla_j)\psi_b(\vec{x}) = 0 \quad . \tag{33}$$

Going over to momentum space we have the eigenvalue equation

$$X_a^{(n)}(\vec{\ell}) = \lambda_n^{-1}\int d^3pK_{ab}(\vec{\ell}, \vec{p})X_b^{(n)}(\vec{p}) \quad , \tag{34}$$

where

$$K_{ab}(\vec{\ell}, \vec{p}) = \frac{if_{acb}p_jA_j^c(\vec{\ell} - \vec{p})}{(2\pi)^3|\vec{p}|^2} \quad , \tag{35}$$

and (33) corresponds to $\lambda_n = 1$.

In terms of the eigenvalues λ_n of the kernel K we can state the determination of the sectors $N(A) = 1, 2, \ldots$ as follows. In $N(A) = 1$

all $\lambda_n^{-1} > 1$. As we cross over the boundary from $N(A) = 1$ to $N(A) = 2$, λ_1 passes through 1. As we cross from $N(A) = 2$ into $N(A) = 3$, λ_2 passes through 1, etc.

We also have here a more precise statement of what a "large gauge field" means: when the norm of the kernel is less than 1, all $\lambda_n^{-1} > 1$. When this norm increases the λ_n one by one march through one taking us from sector to sector.

Now we imagine parametrizing the vector potential by $\kappa(t)$

$$A_j^a(\vec{x}, t) = A_j^{av}(\vec{x}, \kappa(t)) \quad . \tag{36}$$

The eigenvalue of K as a function of time is

$$\lambda_n(t) = \frac{\int d^3q d^3p \overline{X}_a^{(n)}(\vec{q}) K_{ab}(\vec{q}, \vec{p}) X_b(\vec{p})}{\int d^3p \overline{X}_a^{(n)}(\vec{p}) X_a^{(n)}(\vec{p})} \quad , \tag{37}$$

showing that $\dot{\lambda}_n(t)$ is proportional to $\dot{\kappa}(t)$. As κ proceeds from $\kappa(0)$ to $\kappa(T)$, $\lambda_1(t)$ should go from $\lambda_1(0) < 1$ to $\lambda_1(T) > 1$. Somewhere in that interval, say at $\kappa(t_s) = \kappa_s$ the eigenvalue becomes equal to 1 and the Coulomb energy has a $(\kappa - \kappa_s)^{-2}$ singularity.

Return now to the functional integral (32) for the transition amplitude of interest. Using

$$\delta(x) = \lim_{\alpha \to 0} \frac{1}{\sqrt{\alpha}} e^{-\frac{i}{2\alpha} x^2}$$

to place $\delta(\nabla_j A_j^a)$ and $\delta(\nabla_j E_j^a)$ in the exponential, we can perform the gaussian E_j^a integration to find

$$
\begin{aligned}
< A^{(2)}(\vec{x}),\ T \,|\, A^{(1)}(\vec{x}),\ 0> \;=\; & \lim_{\alpha\to 0}\lim_{\beta\to 0} \int dA_j^a \times \\
& \exp\Bigg\{ i \int_0^T dt \Bigg[\frac{1}{2\beta} \int d^3x A_j^a(\vec{x})\, \nabla_j \nabla_k A_k^a(\vec{x}) \\
& + \frac{1}{4} \int d^3x d^3y \dot{A}_j^a(\vec{x},\ t) \left(N_{jk}^{ab}(\vec{x},\ \vec{y}) \right)^{-1} \dot{A}_k^b(\vec{y},\ t) \\
& + \frac{i}{2} \int d^3x \operatorname{tr} \log <x\,|N\,|x> - \frac{1}{2} \int d^3x B_j^a B_j^a \Bigg] \Bigg\} \quad ,
\end{aligned}
\tag{38}
$$

with

$$
N_{jk}^{ab}(\vec{x},\vec{y}) = \frac{\delta_{ab}}{2}\left(\delta_{jk} + \frac{\nabla_j \nabla_k}{\alpha}\right)\delta^3(\vec{x}-\vec{y}) + f_{acd} A_j^d(\vec{x},t) \frac{1}{\Delta_{ce}} A_k^h(\vec{y},t) f_{heb} \quad , \tag{39}
$$

and

$$
\frac{1}{\Delta} = \frac{1}{D_j \nabla_j} \nabla^2 \frac{1}{D_j \nabla_j} \quad . \tag{40}
$$

Using the parametrization in terms of $\kappa(t)$ and truncating the functional integral by exhibiting only $\int d\kappa(t)$ we write the transition amplitude as[10]

$$\int_{\kappa(0)}^{\kappa(t)} d\kappa(t) e^{i\int_0^T \left[\frac{\dot{\kappa}(t)^2}{2} m(\kappa) - V(\kappa)\right] dt} \quad , \tag{41}$$

where $m(\kappa)$ vanishes as $(\kappa - \kappa_s)^2$ near κ_s because of the N^{-1} in (38), while $V(\kappa)$ is at most logarithmically singular there.

The semi-classical approximation follows the variable along the classical path which is given by the Euler-Lagrange equation for

$$L(\dot{\kappa}, \kappa) = \frac{\dot{\kappa}(t)^2}{2} m(\kappa) - V(\kappa) \quad . \tag{42}$$

These equations have the constant of motion (energy)

$$E = \frac{\dot{\kappa}(t)^2 m(\kappa)}{2} + V(\kappa) \tag{43}$$

and the WKB approximation to the transition amplitude in question is essentially

$$\exp i \int_{\kappa(0)}^{\kappa(t)} d\kappa \sqrt{m(\kappa)} \left[\sqrt{2(E - v(\kappa))} - \frac{E}{\sqrt{2(E - V(\kappa))}}\right] \quad , \tag{44}$$

where we are to take $\sqrt{\ }$ into $\pm i\sqrt{\ }$ "under the barrier" ($V(\kappa) > E$) to have a damped exponential as usual. The transition rate (44) is finite even as we integrate through κ_s where λ_1 (or λ_n in general) goes through unity. If we include quark sources as well, then, in essence, the only change is that the quantity $V(\kappa)$ above develops a term which is singular as $(\kappa - \kappa_s)^{-2}$. This comes from a term $J^0 \Delta^{-1} J^0$ in the

Hamiltonian where J^0 is the color charge density of the quarks. With this addition the integral in (44) is still quite finite as we go through κ_s.

What's happening here is that the <u>potential is momentum dependent</u> and at the point κ_s where the potential becomes infinite the momentum is integrable. Consider, for instance, the example (which is really just the Hamiltonian for Equation 42)

$$H(p, \kappa) = \frac{p^2}{2} W(\kappa) + V(\kappa) \quad , \tag{45}$$

where both $W(\kappa)$ and $V(\kappa)$ are singular in the same way at the same point κ_s. This is a mock up of the case of gauge fields plus quarks. For a transition at fixed energy E the momentum is

$$p = \sqrt{2(E - V(\kappa))/W(\kappa)} \tag{46}$$

and the essence of the WKB transition amplitude is

$$\exp - \int_{\kappa_s - \epsilon}^{\kappa_s + \epsilon} d\kappa \sqrt{\frac{2(V(\kappa) - E)}{W(\kappa)}} \quad , \tag{47}$$

which is finite.

This exercise tells us that transitions across the Coulomb barrier are easy even though the barrier is infinite. The essential difference between this situation and that where we have an ordinary potential problem

$$H = \frac{p^2}{2} + V(\kappa) \tag{48}$$

with $V(\kappa) = (\kappa - \kappa_s)^{-2}$ which <u>is</u> a barrier that cannot be crossed, is that the potential for the Yang-Mills coulomb interaction is momentum dependent. Clearly this is a quantum mechanical effect. Classically the field is "stopped" by the Coulomb barrier. In quantum theory when the field $A_j^a(\vec{x}, t)$ gets "big enough" so the kernel (35) develops an eigenvalue unity (equivalently, $D_j \nabla_j$ has a zero eigenvalue), it sails right through the barrier. Our discussion of separating color charges must, therefore, be significantly modified since there appears to be no impenetrable wall to stop the quantum system from tunneling right through.

IV. CONCLUSIONS

This paper has primarily aimed at clarifying and perhaps making more precise various points raised in Reference 4. The ambiguities in Coulomb gauge for non-Abelian theories are connected with the possibility of multiple solutions of the ambiguity equation

$$D_j(A)(\nabla_j g_n g_n^{-1}) = 0 \quad n = 1, 2, \ldots N(A) \quad . \tag{49}$$

Indeed, Gribov showed that even for $A_j = 0$, $N(A = 0)$ is infinite. A heuristic argument indicates that $N(A)$ may be infinite for any A_j. Namely, if one constructs $V_j(\vec{x})$ so it is traceless and anti-hermitean, then

$$g_n(\vec{x}) = \left\{ P \exp \int_0^{\vec{x}} d\vec{y} \cdot \vec{V}(\vec{y}) \right\} g_n(0) \tag{50}$$

where P indicates path ordering, is an acceptable element of the gauge group. $V_j(\vec{x}) = \nabla_j g_n g_n^{-1}$ can be made co-variant divergence free by taking any operator $\mathcal{O}_j$ and any vector W_j and writing

$$V_j = W_j - \mathcal{O}_j \frac{1}{D_k \mathcal{O}_k} D_i W_j \quad . \tag{51}$$

Natural choices for $\mathcal{O}_j$ are ∇_j, D_j, and even a transverse potential B_j. This large number of solutions seriously complicates the functional integral formulation of non-Abelian theories in Coulomb (and Landau) gauges, since for each solution g_n of (49) the Faddeev-Papov determinant receives a contribution. These contributions are in general unequal. Since it is only the sum which is gauge invariant, all terms must be retained. This seems to be a strong hint that the use of Coulomb or Landau gauges in non-Abelian theories may be, in any calculational scheme, restricted to the non-covariant appearing Hamiltonian form.

We have also examined the question of the nature of barrier that the vanishing eigenvalues of the operator $D_j(A)\nabla_j$ appear to raise in the Yang-Mills Coulomb energy

$$H_{coul} = \int d^3x \, \rho \frac{1}{D \cdot \nabla} \nabla^2 \frac{1}{D \cdot \nabla} \rho \quad . \tag{52}$$

If this barrier could not be penetrated by a field configuration at finite energy, we might then have the qualitative picture of confinement outlined in Section II. However, because the potential in the Yang-Mills theory is momentum dependent, the field is not stopped at the singularity in H_{coul}, but in semi-classical approximation sails right through. It appears doubtful then, that the infinities of H_{coul} are connected with the confinement of color. The work of Reference 8 is then to be interpreted as a particular set of field configurations which do not permit color charges to separate indefinitely. However, the field may quite happily choose to avoid such configurations and take a path through H_{coul} which is much milder.

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