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Counting Narrow Levels of Quarkonium

C. QUIGG*
Fermi National Accelerator Laboratory†
P.O. Box 500, Batavia, Illinois 60510

AND

JONATHAN L. ROSNER‡
School of Physics and Astronomy
University of Minnesota, Minneapolis, Minnesota 55455

ABSTRACT

We show rather generally that the number of $^3S_1$ levels of a $(Q\bar{Q})$ system which lie below the threshold for Zweig-allowed decay grows as $a\cdot(m_Q/m_c)^{1/2}$, where m_Q and m_c are the masses of the heavy quark Q and of the charmed quark. Reference to the charmonium system and consideration of specific potential models both indicate a coefficient $a \simeq 2$, implying that for the Υ family ($m_Q \simeq 5$ GeV) three or four quasistable $^3S_1$ levels should lie below threshold.

*Alfred P. Sloan Foundation Fellow; also at Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637.

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Following the discovery of ψ, ψ', and other levels of charmonium,[1] there has been renewed interest in describing the interactions of heavy quarks by means of nonrelativistic potentials. On the basis of a specific potential (and also on heuristic grounds), Eichten and Gottfried[2] argued that for quarks considerably more massive than the charmed quark, the $Q\bar{Q}$ system would have three or more $^3S_1$ states below the threshold for Zweig-allowed decays, rather than the two such states (ψ and ψ') of the charmonium system. This conclusion implied the exciting possibility of extremely complex quarkonium spectra accessible in the study of e^+e^- annihilations into hadrons, should more than four quarks exist in nature.

It is likely that the discovery[3] in the reaction $p+p \to \mu^+\mu^-$ + anything of $\Upsilon(9.4\ \mathrm{GeV}/c^2)$ and of additional structures $\Upsilon'(10.0)$, $\Upsilon''(10.4??)$ at higher mass[4] signals the existence of a new quark Q with mass $m_Q = 5\ \mathrm{GeV}/c^2$. It is natural, therefore, to be optimistic about the rich spectroscopy awaiting the next generation of electron-positron storage rings. However, measured spacing of Υ' and Υ (interpreted as $1\,^3S_1$ and $2\,^3S_1$ $Q\bar{Q}$ levels) is much larger than predicted by the specific potential used in Ref. 2. This has led us to reconsider in more general terms the conclusion of Eichten and Gottfried which is of immediate importance for the interpretation of the upsilon spectrum, as well as for storage-ring futurism. We find a remarkable general result: the number of $^3S_1$ $Q\bar{Q}$ states which lie below the threshold for Zweig-allowed decays (into $Q\bar{q} + \bar{Q}q$ pairs, where q is a light quark) is

$$n \sim a(m_Q/m_c)^{\frac{1}{2}} \quad . \tag{1}$$

The coefficient a appears to be close to 2. This last deduction follows from the $c\bar{c}$ system, in which the second level ψ' lies just below charm threshold. It follows as well in specific potential models, notably that of Ref. 2 and the logarithmic potential shown by us[5] to describe features of both the ψ and Υ families. Taking $m_Q = 5$ GeV/c^2 and $m_c = 1.5$ GeV/c^2, we are therefore led to expect three or four narrow $^3S_1$ Υ states to be prominent in e^+e^- annihilations.

The key to the result (1) is the observation[2,6] that although the dynamics of the $Q\bar{q}$ system cannot be expected to yield to a nonrelativistic approach, the dependence upon m_Q of the lowest (1^1S_0) mass becomes simple as m_Q becomes large. The $(Q\bar{q})$ mass depends upon m_Q in three ways: the additive contribution of m_Q; the hyperfine $(^3S_1 - {}^1S_0)$ splitting which decreases monotonically ($\sim m_Q^{-1}$ in specific models); and the binding which has a feeble dependence through reduced mass effects. Thus the quantity

$$\delta(m_Q) \equiv 2m(\text{lowest } Q\bar{q} \text{ state}) - 2m_Q \tag{2}$$

is expected to approach a finite limit δ_∞ as $m_Q \to \infty$. Furthermore, as we shall show below, it is likely that $\delta(m_c)/\delta_\infty \approx 1$.

Let us now consider the $Q\bar{Q}$ system. It will be convenient to set the zero of energy at $2m_Q$. The threshold for decay of a $Q\bar{Q}$ state into $Q\bar{q} + \bar{Q}q$ is then $\delta(m_Q)$ above the zero point. Let $V(r)$ be any potential which binds $Q\bar{Q}$ states rising at least $\delta(m_Q)$ above $2m_Q$. Any infinitely-rising confining potential satisfies this condition. Then the number n of bound states lying below $\delta(m_Q)$ is given in the semiclassical approximation by

$$\int_0^{r_0} dr\,[m_Q(\delta(m_Q) - V(r))]^{\frac{1}{2}} = (n - \tfrac{1}{4})\pi \quad , \tag{3}$$

where r_0 is the point at which $\delta(m_Q) = V(r_0)$, which <u>becomes independent of m_Q</u> as $m_Q \to \infty$. Dividing both sides of (3) by $m_Q^{\frac{1}{2}}$, we obtain the result (1).

The result (1) is achieved in different ways for potentials of different shapes. For potentials $V \sim r^{\varepsilon}$ the level spacing behaves as $m^{-\varepsilon/(2+\varepsilon)}$. Thus the increase of $n(m_Q)$ as m_Q increases is achieved by packing levels more closely for $\varepsilon > 0$, and pushing them deeper into the well for $\varepsilon < 0$.

Evaluation of the coefficient a in eq. (1) is straightforward in any specific model. An interesting example is provided by the logarithmic potential[5]

$$V(r) = C \ln(r/r_0) \tag{4}$$

which gives rise to $Q\bar{Q}$ level spacings which are strictly independent of m_Q. This is motivated by the apparent equality[4]

$$m(T') - m(T) = m(\psi') - m(\psi) \simeq 0.6\ \mathrm{GeV}/c^2 \quad , \tag{5}$$

which is reproduced with C = 3/4 GeV. At first we ignore variation of $\delta(m_Q)$. An elementary scaling argument[5,6] shows that the energy eigenvalues

$$E_n(m_Q) = E_n(m_c) - \frac{C}{2} \ln(m_Q/m_c) \tag{6}$$

have a simple dependence upon the quark mass. As the quark mass increases, all the energy levels fall deeper into the potential well. Consequently if $\delta(m_Q)$ is fixed, more $^3S_1$ levels appear below the new flavor threshold as m_Q is increased. This behavior is illustrated by the straight line in Fig. 1. Although energies are there referred to the 1^3S_1 (ground-state) energy, the physics is that the ground-state energy is depressed as m_Q increases.

How many new levels appear? From the psion spectrum and eq. (6) we have

$$\delta(m_Q) - E(1^3S_1) = \delta(m_c) - E(1^3S_1) = 2M_D - m_\psi + \frac{C}{2}\ln\left(\frac{m_Q}{m_c}\right) = 0.635\ \mathrm{GeV} + \frac{C}{2}\ln\left(\frac{m_Q}{m_c}\right) \tag{7}$$

while, to good approximation[5]

$$E_n - E(1^3S_1) = C\ \ln\left(\frac{4n-1}{3}\right) \ . \tag{8}$$

We may determine the number of levels below threshold $n(m_Q)$ by equating (7) and (8). The result is

$$n(m_Q) = \frac{1+3e^{(0.635\ \mathrm{GeV})/C}}{4}\left(\frac{m_Q}{m_c}\right)^{\frac{1}{2}} = 2.0\left(\frac{m_Q}{m_c}\right)^{\frac{1}{2}} \quad , \tag{9}$$

which coincides with our estimate made above without reference to a specific potential.

Finally we may estimate the variation of $\delta(m_Q)$ with the heavy quark mass. Although this cannot be done with great reliability, our estimates are comfortingly small. The magnitude of the hyperfine splittings is determined by the charmed mesons, for which $m_{D*} - m_D = 144$ MeV. We assume[2,7] that $Q\bar{q}$ hyperfine splittings are described by QCD and thus are inversely proportional to m_Q. A $^1S_0$ state may therefore be expected to lie a distance

$$\tfrac{1}{2}\delta(m_Q)_{hfs} = \frac{m_c}{m_Q}\cdot\frac{3}{4}(m_{D*} - m_D) = 108\ \mathrm{MeV}\left(\frac{m_c}{m_Q}\right) \tag{10}$$

below the $(^1S_0, {}^3S_1)$ center of gravity. For the highly-excited $Q\bar{Q}$ states we shall neglect hyperfine effects. They should not be larger than a few tens of MeV, and their inclusion would tend to raise slightly our estimate of $n(m_Q)$. We incorporate the effect of the reduced mass

$$\mu(m_Q) = \frac{m_Q m_q}{m_Q+m_q} \tag{11}$$

upon the binding by replacing m_Q/m_c in (6) with $\mu(m_Q)/\mu(m_c)$. Thus we have

$$\begin{aligned} \tfrac{1}{2}\delta(m_Q)_{\text{reduced mass}} &\approx \frac{C}{2}\ln(\mu(m_Q)/\mu(m_c)) \\ &\approx \frac{C}{2}\, m_q\left(\frac{1}{m_Q} - \frac{1}{m_c}\right) \quad . \end{aligned} \tag{12}$$

Combining these two effects with (7) we find

$$\delta(m_Q) - E(1^3S_1) = 2m_D - m_\psi + \frac{C}{2}\ln\left(\frac{m_Q}{m_c}\right) + \left(1 - \frac{m_c}{m_Q}\right)\left(216 \text{ MeV} - \frac{m_q}{m_c}C\right) , \tag{13}$$

which is plotted as the boundary of the shaded region in Fig. 1, with the plausible choice[7] $m_q/m_c = 1/5$. We note that under these assumptions $\delta(m_c)$ is within 10% of δ_∞ so that the difference between the estimates (7) and (13) is unimportant.

We have shown that for a general class of confining potentials the number of narrow $^3S_1$ levels below new-flavor threshold grows as $a(m_Q/m_c)^{\frac{1}{2}}$. This extends the conclusion of Eichten and Gottfried that in a particular charmonium potential heavier quarkonium systems should exhibit spectra richer than that of the psions. Should there exist a new quark with a mass of 20 GeV (accessible at the new electron-positron storage rings), we may expect as many as 8-10 quasistable $^3S_1$ levels!

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FIGURE CAPTION

Fig. 1 Plot of $Q\bar{q} + \bar{Q}q$ threshold relative to the (ground state) 1^3S_1 $Q\bar{Q}$ level as function of ratio of heavy quark mass m_Q to charmed quark mass m_c for a logarithmic potential. Upper curve: reduced-mass and hyperfine corrections included. Lower curve (straight line): reduced-mass and hyperfine corrections ignored. Horizontal lines denote 2^3S_1, 3^3S_1, ... $Q\bar{Q}$ levels in this potential.

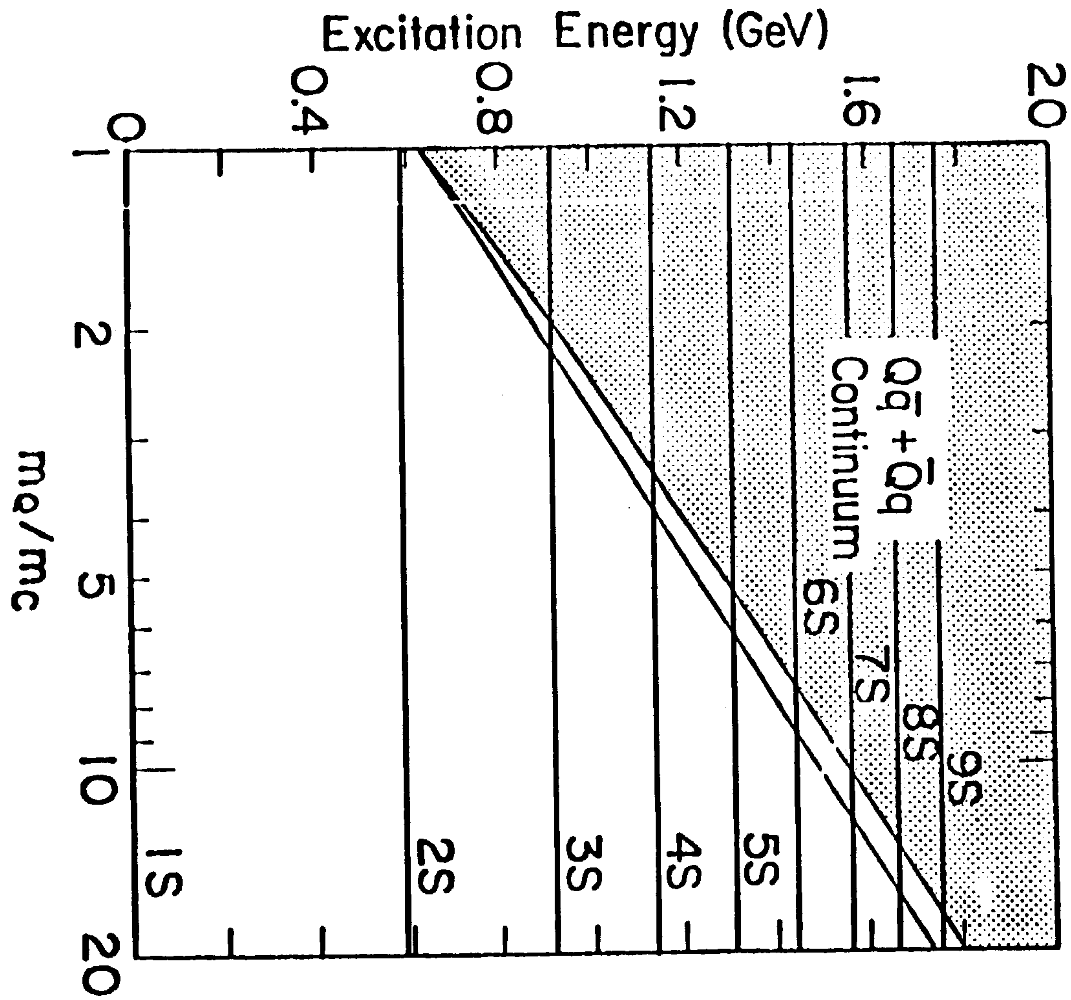
Excitation Energy (GeV)
0
0.4
0.8
1.2
1.6
2.0
mQ/mc
1
2
5
10
20
$Q\bar{q} + \bar{Q}q$
Continuum
1S
2S
3S
4S
5S
6S
7S
8S
9S

Fig. 1