Fermi National Accelerator Laboratory

FERMILAB-Pub-77/32-THY
April 1977

Gravitational Collapse
and Angular Momentum Loss to Neutrinos

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ABSTRACT

We present a model in which neutrinos emitted during gravitational collapse pick up angular momentum by multiple scattering before escape. The slow down of a typical remnant, occuring in about a tenth of a second, is calculated.

*Work supported by the National Science Foundation, Grant No. PHY76-11445.

Operated by Universities Research Association Inc. under contract with the Energy Research and Development Administration

In an earlier letter[1] we suggested that neutrinos emitted during gravitational collapse might carry away angular momentum. This was based on the observation that the neutrinos interact many times before escape and slow down the remnant by picking up some of its angular momentum on their way out. The neutrinos are no longer emitted from the surface with spherical symmetry; instead they come out rotating in the same direction as the star. Here we shall describe a simple model for this process and use it to calculate the slow down of a typical remnant: one solar mass neutron star of radius 10Km.

The Model

We assume that neutrinos are emitted inside a spherical "core" of radius r. The star has an outer radius $R > r$ such that the neutrinos are free beyond R. All interactions take place between creation at r and escape at R. The core is rotating at angular frequency $\vec{\omega}_c$ which may differ in direction or magnitude from the angular frequency $\vec{\omega}$ of the rest of the star, though in much of this paper we will neglect differential rotation and assume axial symmetry around $\vec{\omega}$.

In the absence of rotation we will take a very simple model of neutrino transport, viz. $\vec{c}_\nu = \langle\vec{c}_\nu\rangle$. While the average ν-velocity $\langle\vec{c}_\nu\rangle$ is certainly radial because of spherical symmetry, the actual neutrino velocity $\vec{c}_\nu$ is not-scattering will cause the neutrinos to diffuse out. Assuming radial flow for $\vec{\omega} = 0$ will greatly simplify our calculation when we include rotation, and we expect that with diffusion taken into account properly the effect discussed here will be even more pronounced. The diffusion time is much longer than the radial flow time ($\sim R/c$), and of course the longer a neutrino is trapped inside a star the more chance it has to interact. Note that this approximation does not necessarily mean that we neglect scattering--a neutrino interacting with matter is emitted into a cone with opening angle $2\theta_{\text{scattering}}$, but with equal probability for the azimuthal angle $\phi_{\text{scattering}}$, $0 \leq \phi_{\text{scattering}} \leq 2\pi$. Hence the average direction of scattered neutrinos is along the axis of the cone, which of course is also parallel to the direction of the incoming neutrinos. This is what we mean by radial flow.

With rotation, the axis of the cone is tilted. We shall calculate this tilt by a coordinate transformation to a co-rotating frame.

More specifically, consider a neutrino emitted at some point $\vec{r}$. It undergoes a number of scatterings at $\vec{r}_i$, $i = 2, 3, \ldots$ At each scattering the velocity changes direction from $\vec{c}_{i-1}$ to $\vec{c}_i$ ($|\vec{c}_i| = c$ for all i). Between scattering it travels a distance λ, one mean free path. Therefore

$$\vec{r}_i = \vec{r}_{i-1} + \vec{c}_{i-1} t \tag{1}$$

with $t = \lambda/c$. The "bending" from $\vec{c}_{i-1}$ to $\vec{c}_i$ is to be calculated by the Lorentz transformation mentioned above:

$$\vec{c}_i = \frac{\vec{c}_{i-1} + \vec{v}_i \gamma_i \left[1 + \frac{\gamma_i}{1+\gamma_i} \frac{\vec{v}_i \cdot \vec{c}_{i-1}}{c^2}\right]}{\gamma_i \left[1 + \frac{\vec{v}_i \cdot \vec{c}_{i-1}}{c^2}\right]} \tag{2}$$

where $\vec{v}_i = \vec{\omega} \times \vec{r}_i$, $\gamma_i = (1 - v_i^2/c^2)^{-\frac{1}{2}}$. The initial conditions are specified by $\vec{r}_1 = \vec{r}$, $\vec{c}_1 = c\hat{r}/\gamma_c + \omega_c \times \vec{r}$, $\gamma_c = (1 - (\vec{\omega}_c \times \vec{r})^2/c^2)^{-\frac{1}{2}}$.

Neutrino Trajectories

Equations (1) and (2) completely determine the neutrino trajectories once the initial conditions are given. The trajectories are "spirals" with straight sections of length λ. Our interest is focused on the position $\vec{r}_n$ of the last interaction of the neutrinos in the star and the velocity $\vec{c}_n$ with which they emerge. $\vec{r}_n$ and $\vec{c}_n$ depend on the angles θ and ϕ which define where on the surface of the small sphere the neutrinos originated (the initial conditions). For a given (θ, ϕ) one iterates the coupled equations (1) and (2) n times to get $r_{n+1} > R$. This defines n. The angular momentum lost per solid angle per unit time is given by

$$\frac{d^2\vec{J}_\nu}{dt\, d\Omega} = \frac{1}{4\pi c^2} \frac{dE_\nu}{dt} \vec{r}_n \times \vec{c}_n \tag{3}$$

where dE_ν/dt is the rate of energy loss to neutrinos. The amount of bending (angle between $\vec{r}_n$ and $\vec{c}_n$) is strongly θ-dependent: ν's emitted towards the poles, $\theta = 0$ or π, are not bent at all, while ν's emitted in the equatorial plane, $\theta = \pi/2$, are very much deflected. Similarly $n = n(\theta, \omega, \ldots)$. We will later integrate over θ and ϕ to get $d\vec{J}_\nu/dt$.

Even after one iteration the expressions for $\vec{r}_2$ and $\vec{c}_2$ look quite complicated. We have found closed form solutions to (1) and (2) in two regions of λ-ω-R space:

Region I. $\omega R^2/\lambda c \ll 1$. Here

$$\vec{r}_n = \left[1 + (n-1)\frac{\lambda}{r}\right]\vec{r} + \frac{n}{2}(n-1)\left[1 + \left(\frac{n-2}{3}\right)\frac{\lambda}{r}\right]\frac{\lambda}{c}\,\vec{\omega}\times\vec{r} \quad , \tag{4}$$

$$\vec{c}_n = c\hat{r} + n\left[1 + \left(\frac{n-1}{2}\right)\frac{\lambda}{r}\right]\vec{\omega}\times\vec{r} \tag{5}$$

where $n = 1 + \mathrm{Int}((\mathbf{R}-r)/\lambda) \approx R/\lambda$. We have let $\vec{\omega}_c = \vec{\omega}$.

Region II. $\omega R^2/\lambda c \gg 1$. Here

$$\vec{r}_n = \vec{R} = \hat{r}R \quad , \tag{6}$$

$$\vec{c}_n = c\frac{\vec{\omega}\times\vec{R}}{|\vec{\omega}\times\vec{R}|} \quad . \tag{7}$$

This is a region where $n \to \infty$, $\lambda \to 0$, with $n\lambda$ kept fixed. We have used axial symmetry in writing down (6) and (7).

The Angular Momentum of the Neutrino Gas

Integrating over the solid angle in equation (3) one gets $d\vec{J}_\nu/dt$, the total rate of angular momentum loss to neutrinos. We shall assume that the neutrinos are created uniformly over the small sphere r:

$$\frac{d\vec{J}_\nu}{dt} = \frac{1}{4\pi c^2}\frac{dE_\nu}{dt}\int \vec{r}_n \times \vec{c}_n \, d\Omega \, . \qquad (8)$$

It will be useful to introduce two quantities which have simple physical interpretations and which relate $d\vec{J}_\nu/dt$ to dE_ν/dt:

$$\frac{d\vec{J}_\nu}{dt} = \frac{\pi R}{4c}\frac{dE_\nu}{dt} \langle \sin \alpha \rangle \hat{\omega} = \frac{1}{c^2}\frac{dE_\nu}{dt} k^2 \vec{\omega} \quad . \qquad (9)$$

α is the average bending angle of the neutrinos as they leave the star, and k may be thought of as the "radius of gyration" of the neutrino gas. It is ω-dependent. In region I it is given by

$$k_0^{\,2} = \frac{2}{3} n \left\{ \frac{1}{3}(n-1)(n-1/2)\lambda^2 + (n-1)\lambda r + r^2 \right\} \approx \frac{2}{9}\frac{R^3}{\lambda} \, , \qquad (10)$$

and in region II,

$$k_\infty^{\,2} = \frac{\pi c R}{4\omega} \quad . \qquad (11)$$

The value of k in the overlap region, $\omega R^2/\lambda c \sim 1$, lies between k_0 and k_∞, and was calculated numerically by integrating over the solid angle with the integrand $\vec{r}_n \times \vec{c}_n$ obtained in each element of solid angle by iterating equations (1) and (2) until $r_{n+1} > R$ was reached.

In Fig. 1 we plot k/R as a function of ω for R = 10 Km, r = R/10, and two values of λ. In region I k/R is independent of ω but depends on λ. In region II the opposite is true. Here, of course, $\alpha \to \pi/2$. By a dimensional argument k/R "scales" in the sense that $k/R = f(\lambda/R, \omega R/c, r/R)$. It is found to be not very sensitive to the ratio r/R as long as $r \ll R$ (see Eq. 10). Using this scaling property Fig. 1 can be extended to cover other values of λ, ω, and R.

Slow Down of the Star

Having calculated the rate of angular momentum loss to neutrinos, we can proceed to find out how a star slows down because of it. All the quantities discussed above, in particular ω and k, are really functions of time. Including the possibility that R also might change, the differential equation one has to solve is

$$\frac{1}{\omega}\frac{d\omega}{dt} + \frac{1}{I}\frac{dI}{dt} + \frac{k^2}{Ic^2}\frac{dE_\nu}{dt} = 0 \qquad (12)$$

where $I = 2/5\, MR^2$.

In general this equation of conservation of total angular momentum must be solved numerically. It is useful, however, to look at the solutions in regions I and II where k is known analytically, and to take R = constant to separate the effect of the neutrinos from the spin-up due to collapse. It also turns out to be a very good approximation in a realistic case where the effect of the neutrinos comes mostly after the collapse is halted.

In region I,

$$\omega(t) = \omega(0) \exp \left\{ -\frac{k_0^2}{Ic^2} \left[E_\nu(t) - E_\nu(0) \right] \right\} \quad , \tag{13}$$

and in region II,

$$\omega(t) = \omega(0) - \frac{\pi R}{4Ic} \left[E_\nu(t) - E_\nu(0) \right] \quad . \tag{14}$$

We have used $E_\nu(t) = \int^t dE_\nu/dt \; dt$.

The solution to Eq. (12) for a realistic case is shown in Fig. 2. The general behavior of R and dE_ν/dt, also shown in Fig. 2, is suggested by recent calculations on gravitational collapse.[2] The total neutrino energy emitted is about 5.5×10^{52} ergs. For the mean free path, $\lambda = 1/N\sigma$, we have used $\sigma = 10^{-42}$ cm^2, $N = 2.8 \times 10^{38}$ $(R/10^6 \text{ cm})^{-3}$ nucleons/cm^3, corresponding to one solar mass. For a detailed discussion of the relevant parameters we refer to Tubbs and Schramm.[3]

In about .04 seconds the star collapses from $R = 14 \times 10^6$ cm to 1×10^6 cm. During this time its rotation goes up by a factor $(14)^2 \approx 200$. The neutrinos have little effect on this initial spin-up because most of them are still to come after the collapse is halted ($t = 0$). After $t = 0$ and in about .1 second the neutrinos slow down the star by a factor of about 30. As indicated, we have taken $\omega = 2\pi$ rad/sec. at $t = -.04$ sec. About 6×10^{50} ergs of rotational energy are lost to the neutrinos from $t = 0$ to $t \approx .1$ sec.

This slow down proceeds from region II to region I. Around $t = 0$ ω is large because of the spin-up during the collapse. It starts to decrease linearly with E_ν (see Eq. 14) until it is small enough that region I is reached. The decrease is now exponential (see Eq. 13) and continues until the neutrino burst is over. Had the neutrinos been emitted earlier the process would go from region I to region II and then back to region I.

Remarks

1) That a large amount of energy must be released in the form of neutrinos during gravitational collapse is widely accepted. We emphasize that this is all that is required for the proposed theory of slow down. Whether the supernova explosion, if it occurs, is triggered by the neutrino pressure or by a shock wave or by a combination of both is still a question very much debated[2] and should not be confused with the question of angular momentum. Even if the explosion is powered by a totally new mechanism and the neutrino pressure alone is not sufficient to blow-off the envelope, the angular momentum picked up by the neutrinos will substantially slow down the remnant. The reason is the following: a neutrino-triggered explosion is a very delicate problem highly sensitive to the time structure of dE_ν/dt, but the slow down process is primarily a function of the total energy E_ν. The neutrino luminosity in the example treated here is barely sufficient for a supernova explosion. Had the 5.5×10^{52} ergs been emitted over a longer period and/or at a somewhat later time, then their effect

on the explosion would have been negligible. The slow down, on the other hand, would simply have been prolonged and/or delayed.

2) This slow down depends on initial rotation (see Eq. 14). Very high initial rotation is not much slowed down unless E_ν is proportionately larger.

3) Neutrinos from rotating cores take a longer time to escape which of course decreases the neutrino pressure on the mantle. This remark and the previous one suggest that if neutrinos play any active role in blowing off the stellar envelope, then this role is very much affected by rotation.

4) We have made approximations too numerous to discuss in any detail here. We just mention a few: (a) there are different kinds of neutrinos and antineutrinos, emitted at different energies, hence having different mean free paths because the cross-section depends on the energy as well as the type of the neutrino.[1,4] (b) There are substructures in dE_ν/dt on the millisecond level not shown in Fig. 2, coming from neutronization or thermal processes. (c) The neutrino trajectories are more complicated, and they spend even more time inside the star than what our model indicates, viz. $n\lambda/c$. All these can be effectively treated in a Monte Carlo program for neutrino transport properly modified to include rotation.

(5) It is well known that pulsars are rotating slower than what simple gravitational collapse predicts. In the example treated here a star with a period of one second spins up to 200 times faster by the time the collapse is halted. No pulsar rotates that fast even after allowing for its slow down

as measured now ($\dot{T} \approx 10^{-15}$). We have seen, however, that because of angular momentum loss to neutrinos the newly born pulsar loses some 6×10^{50} ergs of rotational energy in the first tenth of a second or so, slowing down to a period of about 0.15 second, well within the range of observed pulsar periods.

Elsewhere we will discuss a possible scenario for black hole versus neutron star formation, and try to take into account magnetic fields.

I wish to thank Professor B. Lee for his kind hospitality at Fermilab.

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[2] J. R. Wilson, in Proceedings of the International School of Physics "Enrico Fermi", Varenna, Italy, 1975, to be published. D. N. Schramm, in Proceedings of the 2nd Dumand Summer Workshop, University of Hawaii, 1977 (Fermilab publication edited by A. Roberts).

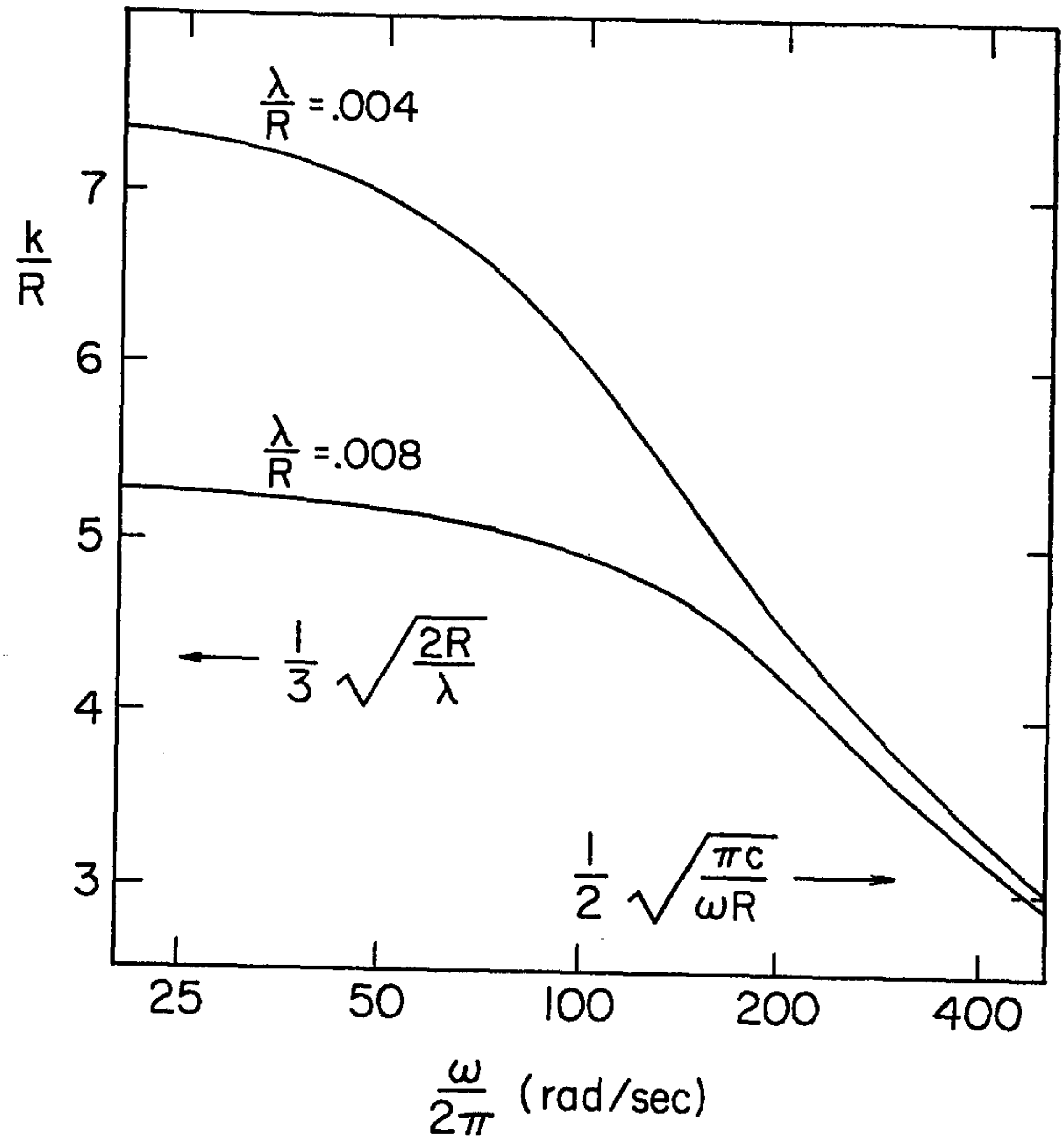
[3] D. L. Tubbs and D. N. Schramm, Astrophys. J. 201, 467 (1975).

[4] The bulk of the cross section, $\sim 10^{-42}$ cm^2, comes from elastic scattering via weak neutral currents. See K. O. Mikaelian, to be published.

FIGURE CAPTIONS

Fig. 1: The neutrino radius of gyration k in units of R as a function of ω. Region I (II) is indicated by the left (right) pointing arrow labeled with the value of k/R in that region. $R = 10$ Km, $r = R/10$.

Fig. 2: The angular frequency $\omega(t)$ in units of 2π rad/sec. as a function of time. Also plotted are R(t) in units of 10^6 cm and the neutrino luminosity $dE_\nu/dt\ (t)$ in units of 10^{51} ergs/sec. The collapse is assumed to halt at $t = 0$.



$\frac{\lambda}{R}$ =.004
$\frac{\lambda}{R}$ =.008
$\frac{k}{R}$
7
6
5
4
3
$\frac{1}{3}\sqrt{\frac{2R}{\lambda}}$
$\frac{1}{2}\sqrt{\frac{\pi c}{\omega R}}$
25
50
100
200
400
$\frac{\omega}{2\pi}$ (rad/sec)

Fig. 1

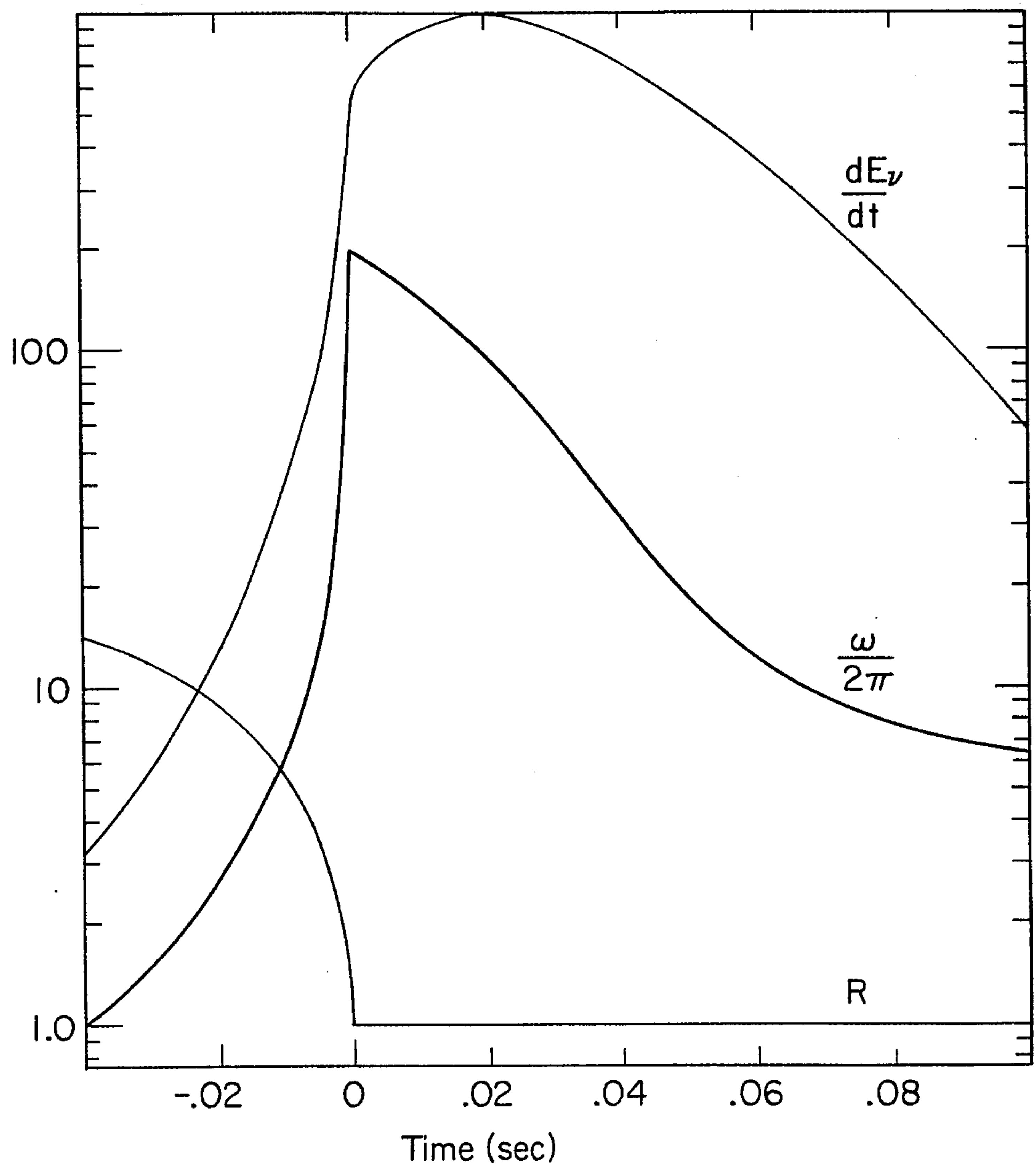


Fig. 2