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Why is There Charm Strangeness Color and All That?

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I. INTRODUCTION

Once upon a time physicists believed that matter was made of protons and electrons. Then the neutron was discovered. There were now two particles, the proton and neutron, which were very similar, yet they were also different. Now there are many particles classified in groups containing members which are similar and also different. Exactly how are they similar? Exactly how are they different? Why do particles appear in such groups? These are some of the fundamental questions to be explored in these lectures.

The neutron and proton have similar masses and strong interactions. They have different electric charges and electromagnetic interactions. The similarity of their strong interactions is expressed formally by the principle of charge independence of nuclear forces and by the symmetry of isospin invariance. The symmetry is broken by the electromagnetic interactions which do not conserve isospin but only the z component or electric charge. This symmetry breaking removes the degeneracy of the nucleon doublet, chooses the eigenstates of electric charge as the physical particles and introduces a mass splitting between them.

The nucleon example shows the two kinds of internal quantum numbers now used to classify particles:

1. Additive quantum numbers, conserved like charge or approximately conserved like strangeness.

2. "Non abelian" quantum numbers like isospin which label families of particles. These are associated with operators which change the members of a given family into one another. They thus do not commute with the charge operators and are called non-abelian.

The non-abelian quantum numbers define families or supermultiplets of related particles. The additive quantum numbers label the members of the families and distinguish between them. Such a multiplet structure arises naturally in any model of hadrons built from basic building blocks in the same way that nuclei are built of nucleons. The mass number and charge of a nucleus are linear combinations of the number of neutrons and the number of protons in the nucleus. The isospin of a nucleus is determined by the permutation symmetry of the basic building blocks. In models where quarks are assumed to be the basic building blocks of hadrons there are several different types of quarks having different values for additive quantum numbers.

The internal degrees of freedom which label the quantum numbers of quarks are called flavors and colors. The values of the additive quantum numbers for any given hadron are linear combinations of the numbers of quarks of a given flavor and color, in the same way that the additive quantum numbers for a nucleus are related to the number of neutrons and protons. The non-abelian quantum numbers are related to permutation symmetries and the behavior under transformations which change the color and flavor of quarks. But unlike nuclear physics where the nucleons are known and their properties and quantum numbers have been measured, quarks have not been observed. Thus the additive and non-abelian quantum numbers of hadrons were discovered experimentally and are well established independently of the validity of the quark model. This raises the question, Why do hadrons have abelian and non-abelian quantum numbers which suggest that they are made of quarks when quarks are not observed as free particles in nature?

Some examples of the additive quantum numbers and the associated non-abelian symmetries are listed in Table 1.1. The question marks

Table 1.1 Additive Quantum Numbers and Non-Abelian Symmetries

Additive Quantum Numbers	Non-Abelian Symmetries
Electric Charge	Isospin
Strangeness	SU(3) U spin
Charm	SU(4)
?	Color
Baryon Number	? Supersymmetry?
Lepton Number	?
Electron Number	?
Muon Number	?

S P I N

indicate cases where either an additive quantum number or a non-abelian symmetry are known but the companion quantum number has not been established and it is not clear whether it exists or is observable.

Off in the corner is spin on the boundary between internal and external degrees of freedom. Although intrinsic spin is a property of a particle and is determined by its nature or intrinsic structure, it is also a physical angular momentum and can be rotated by interactions in space time. Rotational invariance is a symmetry which combines rotations in space time with rotations of the intrinsic spins of the particles. In a nonrelativistic theory, one can postulate symmetries in which the dynamics are invariant under separate rotations of intrinsic spin and space time. The impossibility of such a separation in a relativistic theory has led to many difficulties in including spin together with internal symmetries[1] in symmetry groups like SU(6). These difficulties are outside ths cope of the present lectures and will not be discussed further.

The transformations in the space of these internal degrees of freedom are described by symmetry algebras. These are well known in other areas of physics but they appear in particle physics a very different way. This is illustrated in Table 1.2. Conventional applications begin in space time, then go to Hilbert space and then to the laboratory.[2] One begins in space time with a symmetry principle like rotational invariance which requires the equations of motion to be invariant under

Table 1.2 Symmetry Algebras in Physics

Space time Conservation Laws		Hilbert Space Operator Algebras		Laboratory Multiplet Structure of Spectrum
Rotations $p_x \to p_x \cos\theta + p_y \sin\theta$ Conservation of angular momentum	→	$\psi_\alpha \to \Sigma_\beta C_{\alpha\beta}\psi_\beta$ $[J_x, J_y] = iJ_z; [J^2, J_z = 0]$	→	J, M $J = n, n+1/2$ $M = -J, \ldots +J$
Charge Independence of Nuclear Forces	←	Isospin Transformations $\begin{pmatrix} n \\ p \end{pmatrix} \to \begin{pmatrix} \cdots & \vdots & \cdots \end{pmatrix} \begin{pmatrix} n \\ p \end{pmatrix}$	←	Neutrons and Protons have similar properties
Quarks?	←	SU(3) Algebra...SU(6) 8 Generators	←	Isospin-Strangeness Multiplets...Spin
Gauge Theories Unifying Weak and E. M.	→	GIM Mechanism...SU(4)	←	Peculiar Weak Currents. No $\Delta Q=0; \Delta S=1$
			↘	Charmed Particles Charmonium
Quark Confinement Condenser Plate Model Non-Abelian Gauge Theories	←	Color Degree of Freedom $SU(3)_{color}$	←	Baryons have wrong statistics
↑		Throw wrong states out of Hilbert Space	←	Quarks are not seen
Asymptotic Freedom Parton Models--Free Quarks	←	←	←	Scaling in Deep Inelastic Scattering
Supersymmetries?	→	Graded Lie Algebras	←	Baryons and Fermions exist

certain transformations. The dynamical variables describing the system are classified according to their behavior under these transformations; e.g. as scalars, vectors, tensors, etc. under rotations. Conservation laws like conservation of angular momentum are seen to follow from these invariance principles.

The next step extends the classical implications of symmetries to the quantum theory where states of the system are described by vectors in Hilbert space and dynamical variables by operators. To each symmetry transformation in space time there corresponds in Hilbert space a linear transformation of state vectors into one another. These state vectors can be classified into groups called multiplets or representations of the symmetry algebra which form closed sets transforming into one another under the symmetry. One also finds operators like the angular momentum operators which generate the symmetry transformations. The commutation relations among these operators generate an algebra. Analysis of the algebra leads to new operators like J^2 which commute with all of the generators and determine the structure of the multiplets whose states transform into one another under the algebra.

The next step is into the laboratory to examine those consequences of the symmetry algebra description in Hilbert space which are directly verifiable in experiment. The invariance of the Hamiltonian under symmetry transformations means that all eigenfunctions of the Hamiltonian connected by symmetry transformations must be degenerate.

The observed spectrum of states thus shows a multiplet structure with states labeled by quantum numbers determined by the symmetry algebra. In the case of rotational invariance, the multiplet structure consists of states labeled by quantum numbers J and M. The non-abelian quantum number J labels the entire multiplet which consists of 2J + 1 states and the additive quantum number M is the eigenvalue of the operator J_z and takes on values from -J to +J in steps of unity through the multiplet.

In particle physics everything goes backwards. We do not start by an invariance principle in space time which requires invariance under isospin transformations and end with the prediction of isospin multiplets like the proton and the neutron. We start at the bottom and observe a multiplet structure of the spectrum. There are two states, the neutron and proton with very similar properties. We then go back into Hilbert space and ask what are the transformations which would give rise to the observed multiplet structure. We find the SU(2) algebra which transforms neutrons and protons into one another. We then ask what kind of description in space time with a Lagrangian or Hamiltonian would naturally incorporate the symmetry that leads to invariance under these transformations in Hilbert space. The answer in this case is a model in which all complex nuclei are made from an elementary doublet building block, the nucleon, if the forces which bind nucleons together to make nuclei are charge independent.

The next multiplet structure observed in the laboratory was groups of several isospin multiplets having different values of strangeness and the same eigenvalues for all other conserved quantum numbers and similar masses. The search for the right symmetry algebra to describe this multiplet structure in Hilbert space took a long time because there were no obvious elementary building blocks, like the nucleon in nuclear physics, and there was no single obvious candidate for the symmetry group. The correct SU(3) algebra was eventually found and called the eight-fold way because it has eight generators and the lowest lying baryon and meson states were classified in the octet representation, the same representation as the generators. The search for a dynamical model which would lead to this symmetry in Hilbert space began with the puzzle of why the symmetry of SU(3) should describe a system with eight basic baryons and eight basic mesons rather than some group of transformations in an eight dimensional vector space. One answer was that the mesons and baryons were not elementary objects but were composites built from yet unknown basic building blocks with only three states. This elementary triplet, named the quark, is very peculiar because it has fractional electric charge and baryon number, and because it still has not been found.

Soon after the SU(3) symmetry came SU(6) which followed from the observation that SU(3) multiplets with different spins fit together into supermultiplets of the SU(6) algebra, as one would expect for composite models with basic building blocks having three flavors and spin one half.

The next set of symmetries were discovered neither forward nor backward in the chain "space-time-Hilbert-space-laboratory," but via more complicated paths. One began in space-time with gauge theories unifying weak and electromagnetic interactions[3] and in the laboratory with the observation that there were no strangeness-changing neutral weak currents. The addition of a fourth quark flavor[4] and an SU(4) symmetry with the GIM mechanism[5] for suppressing neutral strangeness-changing transitions was motivated by the peculiar structure of the weak currents observed in the laboratory and led to new predictions that charmed particles and charmonium states should be observed in the laboratory. The SU(4) model received a new impetus when the neutral strangeness-conserving weak currents were observed.[6] There were now charged currents both strangeness conserving and strangeness changing, but the neutral current conserved strangeness. The charm model gave this kind of current in a very simple way, while no other model gave such predictions. The completely independent theoretical discovery that gauge theories unifying weak and electromagnetic interactions were renormalizable[7] led to a renewed interest in these theories and the subsequent experimental discoveries of neutral currents and charm gave a strong push to the development of gauge theories.

The color degree of freedom and color symmetry[8] was motivated by three different discoveries in the laboratory. 1) That the baryon spectrum is described simply in the quark model only if quarks have the wrong statistics. 2) That free quarks have not been discovered and 3) The scaling phenomenon discovered in deep inelastic lepton scattering. In the Hilbert space, one finds that the statistics problem can be solved by introducing a new color degree of freedom with three colors and requiring the low lying baryon states to be singlets in color SU(3). The observation that quarks, diquarks, or other states with fractional electric charge have not been seen is explained by pushing all states which are not color singlets up very high in mass or throwing them out of Hilbert space all together. The latter is the limit of pushing them up in mass to the point where they have infinite mass. A search for a dynamical theory described in space time which would have these properties in Hilbert space led to non-abelian gauge theories which depressed all color singlet states and might lead to quark confinement, and the pushing up of all non-singlet states to infinite energy. This happens exactly in a 1 + 1 dimensional model[9] where a quark-antiquark pair are like a pair of condenser plates and separating the plates requires infinite energy.

A different path to color and non-abelian gauge theories started with the observation that the experimentally observed scaling could be obtained from parton models in which quarks behaved as free point-like objects in deep inelastic scattering. The question of how quarks can be so strongly bound that they can never escape, while nevertheless behaving as quasi-free particles led to the discovery of asymptotic freedom[10] and infra-red slavery in which quarks interacted with weak short range forces and strong long range forces. It was then found that the only theories which had this asymptotic freedom property were just the same non-abelian gauge theories needed to solve the quark statistics problem and the saturation of bound states at the quark-antiquark and three-quark levels.

New speculations of possible additional symmetries are motivated by the existence of the additive quantum numbers of baryon number and lepton number.[11] There are suggestions that states having different eigenvalues of these quantum numbers could be grouped into larger supermultiplets in a new scheme which would eventually unify all of particle physics. The inclusion of states with even and odd baryon number requires a different mathematical structure from the discrete symmetries and Lie algebras used for conventional symmetries. The appropriate algebra to use in Hilbert space is called a graded Lie algebra. These are related to dynamical symmetries in space-time known as supersymmetries.[12]

The unification of states having different baryon and lepton numbers but without mixing bosons and fermions has been explored with the aim of putting quarks and leptons, the basic building blocks of all particles, into a unified scheme. These new speculations on supersymmetry and quark-lepton universality have not yet led to any verified experimental predictions and are still at a very early stage of development. They will not be discussed further in these lectures. They are discussed elsewhere at this course.[1] Hopefully they will lead ultimately to the answer to the question, "Why is there charm strangeness, color and all that?".

II. STRANGENESS, CHARM AND MASS SPLITTINGS[13]

Because symmetries are introduced backwards in particle physics, there is no unambiguous way to introduce symmetry breaking. In conventional applications like rotational invariance in atomic physics, the symmetry is broken by a well understood mechanism, such as an external magnetic field, whose transformation properties under rotations are known. The symmetry algebra can then be used to calculate the splittings of levels and transition matrix elements. But in particle physics there is no underlying theory to specify the transformation properties of the symmetry breaking interactions.

One starts in the laboratory by noting that pions and kaons have different masses, and that additional strangeness goes with increasing mass. By analogy with the breaking of rotational invariance with a magnetic field that transforms like a vector under rotations, one can assume that the breaking of SU(3) symmetry transforms like the SU(3) analog of a vector, namely an octet. This gives the Gell-Mann-Okubo mass formula. But there is no theory to tell whether the formula applies to linear masses, quadratic masses, some exotic power of the mass, the S-matrix, or to "reduced" matrix elements with certain kinematic factors removed. The original folklore suggested linear mass formulas for baryons and quadratic formulas for mesons. These gave good agreement with experiment for SU(3) and SU(6) mass formulas. But the quark model gave results which related baryon mass splittings to meson mass splittings, in particular, the naive assumption that the difference between strange and nonstrange quarks relates meson and baryon splittings as well as mesons and baryons among themselves. Within the meson and baryon supermultiplets these quark model relations are equivalent to SU(6) relations. But between mesons and baryons they give something new, which agrees with experiment when linear masses are used. The situation was summarized at the 1966 Berkeley conference[14] by the "crazy mass formula"

$$K - \pi \overset{Q}{=} K^{*} - \rho \overset{L}{=} \Sigma^{*} - \Delta \overset{L,Q}{=} \Xi - \Sigma \quad , \tag{2.1}$$

where the L above the equality implies that linear masses should be used and the Q above the equality implies that quadratic masses should be used.

While there are many ways to derive some of these equalities, no credible model includes both the linear and quadratic relations involving the same vector meson mass splitting. But the experimental agreement with the crazy formula is sufficiently impressive to suggest that it cannot be wholly accidental.

The discovery of charm allows a similar formula to be written for the charmed states by simply replacing all strange quarks in (2.1) by charmed quarks. The result is

$$D - \pi \overset{Q}{=} D^* - \rho \overset{L}{=} C_1^* - \Delta = \qquad , \tag{2.2}$$

where the last equality is left open since the doubly charmed baryon analogous to the Ξ has not yet been found. This formula also agrees with experiment, as shown in Table 2.1. Thus changing a nonstrange quark in the ρ to a strange or to a charmed quark produces a linear mass shift which is equal to that produced by the corresponding change of a quark in the Δ, while the shift in squared mass is equal to that produced by the corresponding quark change in the pion.

An interesting relation between the spin splittings of the masses of strange and nonstrange baryons was given by Federman, Rubinstein and Talmi[15] in 1966

$$(1/2)(\Sigma + 2\Sigma^* - 3\Lambda) = \Delta - N \quad . \tag{2.3}$$

Experimentally the left and right hand sides of this relation are 307 and 294 MeV, which is rather good agreement. This relation follows from the assumption that the mass differences are due to two-body forces which are spin dependent. The right hand side is just (3/2) the difference between the interaction of two nonstrange quarks in the triplet and singlet spin states when these quarks are bound in a nonstrange baryon. The left hand side is the same difference for a nonstrange quark pair bound in a hyperon (the particular linear combination chosen causes the contribution from the strange quark interaction to cancel out). The experimental agreement indicates that the assumptions of two-body forces and SU(6) spin couplings in the wave functions are good approximations.

Here again the relation can be extended to charm by replacing strange quarks everywhere with charmed quarks,

$$(1/2)(C_1 + 2C_1^* - 3C_0) = \Delta - N \quad . \tag{2.4}$$

Since the present experimental information on charmed baryons[15] gives a mass of 2260 for the C_0 and a mass of 2500 for a broad peak interpreted to be the unresolved $C_1 - C_1^*$ combination, it is convenient to rewrite Eq. (2.4) as

$$(C_1 + 2C_1^*)/3 = C_0 + (2/3)(\Delta - N) \quad . \tag{2.5}$$

The left hand side is a weighted average of the C_1 and C_1^* masses, which can be roughly approximated by the value 2500 MeV for the unresolved peak. The left hand side is 2456 MeV, which is in reasonable agreement. So the spin interactions of the ordinary u and d quarks in charmed hadrons are the same as in nucleons and hyperons.

We see that charm really behaves very much like strangeness, and that we don't understand it either!

Table 2.1 Experimental Tests of Crazy Mass Formula

a) Strangeness Splittings

	$K - \pi$	$\overset{Q}{=} K^* - \rho$	$\overset{L}{=} \Sigma^* - \Delta$	$\overset{L,Q}{=} \Xi - \Sigma$
ΔM(GeV)	0.35 GeV	0.12	0.15	0.12
ΔM^2(GeV)^2	0.22	0.20		

b) Charm Splittings

	$D - \pi$	$\overset{Q}{=} D^* - \rho$	$\overset{L}{=} C^* - \Delta$
ΔM(GeV)	1.72	1.23	1.26 (if $M_{C^*} = 2.5$)
ΔM^2(GeV)^2	3.3	3.4	

III. HIGH ENERGY SPECTROSCOPY[17]

3.1 Introduction

New particles are awaiting discovery with new accelerators, but it is not clear how to look for them, particularly since the most exciting new discoveries have unexpected and surprising properties. Suggestions from theorists are of dubious value. Even when they are right their advice is usually useless and following it exactly usually leads to missing something crucial. But something equally crucial can be missed by ignoring their advice. After each discovery it usually turns out that some theorist predicted it. But dozens of equally plausible suggestions also made at the same time led nowhere and it was by no means obvious which approach would be fruitful. This makes life difficult for experimentalists and program committees trying to decide what experiments to do. But if their tasks were easier and the outcome of experimental investigations could be predicted in advance, research would be much less exciting.

The recently discovered new charmonium spectroscopy presents an instructive example of these difficulties. At the 1975 Palermo Conference I was given credit[18] for predicting the discovery of these particles on the basis of the analysis[19] shown in Table 3.1 of the new particle search proposals in 1972 at Fermilab. The conclusions were that the searches for quarks, monopoles, tachyons, etc. were not apt to lead anywhere and that the really exciting search would discover a

TABLE 3.1 Guide to inconclusive experiments and hypothetical particles.

Particle	Who needs it?	If not found, so what?	Craziness index	Signature
		I. PEDAGOGICAL EXAMPLES		
Ω^-	MGM & YN	Kills SU(3)	No	Good
$M^0(150) \to 5\gamma$	Nobody, but why not?	Nobody cares	Not particularly	Missing mass
		II. PROPOSED SEARCHES AT NAL		
Tachyons	Nobody, but why not?	Nobody cares. Try harder	Very	Good
Quarks	Dalitz	Try harder	Fair	Good (fractional charge)
Monopoles	Dirac-Schwinger	Try harder	Moderately	Good
Intermediate bosons	Yukawa	Try harder, but credibility falls	No	Good
Heavy leptons	Nobody, but why not?	Look elsewhere (spectroscopy)	No	Good
Partons	Bjorken-Paschos	Ask Bjorken-Paschos	No	Good
Han-Nambu triplets	Dalitz might settle for these	Try harder	Less than quarks	Missing mass best
Superheavy nuclei	Nuclear physicists	Try harder	No	Chemistry. Not clean
		III. THE REALLY EXCITING SEARCH		
?	Nobody has thought of it	It will be found; it's <u>there</u>	Who knows, the theorists have not thought of it yet	

particle not listed in these proposals and which the theorists had not thought of. This prediction is not strictly correct if the new particles[20] discovered since November 1974 are indeed bound states of charmed quarks and antiquarks as they seem to be today. Such states were proposed by theorists[4] a long time ago and their properties were investigated in detail. However, in 1972 there were no charm search proposals at Fermilab. Even in the summer of 1974 when charm searches suddenly became fashionable and theorists suggested ways of looking for charm,[21] there was no suggestion that charmonium or hidden charm would be found long before charm itself or that the most fruitful search would be for very narrow states produced in electron-positron annihilation. The reason why these suggestions were not made is instructive. Two crucial missing links in our understanding of hadron properties prevented the appropriate suggestions from being made and taken seriously. These were the existence of neutral weak currents[6] and the mysterious selection rule attributed to Zweig, Okubo, Iizuka and others.[22, 23, 24, 25]

In 1971 hadron spectroscopy was well described by the conventional quark triplet with three quarks and no fourth quark was needed to describe the observed states. The motivation for charm came entirely from weak interactions where a number of attractive looking theories encountered difficulties in predicting the existence of neutral weak currents[26] in flagrant contradiction with experiment. The introduction of a fourth charmed quark with the GIM mechanism[5] cancelled out all the strangeness changing neutral currents and removed the disagreement with experiment. But the strangeness conserving neutral currents were not cancelled and there was no experimental evidence for such weak neutral currents. There was also no convincing evidence against them, but most particle physicists assumed that this was simply a problem of experimental techniques. Sensitive experiments testing strangeness-changing neutral currents were much easier than tests of strangeness-conserving neutral currents, and there was no obvious reason why one should be absent while the other was present. Thus a model which looked attractive to theorists did not seem attractive to experimentalists because it predicted all kinds of unobserved experimental results and then had to introduce various ad hoc cancellations to get rid of them. Furthermore the same theorists of the Harvard group who proposed the charm model to get rid of strangeness changing neutral currents had more complicated models[27] with additional heavy leptons that could get rid of all neutral currents. There was a general proliferation of models each introducing either new quarks, new leptons or new ad hoc couplings of electromagnetic and weak currents. They were all equally believable and each suggested different experiments to test its validity. It was hard for an unprejudiced experimentalist to know which model should be taken seriously or whether the whole picture of gauge theories was worth considering seriously at all.[28]

Everything changed with the discovery of the weak neutral currents.[6] It was now clear that nature had placed the strangeness conserving and strangeness violating neutral currents on a completely different basis and the most natural explanation for this difference came from the GIM mechanism[5] which required the existance of charm. So the charm model suddenly jumped from being one of many dubious theoretical models with ad hoc assumptions not justified by experiment to the simplest and most reasonable model available which would explain a very striking and important new experimental result.[29] Attention immediately turned to charm searches.

The charmonium states, bound states of a charmed quark-antiquark pair were also predicted, and it was also realized that the decay of these states would be inhibited by the same OZI selection rule which prevents a strange quark-antiquark pair from disappearing in the ϕ meson decay to produce final states without strange quarks. However, estimates of the suppression factor were off by a large factor because the width of the $\phi \rightarrow \rho\pi$ decay was the only experimental evidence available for the strength of transitions violating the selection rule. Why the charmonium states are so much narrower is still not understood.

It is now 2 1/2 years since the J particle was produced at Brookhaven by Sam Ting and collaborators. But even though we recognize the importance of Ting's discovery and great effort has gone into subsequent investigations we still know very little about the production mechanism for the J in those experiments.

The OZI rule allows this J production only with an accompanying pair of charmed particles. But there is no evidence for this charmed pair, and the J production seems to go via some mechanism[25] which violates the OZI rule.

Except for this absence of charmed pairs we know very little about the final state in the reaction which includes the J. Thus, it is very difficult to estimate production cross sections for other new objects in hadronic experiments and any extrapolation of Ting's results for such estimates contain so many unknown factors that they are extremely unreliable. Since the narrow width of the J is not understood all estimates of the strength of couplings of new objects to ordinary hadron channels are unreliable. Future experiments might provide new insight into these fundamental uncertainties.

All properties of the charmonium states were predicted well except for the most striking property, the very narrow width which was crucial in their discovery. Similar theoretical considerations and difficulties can be expected to arise in predicting the properties of states to be discovered with new high energy accelerators. So theoretical guidelines should not be dismissed but should be considered with the view that they may be even 90% correct, but a crucial 10% may be missing.

3.2 Signal and Noise in High Mass Spectroscopy

Resonances with masses in the several GeV range have very many open decay channels. Their branching ratios into any one exclusive channel are of the order of 0.1%. Since the signature for the detection of such a resonance generally picks a particular decay mode, the signal is proportional to the branching ratio and is very small. The crucial factor in discovering and confirming such high mass resonances is the signal to noise ratio.

It is useful to define a figure of merit F(P,T) for the production of particle P, by observing a characteristic T of the final state which may either be used as a trigger or as a signature for picking out events. The trigger T may be either the full final state like the electron pair in the decay of the J, or one of the particles produced inclusively in the decay such as a single muon. The figure of merit is defined by the relation

$$F(P,T) = \sigma(P+X) \cdot BR(T)/\sigma(T+X) \tag{3.1}$$

where $\sigma(P+X)$ and $\sigma(T+X)$ denote the cross sections inclusive for production of the particle P and the trigger T in the reaction under consideration and BR(T) denotes the branching ratio for the appearance for the trigger T in the decay of the particle P.

Examination of Eq. (3.1) shows that the optimization of the figure of merit may best be achieved by finding a trigger T with low inclusive production. The characteristics of the signal appearing in the numerator will not be changed very much by choosing a different trigger or a different production mechanism. However, the denominator may be reduced by a large factor by choosing a trigger for which the background is low. Possibilities for improving F(P,T) by reducing the noise seem to be more favorable than by enhancing the signal. We examine three possible approaches to noise reduction.

1) Production of a low noise signal. The signal can be produced by a mechanism which naturally has a low background, as in the production of the Ψ as a very narrow resonance in e^+-e^- annihilation.

2) A low noise signal signature. An exclusive decay channel can be found which has a low production background as in the detection of the J particle by its leptonic decay mode. The particular case of ϕ signatures is of interest.

3) Use of background signature. Since many partial waves in the background can appear at the high mass available and only a few in the signal, the background may have a characteristic structure which enables cuts in selected kinematic regions of the multiparticle phase space to reduce the noise by a large factor.

Production of Low Noise Signal

The production of a new particle with a very low background is possible for a narrow s-channel resonance whose cross section is very much enhanced over the background in a narrow energy region. This approach can be used only for the production of resonances having the quantum numbers available in the initial state. It is particularly suitable for the production of vector meson resonances in electron-positron annihilation.

For states which do not have the quantum numbers of the photon or of the meson-baryon, nucleon-nucleon or nucleon-antinucleon system, some possibilities exist for production via the decays of states which do have these quantum numbers; e.g. in the production of the positive parity charmonium states by radiative decay of the ψ' and the production of charmed particle pairs by the decays of higher vector resonances.

For states not easily produced in this way and available only in inclusive production there is no simple mechanism for reducing the multiparticle background by choice of a particular production mechanism. This applies to most cases of hadronic resonance production, as in J production where no one production mechanism seems to be superior by any large factor.

Low Noise Triggers and ϕ Signature Spectroscopy

The triggers which have low inclusive production cross section in normal hadronic processes include photons and leptons produced by electromagnetic interactions. These are suppressed by powers of α relative to hadron production. Some examples are the lepton pairs used as the signature for the discovery of the J particle, the photons used as a signature to discover even parity charmonium states produced by the decay of the ψ' and the two-photon and multiphoton channels used for the possible detection of the pseudoscalar mesons.

In addition to these electromagnetic triggers which have already been used successfully, particles like the ϕ and f' which are suppressed by the OZI rule in nonstrange hadron reactions might be used successfully. These appear as signatures for states whose branching ratios into decay channels involving ϕ and f' are not suppressed by significant factors over other decays. ϕ signature spectroscopy looks attractive for states decaying into a ϕ because inclusive ϕ production without kaons is forbidden for nucleon-nucleon and pion-nucleon reactions and the background should be small. Typical suppression factors observed experimentally for ϕ production are a factor of 500 below ω production in pion-nucleon reactions[30] at 6 GeV/c or a factor of 100 below pion production at Fermilab energies.[31] The ϕ is easily detected in the $K^+ K^-$ decay mode at high energies because the Q of the decay is so low that both kaons will pass together in the same arm of a spectrometer and will not trigger a Cerekov detector set for pions.[31] An even smaller background would be expected in $\phi\phi$ spectroscopy for states expected to decay into two ϕ's. Examples of such states are isoscalar bosons even under charge conjugation which have the structure of a quark-antiquark pair, either strange, charmed, or some new heavy quark.

"Strangeonium" states of a strange quark-antiquark pair are allowed by the OZI rule to decay into $\phi\phi$ and should have a comparatively strong branching ratio. Such strangeonium states are of general interest since no such states above the ϕ or f' are well known. Our present knowledge of charmonium spectroscopy is at present much better than strangeonium because the low noise electromagnetic signature of lepton pairs and photons enables charmonium to be seen much more easily. Even if $\phi\phi$ spectroscopy does not lead to the discovery of any new charmonium or "x-onium" states made from heavy quarks of type x, the development of strangeonium spectroscopy would add to our understanding of hadron dynamics.

The ϕ decay of charmonium or x-onium is singly forbidden by OZI or other quark line rules and is therefore on the same footing as all other hadronic decays which are also at least singly forbidden. Estimates of the $\phi\phi$ branching ratios for these particles are of the order of 0.1%, which is probably only a small factor below the $\rho\rho$ branching ratio. The $\phi\phi$ background should be very much lower than the $\rho\rho$ background and therefore can provide a fruitful trigger for such states.* The most interesting of such states at present are the pseudoscalar states of charmonium or of the new heavier quarks if they are there.

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*One estimate for $(x\bar{x})_{C=+} \to \phi\phi$, is based on the analogy with $\psi \to \phi\eta$ which also involves annihilation of a heavy quark pair and creation of two strange quark pairs. Another is based on the analogy $\psi \to \rho\pi$ and $(x\bar{x}) \to \rho\rho$ and used SU(3) to relate $\phi\phi$ to $\rho\rho$.

Single ϕ spectroscopy would be useful also in observing decays of higher strange resonances such as K^*, Λ^*; Σ^*, Ξ^*; * and ϕ^* which could decay into lower resonances with the same quantum numbers by ϕ emission above the threshold. Nonstrange baryon resonances at high masses have been observed by the technique of pion-nucleon phase shift analysis. ϕ spectroscopy may enable the discovery of corresponding resonances with different quantum numbers not accessible to phase shift analysis.

States like the $F^{\pm}$ meson containing both charm and strangeness might be observed by the decay into a ϕ and a pion or lepton pair. The $\phi\pi$ decay mode might also be useful in the search for the exotic four-quark states discussed in section VI.

The $\phi\pi$ decay mode is particularly interesting in searches for new objects, because $\phi\pi$ decay is forbidden by the OZI rule for <u>any</u> boson constructed from a quark-antiquark pair. Thus resonances in the $\phi\pi$ system indicate either a new object like a four-quark system, an OZI-violating strong decay of a conventional boson, or a weak decay into a system containing a strange quark-antiquark pair.

A partial list of states which might be detected by ϕ signature spectroscopy are

<u>Single ϕ spectroscopy:</u>

$$K^* \to K + \phi \tag{3.2a}$$

$$\Lambda^* \to \Lambda + \phi \tag{3.2b}$$

$$\Sigma^* \to \Sigma + \phi \tag{3.2c}$$

$$\Xi^* \to \Xi + \phi \tag{3.2d}$$

$$\Omega^* \to \Omega^- + \phi \tag{3.2e}$$

$$\phi^* \to K + \bar{K} + \phi \tag{3.2f}$$

$$F^{\pm} \to \pi^{\pm} + \phi \tag{3.2g}$$

$$F^{\pm} \to \text{leptons} + \phi \tag{3.2h}$$

$$\tilde{F}^{o}_{I} \to \pi^{o} + \phi \tag{3.2i}$$

$$A^{s}_{\pi} \to \pi^{o} + \phi \tag{3.2j}$$

ϕ-ϕ spectroscopy

$$\eta_c \to \phi + \phi \tag{3.3a}$$

$$\text{charmonium } (c\bar{c})_{C=+} \to \phi + \phi \tag{3.3b}$$

$$\text{strangeonium } (s\bar{s})_{C=+} \to \phi + \phi \tag{3.3c}$$

$$\text{x-onium } (x\bar{x}, \text{ where } x \text{ is a new heavy quark})_{C=+} \to \phi + \phi \tag{3.3d}$$

Above 3 GeV the possibility of observing 3ϕ decay arises. Vector meson states like the ψ' and other higher members of the ψ family can decay into three vector mesons. The dominant 3V final state would be $\omega\rho\rho$ but 3ϕ would be of the same order of magnitude in the SU(3) symmetry limit. The 3ϕ state would have a unique signature and a very low background.

The use of ϕ triggers can thus lead to various kinds of interesting physics. The first step is the understanding of ϕ production itself, by examining the other particles produced along with the ϕ and looking for ϕx resonances. Understanding the mechanisms for ϕ production can provide insight into models for particle production, even if no new phenomena or resonances are found. But chances are that some part of the production will be due to decays of higher resonances, and at this stage any resonance with a ϕ-decay mode is interesting.

Background Signatures

The signal to noise ratio can be improved by the alternative approach of characterizing peculiar signatures for the background in order to enable its removal from the signal. This approach is based on the fundamental difference between the spectroscopies of the high mass resonances and old low-lying resonances. The conventional low-lying resonances show up as peaks in cross sections with particular decay angular distributions against a comparatively smooth and structureless background. At high mass the background may have a more striking and easily identified structure than the signal.

High mass resonances are states of low angular momentum decaying primarily into multi-particle channels. Their decays reflect the low angular momentum by containing very few parital waves all having relatively low angular momentum. The background on the other hand can have very large angular momenta and a sharp structure in momentum and angular distributions are present in the signal. A small portion of the multi-particle phase space could include a very large portion of background events. In this case the signal to noise ratio would be improved by a cut excluding this small volume of phase space. The exact kind of cut to be effective depends on the individual case and could be most easily decided by examining the background and looking for its most striking features.

Consider for example the search for a new particle in a particular four-particle decay channel by looking for peaks in the mass spectrum, e.g. looking for a charmed baryon decaying into $\Lambda 3\pi$. The problem is how to use the angular distributions of these four particlesin the center-of-mass system of the four particle cluster (hopefully the rest system of the new particle) as a means of distinguishing between signal and background. Three axes are relevant for examining the angular distributions, (1) the direction of the incident beam momentum, (2) the direction of the momentum of the four-particle clusters, and (3) the normal to the production plane. Signatures which characterize the new particle appear most clearly in angular distributions with respect to the direction of the momentum of the four-particle cluster or with respect to the production plane. But signatures for the noise will show up in angular distributions with respect to the incident beam direction.

Background from uncorrelated particles whose mass happen accidently to fall in the desired range should have angular distributions with respect to the incident beam direction similar to those for single-particle inclusive productions. They should be peaked in the forward and backward directions with a rapidly falling cutoff in transverse momentum. Background events could show forward-backward asymmetry or a tendency to be concentrated in cones forward and backward relative to the direction of the incident beam. The signal from decay of a D meson of spin zero should show a completely isotropic angular distribution with respect to any axis. Particles of non-zero spin might have some anisotropy in their angular distributions if they are polarized in production. But these will involve only low order spherical harmonics and will not concentrate large numbers of events in a small region of phase space. Thus a cut eliminating events in which one or more particles appear within a narrow cone forward and/or backward with respect to the incident beam direction could reduce the background considerably with a negligible effect upon any signal coming from the decay of a low angular momentum state.

As an example consider a four particle decay into a baryon and three pions of a state produced by a high energy accelerator beam hitting a fixed target. This state appears as a four particle cluster with a low mass in the several GeV region but with total laboratory momentum in the 100 GeV range. In the center-of-mass system of the cluster the momenta of the baryon and of the pions are all small and of the same order of magnitude. In the laboratory the baryon has a much larger momentum than the pions because of the effect of the mass on the Lorentz transformation. If the baryon is not a proton and cannot be a leading particle the inclusive momentum distribution for the baryon and the pions can be expected to be very different in the relevant ranges. In particular the momentum distribution for high momentum hyperon or anti-hyperons could be falling rapidly in this region while the momentum distribution for relatively low momentum pions could be rising. This would appear in the center-of-mass system for the multi-particle cluster as baryons being preferentially emitted backward and pions preferentially emitted forward. Cutting out events in which all pions are in the forward hemisphere would thus appreciably reduce the background, but would only remove one eighth of the signal. Using a cone instead of a hemisphere would interfere even less with the signal and still substantially reduce the background.

IV. QUARKONIUM SPECTROSCOPY[17]

Among the new exciting states hopefully waiting to be discovered are sets of positronium-like mesons made of a quark-antiquark pair with the same flavor. These include "strangeonium" states like the ϕ and f' of a strange quark-antiquark pair, charmonium states like the J/ψ family, and states made from quarks of new flavors as yet undiscovered.

4.1 Flavor Dependence of the Spectrum

Strangeonium ($s\bar{s}$) spectroscopy is still in its infancy, and is not yet as well developed as charmonium spectroscopy, even though strangeness was known over two decades before charm. The reason for the comparatively slow development of strangeonium spectroscopy is the absence of a good signature having a high figure of merit like the electromagnetic signatures used to detect charmonium states. The dominant decay modes of the strangeonium states are $K\bar{K}X$ which are allowed by the OZI rule and which also appear in the background. As a result the higher strangeonium states are expected to be broad, have comparatively low branching ratios to electromagnetic channels, and no striking signature different from background below the $\phi\phi$ threshold.

Charmonium ($c\bar{c}$) has given rich experimental results because the dominant OZI allowed decay channel, $D\bar{D}$, is closed for a large set of low-lying states including the radially excited s-wave (the ψ') as well as the lowest p states. Thus these states are all narrow and have appreciable branching ratios and couplings to electromagnetic channels like e^+e^-, $\mu^+\mu^-$, $\gamma\gamma$ and γX. The vector mesons states are therefore easily produced in e^+e^- annihilation and photoproduction experiments, and can also be detected by leptonic decay modes if produced by other means. Other states can be produced by cascade decays of the higher vector mesons and recognized by the presence of photons from the decay which produced them or from their own decays.

Higher x-onium states from heavier quarks with new flavors are expected in many theoretical models, and evidence for such a state has been reported.[32] Eichten and Gottfried[33] have pointed out that such states should show an even richer spectrum than charmonium, because of theoretical arguments showing that more states lie below the OZI-allowed threshold for increasing quark mass. This threshold for the decay of an $(x\bar{x})$ meson is at twice the mass of the lowest $(x\bar{u})$ state; e.g. $2M_K$ for strangeonium and $2M_D$ for charmonium. Eichten and Gottfried argue that the lowest vector state, analogous to the ϕ for strangeonium and the ψ for charmonium, is farther below the threshold as the quark mass increases, continuing the trend seen in the ϕ and the ψ. Thus the range of excitation energy available for narrow OZI-forbidden resonances increases with quark mass.

4.2 Quarkonium production mechanisms

Quarkonium production for states with flavors absent in the initial state is forbidden in strong interactions by the OZI rule. Electromagnetic

$(x\bar{x})$ pair creation is not suppressed and is comparable to other $(q\bar{q})$ production if the x-quark has an electric charge. However, the production of $(x\bar{x})$ from a single photon occurs only for states with the same quantum numbers as the photon, namely odd-C vector mesons.

Processes involving the Pomeron might not be suppressed by OZI. In the SU(3) limit the Pomeron couples equally to strange and nonstrange quarks, and a factorizable Pomeron carries no information on strangeness from one vertex to another. This is borne out by the total cross section for ϕN scattering, which has no OZI-suppression factor, and is only lower than $\sigma(KN)$ by the same amount that $\sigma(KN)$ is below $\sigma(\pi N)$. This small effect is naturally understood as SU(3) breaking in the couplings of the Pomeron to strange and nonstrange quarks, and is not related to the connected and disconnected quark diagrams of the OZI rule. Thus in a multiperipheral process, the f' is emitted by a Pomeron about as easily as any other tensor meson. In the particular case of double Pomeron exchange,[34] one should expect to see f' production comparable to f production. In a Mueller diagram for the central region,[21] one should also expect comparable ϕ and ω production and comparable f and f' production if the Pomeron is approximately an SU(3) singlet as commonly believed.

There is no contradiction in the violation of OZI rule by the Pomeron, since the connected quark diagrams used to describe Reggeon exchanges do not apply to the Pomeron. However, in models where the

Pomeron is "built" from other trajectories,[35] there may be some "memory" of quantum numbers propagated a small distance down the multiperipheral chain and a consequent respect for OZI at moderate energies and low multiplicities. This question is still open. It could be tested by looking for the f' in processes where the f is produced by a mechanism which seems to be double Pomeron exchange, or by looking at the ϕ/ω ratio in the central plateau.

Experimental data on ψ photoproduction seem to indicate that the coupling of the ψ to the Pomeron is considerably less than that of ordinary strange and nonstrange mesons. This must be taken into account in estimating production cross sections for new particle production by Pomeron exchange. But this flavor dependence in Pomeron couplings should not be confused with the OZI rule which is determined by the topological character of quark diagrams.

Hadronic production of quarkonium states may have a very different dependence on the spin and parity quantum numbers than electromagnetic production, which favors vector mesons. There are suggestions that the OZI rule holds much better for vector mesons than for pseudoscalars. In QCD, where the rule is broken by annihilation of a quarkonium pair into gluons, three gluons are required to annihilate a vector state, while a pseudoscalar can go into two gluons. There are also experimental arguments which show that OZI violating processes are stronger in the pseudoscalar state than in the vector state. The absence of ideal mixing in the lowest pseudoscalar nonet is evidence for OZI violation, since the interaction which mixes strangeonium and nonstrangeonium effectively[24] violates OZI. More recently there is experimental evidence from radiative decays that the OZI-violating transition between charmonium states and light quark states is stronger in the pseudoscalar state than in the vector state.[36]

In radiative decays of charmonium to a photon and light quarks, there are two possible transitions (a) The photon is emitted by the charmonium system before the transition into light quarks. In this case the photon cannot carry away isospin and the final light quark state must have isospin zero; (b) The photon is emitted by the light quark system after the OZI-violating transition of the charmonium into light quarks.

$$(c\bar{c}; I=0, J^P=1^-) \rightarrow (c\bar{c}; I=0, J^P=J_f^{P_f}) + \gamma \rightarrow (q\bar{q}; I=0, J^P=J_f^{P_f}) + \gamma \tag{4.1a}$$

$$(c\bar{c}; I=0, J^P=1^-) \rightarrow (q\bar{q}; I=0, J^P=1^-) \rightarrow (q\bar{q}; I=I_f, J^P=J_f^{P_f}) + \gamma \quad . \tag{4.1b}$$

In case (a) the photon carries away its angular momentum and parity before the OZI violation, and the violation occurs in a system having the space-spin quantum numbers of the final state. In case (b) the OZI violation occurs in a system having the space-spin quantum numbers of the initial state before the photon carries away angular momentum and parity. The photon can now carry away isospin zero or one, and the final state can be both isoscalar and isovector. Thus the isospin properties of the final state contain information on the space-spin state in which the OZI violation occurred.

In the particular case of $\psi \to P\gamma$ decays, the $\pi^0\gamma$ state can only be produced by the transition (4.1b) with emission of an isovector photon after the OZI violation has occurred in the initial vector state. The $\eta\gamma$ and $\eta'\gamma$ states can be produced by either transition (4.1a) or (4.1b) with isoscalar photons emitted either before or after OZI violation. Experimentally the $\eta\gamma$ and $\eta'\gamma$ decays are much stronger than the $\pi^0\gamma$ decay,[36] by a factor of about 30. So OZI violation in the pseudoscalar state seems to be much stronger than in the vector state.

We can use this information to estimate the production of the pseudoscalar charmonium state η_c in pp collisions. Assuming that the difference between η_c production and J production is only in the OZI violating charmed pair creation, and that the difference between the strength of the violation in vector and pseudoscalar states is given by the argument of radiative decays above, we obtain

$$\frac{\sigma(pp \to \eta_c X)}{\sigma(pp \to JX)} \sim \left| \frac{A(q\bar{q} \to c\bar{c};\ J^P = 0^-)}{A(q\bar{q} \to c\bar{c};\ J^P = 1^-)} \right|^2 \sim \frac{BR(\psi \to \eta'\gamma)}{BR(\psi \to \pi^0\gamma)} \sim 30 \quad . \tag{4.2}$$

4.3 How to Look for New Quarkonium States

The charmonium experience shows that e^+e^- colliding beams provide a very effective means for discovering and studying the properties of vector mesons which are directly produced as s-channel resonances, and of other states produced by electromagnetic decays of these vector mesons. Hadronic beams can produce these vector states, but very little information about their properties are obtained in a simple way because of the enormous background. If the SPEAR and DESY results were not available to complement the information obtained from the Brookhaven experiment, we would know very little about the nature of the J particle, and there would be very little evidence that it is indeed a charmonium state.

Hadronic beams might provide additional information on the properties of other states not easily seen with e^+e^-, such as the pseudoscalars. So far the η_c has been seen only in one experiment at DESY and only in the $\gamma\gamma$ decay mode. There is interest in seeing the hadronic decay modes, and any ingenious method for seeing such decay modes with hadronic production would constitute a real breakthrough in x-onium spectroscopy. If the estimate (4.2) of the hadronic production cross section is reasonable, there may be some hope for detecting the η_c via the $\phi\phi$ decay mode after production in pp collisions. The figure of merit for this process can be estimated by comparison with the detection of the J in the e^+e^- decay mode.

$$\frac{F(\eta_c, \phi\phi)}{F(J, ee)} = \frac{\sigma(pp \to \eta_c X)}{\sigma(pp \to JX)} \cdot \frac{BR(\eta_c \to \phi\phi)}{BR(J \to ee)} \cdot \frac{(pp \to eeX)}{(pp \to \phi\phi X)} \cdot \qquad (4.3)$$

Since the decay $\eta_c \to \phi\phi$ is similar in nature to the decay $J/\psi \to \phi\eta$, we can assume

$$BR(\eta_c \to \phi\phi) \sim 2\,BR(J/\psi \to \eta\phi) \sim (1/35)\,BR(J \to ee) \quad , \qquad (4.4)$$

where we have introduced a factor 2 because only about 50% of the η wave function, the $s\bar{s}$ piece, contributes to the $\eta\phi$ decay mode of the J/ψ, and we have substituted the experimental values for the branching ratios. Combining Eqs. (4.2), (4.3) and (4.4) then gives

$$\frac{\sigma \cdot BR(pp \to \eta_c X \to \phi\phi X)}{\sigma \cdot BR(pp \to JX \to eeX)} \sim (30/35) \sim 1 \quad , \qquad (4.5a)$$

$$\frac{F(\eta_c,\ \phi\phi)}{F(J,\ ee)} \sim \frac{\sigma(pp \to eeX)}{\sigma(pp \to \phi\phi X)} \quad . \qquad (4.5b)$$

Thus if the $\phi\phi$ background is no more than the lepton pair background, it should be just as easy to see $\eta_c \to \phi\phi$ as it is to see $J/\psi \to$ lepton pairs.

Results from the double arm spectrometer experiment at Fermilab[31] showed no $\phi\phi$ events, while the same run observed about 100 events of $J/\psi \to \mu^+\mu^-$. This is still consistent with the result (4.5) of equal signal/noise and comparable signals for the two processes, because the spectrometer had a much lower acceptance for ϕ's than for muons. The absence of any $\phi\phi$ signal confirms that the background is low, and that any further experiments with increased sensitivity might see a small signal without appreciable background. Note that even 3 events for $\phi\phi$ at

2.8 GeV with no background would constitute serious supporting evidence for the existence of the η_c, whereas several hundred events in another decay mode against a background of thousands of events would be ambiguous.

Similar arguments would apply to the detection of higher x-onium pseudoscalars via the $\phi\phi$ decay mode. Note that x-onium pseudoscalars above 6 GeV would also have a $\psi\psi$ decay mode which might be detectable in a four lepton final state.

The $\pi A2$ decay mode of the η_c has also been suggested as a possible useful signature.[23] A detailed analysis of the hadronic decays of the η_c has been given by Quigg and Rosner.[37]

The recent beautiful experiment at DESY reported by Schopper[38] showing evidence for the F and F^* mesons[39] is an example of how choosing an appropriate signature minimizes background and gives serious evidence for these particles with only a few events. The signature in this case was three photons and a pion, with one photon having a low energy and the other two having the mass of the η.

A similar kind of signature might be used to find the η_c in the decay

$$\psi \to \gamma\eta_c \to \gamma\pi^{\pm}A2^{\mp} \to \gamma\pi^{+}\pi^{-}\eta \to \gamma\pi^{+}\pi^{-}\gamma\gamma \quad . \tag{4.6}$$

This would give three photons, one of 300 MeV (or less if the η_c is not at 2.8 GeV but higher) and the other two having the mass of the η, and two additional charged pions.

V. COLOR

5.1 Who Needs Color?

Many reasons have been proposed for introducing color, and not all of them are compatible. Color is needed by

1) People who like ordinary fermi statistics for quarks[40] and do not like baryon models with three spin-1/2 quarks in symmetric rather than in antisymmetric states.

2) People who like integral electric charge.[41]

3) People who believe Adler's argument for color,[42] based on the current-algebra-PCAC calculation of the decay $\pi^0 \to \gamma\gamma$. Adler's result is proportional to the sum of the squares of the charges of all elementary fermions in the theory. The numerical experimental value for the width of this decay agrees with predictions from a 3-color model and disagrees with models having no color degree of freedom.

4) People who want to push up the ratio $R \equiv e^+e^0 \to$ hadrons/ $e^+e^- \to \mu^+\mu^-$, whose present experimental value exceeds the prediction from the simple quark model.[43] The addition of new internal degrees of freedom pushes this ratio up, just as in $\pi^0 \to \gamma\gamma$.

5) People who worry about the saturation of hadrons at the quark-antiquark and three-quark levels and want a model which explains why states like $qq\bar{q}$ and $4q\bar{q}$ are not found. Colored models provide a natural description of this saturation.[8,44,45]

6) People who like non-Abelian gauge theories and quark confinement.[46] However these people require the color symmetry to be an exact symmetry of nature not broken by weak or electromagnetic interactions. They are unable to incorporate integrally-charged quarks into this framework and must have fractional charges.

7) People who like to explain the $\Delta I = \frac{1}{2}$ rule by a Fierz transformation[47] of the four fermion V-A interaction.

The three-triplet model, originally suggested to allow the three quarks in a baryon to have a symmetric wave function without violating Fermi statistics,[8,40] is now called a model with "red, white and blue" quarks. For those who find this American chauvinism distasteful, we recommend the "Equal Opportunity Quark Model" (EOQM) which has equal representation of black, white and yellow quarks.

For three colors and n flavors a symmetry group SU(3n) can be defined which treats all quarks on an equal footing. This has a subgroup $SU(3)_c \times SU(n)_f$. There is no evidence for the rich hadron spectrum corresponding to the presence of states classified in nontrivial representations of $SU(3)_c$. The observed hadrons are assumed to belong to the trivial singlet representation of $SU(3)_c$ and "color excitations" of higher representations are either postulated not to exist or are assumed to have a high mass. The color-excited states contain exactly the same colored quarks as the observed hadrons, they differ only in having a different permutation symmetry in the space of the colors. This is in contrast to states with quarks of new flavors, which can be pushed up in mass by simply postulating a higher mass for the new flavored charmed quarks. Color excitations can be pushed up only by having the interaction between quarks depend on the permutation symmetry in color-space since different colored quarks all have the same mass. Interactions which confine quarks have this property.

Models with quark confinement have an interaction between quarks which increases with distance[48] so that an infinite energy is required to separate a pair. The simplest example of the confinement is the Coulomb interaction in a 1 + 1 dimensional world.[49] A quark-antiquark pair behave like a pair of condenser plates in this world, and the force between them remains constant as they are separated. The potential varies linearly with distance and infinite energy is required to separate the pair. Before this happens, enough energy is present in the field to allow a new pair to be created. The lines of force connecting the two original quarks are broken by the new pair, and the members of the new pair couple to the corresponding members of the old pair to make two separated bound states with no force between them.

In three spatial dimensions, quarks are not condenser plates, and the lines of force connecting a quark-antiquark pair can spread out in the other two dimensions. In ordinary QED this gives the conventional $(1/r)$ Coulomb potential which does not require infinite energy to achieve a separation. In QCD it is hoped that the non-Abelian character of the gauge theory produces "infra-red slavery" which perhaps confines the lines of force to a tube and makes the system behave like a one-dimensional system. This gives the linear potential conventionally used for confinement. But so far there has been no real proof that the gauge theories really predict quark confinement or linear potentials. The potential may have a different form, and may not confine. A potential weaker than kr like the logarithmic potential[50] would still give confinement. A potential which requires a very large energy (e.g. hundreds of TeV) to separate quarks would not permanently confine quarks, but would be equivalent to confinement for experiments in the 1 TeV energy region.

In our discussions we consider the possible existence of free quarks with a very heavy mass. This then includes the case of quark confinement as the limit in which the free quark mass goes to infinity. Note that this free quark mass is not the same as the quark mass used in model calculations. The difference is easily seen in a model where lines of force of the "color field" join a quark-antiquark pair. As the pair is separated, the lines of force cover a greater volume and more energy is present in the field. If the quarks can actually be separated with a finite high energy, this energy remains in the field around the two free quarks, and covers a comparatively large volume because of the long range of the color force. The mass of the free quark therefore comes from the strong long range color field around it. When a quark is bound in a hadron, its color field is confined to the volume of the hadron and contains much less energy. Thus the mass of the bound quark is very much less than the mass of the free quark.

In colored quark models, the color may or may not be directly observable. In models where color is not observable, all quarks which differ only in color and otherwise have the same quantum numbers must have the same properties. In other models quarks of different colors have different observable properties, e.g., different electric charges. This possibility has been used to construct models with quarks of integral electric charges. Such integrally-charged colored quarks cannot satisfy the Gell-Mann-Nishijima relation and must have nonzero eigenvalues of a new additive quantum number which appears in the modified Gell-Mann-Nishijima formula. The electromagnetic current then has a component which is an $SU(3)_f$ singlet and which is not a singlet in $SU(3)_c$. There is a definite conflict between the use of integral charges and the use of color as an exact symmetry of nature in a non-Abelian gauge theory. If quarks of different colors have different electric charges, then the electromagnetic interaction breaks the color symmetry and it is not exact. Thus there are two incompatible approaches to color: 1) Quark confinement with fractionally charged quarks; 2) Quark liberation with integral charges. The truth might well be in between.

5.2 The Deuteron World

Some insight into the colored quark models is given by the analogy of a world in which all low-lying nuclear states are made of deuterons and have isospin zero, free nucleons have not yet been seen and experiment has not yet attained energies higher than the deuteron binding energy or the symmetry energy required to excite the first $I = 1$ states. In this isoscalar world where all observed states have isospin zero the isovector component of the electromagnetic current would not be observed since it has vanishing matrix elements between isoscalar states. The deuteron energy level spectrum (something like that of a diatomic molecule) would indicate that the deuteron was a two-body system, but there would be no way to distinguish between the neutron and the proton. The deuteron would thus appear to be composed of two identical objects which might be called nucleons. Since the deuteron has electric charge +1, the nucleon would be assumed to have electric charge +1/2. Furthermore, the nucleon would be observed to have spin 1/2 and be expected to satisfy Fermi statistics. However, the ground state of the deuteron and all other observed states would be found to be symmetric in space and spin. Thus, the nucleon would appear to be a spin 1/2 particle with fractional electric charge and peculiar statistics.

Some daring theorists might propose the existence of a hidden degree of freedom expressed by having nucleons of two different colors. There would be a hidden SU(2) symmetry (which might be called isospin) to transform between the two nucleon states of different colors. All the observed low-lying states would be singlets in this new color (or isospin) SU(2). Since the color singlet state of the two-particle system is antisymmetric in the color degree of freedom, the Pauli principle requires the wave function to be symmetric in space and spin, thus solving the statistics problem.

The direct analog of this deuteron problem in hadron quark models is the quark model for the Ω^-. In the conventional quark model, the Ω^- consists of three identical strange quarks (called λ-quarks by some people and s-quarks by others), with their spins of 1/2 coupled symmetrically to spin 3/2. Since the electric charge of the Ω^- is -1, the strange quark is required to have charge -1/3, and it is also required to have peculiar statistics because the system of three identical particles has a symmetric wave function in all known degrees of freedom. Some daring theorists have therefore proposed the existence of a hidden degree of freedom expressed by having strange quarks of three different colors,[7] and a hidden SU(3) symmetry to transform between the three strange quark states of different colors. All the observed low-lying states are singlets in this $SU(3)_{color}$ group. Since the color-singlet state of the three-particle system is antisymmetric in the color degree of freedom, the Pauli principle requires the wave function to be symmetric in the other degrees of freedom, in agreement with experiment and ordinary Fermi statistics. It is also possible to give these colored strange quarks different integral electric charges, one with charge -1 and two neutrals, by analog with the nucleons in the deuteron. However, as we are concerned primarily with strong interactions, we need not choose between models having different electric charges for colored quarks.

We have chosen the example of the Ω^- for this discussion to simplify the treatment of the flavor degree of freedom by considering only strange quarks. When all flavors are considered, there are three colors for each flavor, and $3n_f$ quarks altogether. There are two SU(n) groups, the flavor $SU(n)_f$ and the color SU(3), which are combined into the direct product $SU(n)_f \times SU(3)_{color}$.

5.3 The Whys of Quark Model Predictions of the Hadron Spectrum

Let us now consider some "whys" posed by one of the outstanding "successes" of the quark model, the prediction of the hadron spectrum. The empirical rule that all observed hadron bound states and resonances have the quantum numbers found in the three-quark and quark-antiquark systems is in remarkable agreement with experiment. Since no alternative explanation or description has been given for this striking regularity in the hadron spectrum, this rule may constitute evidence for taking quarks seriously. The quark model also predicts the energy level spectrum of the states constructed from the three-quark and quark-antiquark systems and observed experimentally as hadron resonances. These predictions also seem to be in reasonable agreement with experiment, but pose additional questions.

Why is the observed baryon spectrum fit only by the symmetric quark model[51] which restricts the allowed states of the three-quark system to those being totally symmetric under permutations in the known degrees of freedom rather than totally antisymmetric, as one expects for fermions? This can be explained by assuming that quarks obey peculiar statistics, or that there is a hidden degree of freedom sometimes called "color." But this requires the additional ansatz that all observed hadrons are color singlets. Why and why only 3q and $q\bar{q}$? Why not other configurations? Why does the low-lying meson spectrum show all the states "predicted by the quark model" without any supplementary conditions and with no allowed states conspicuously absent?

There is an inconsistency between the observation of bound states in all channels for $q\bar{q}$ scattering and the absence of bound states with quantum numbers of $2q\bar{q}$ and $3q\bar{q}$. If the quark-antiquark interaction is attractive in all possible channels, as indicated by the presence of bound states, an antiquark should be attracted by any composite state containing only quarks, like a diquark or a baryon, to make a bound state with peculiar quantum numbers that have not been observed.

In our discussion, we assume that free quarks are very heavy, and we consider only effects on the mass scale of the quark mass. All observed particles have zero mass on this scale. The observed hadron spectrum is a "fine structure" which we are unable to resolve in this approximation. This is a reasonable approach, since as long as we are not treating spin in detail, we are unable to distinguish between a pion and a ρ meson, and are neglecting mass splittings of the order of the $\rho - \pi$ mass difference. We therefore are only able to discuss whether a particle has "zero mass" and appears as an observed hadron, or whether it has a mass of the order of the quark mass and should not have been observed.

The question why only 3q and $q\bar{q}$ can be stated more precisely in terms of the following three whys:

1. The triality why. With attractive interactions between quarks and antiquarks, why are three quarks and an antiquark not bound more strongly than a baryon or two quarks and an antiquark bound more strongly than a meson? Note that we are not asking about four quarks vs. three quarks. Symmetry restrictions such as the Pauli principle with colored quarks can prevent the construction of a four quark state which is totally symmetric in space, spin and unitary spin. But there is no Pauli principle which prevents an antiquark from being added to a system of three quarks in all possible states. Thus if each quark in the baryon attracts the antiquark, some additional mechanism must be found to prevent it from being bound to the quark system.

2. The exotics why. Even assuming some mysterious symmetry principle which prevents fractionally charged states from being seen, why are there no strongly bound states of zero triality, like those of two quarks and two antiquarks or four quarks and one antiquark? Note

that we are not discussing the Rosner "baryonium" exotics which are baryon-antibaryon resonances decoupled from the two meson system or Jaffe exotics bound by spin forces. We are discussing states like an I = 2 dipion resonance or bound state with a mass near the mass of two pions. If the quarks and antiquarks in two pions attract one another, why is there no net attraction between two positive pions to produce a bound state or a resonance very near threshold?

3. The diquark or meson-baryon why. Why is the quark-quark interaction just enough weaker than the quark-antiquark interaction so that diquarks near the meson mass are not observed, but three-quark systems have masses comparable to those of mesons? Vector gluons which are popular these days would bind the quark-antiquark system, but the force they provide between identical quarks is repulsive. Scalar or other gluons which are even under charge conjugation bind both the quark-antiquark and diquark systems equally. If the quark mass is very heavy, the single quark-antiquark interaction in a meson must cancel two quark masses, while the three quark-quark interactions in the baryon must cancel three quark masses. This suggests that the quark-quark interaction is exactly half the strength of the quark-antiquark interaction.[52] Such a result can be achieved by a suitable mixture of vector and scalar interactions, but it is not very satisfying to obtain such a simple fundamental property of hadrons by a model which fits it with an adjustable parameter.

In all of this discussion, we are considering one-particle states, with the assumption that multiparticle states exist which contain separated particles each having the properties we are trying to explain. Multiparticle states pose additional problems. The allowed spectrum for multiparticle states is not specified by a set of allowed quantum numbers, but by the condition that their constituent particles individually have allowed quantum numbers. Thus the whys cannot be answered by general symmetry principles which apply to all states. The triality why is not answered by a symmetry principle forbidding all states which do not have zero triality, because multiparticle states of zero triality must also be forbidden if they are made of particles which individually have nonzero triality. Similarly, the exotics why is not answered by a symmetry principle forbidding all states with exotic quantum numbers because multiparticle exotic states made from nonexotic particles are allowed. Thus any treatment which attempts to answer these whys must discuss both single-particle and multiparticle states, and must consider the space-time properties which distinguish between them. Algebraic arguments involving only internal symmetry groups cannot be sufficient.

Our three whys involve only the strong interactions which do not depend upon the couplings of quarks to the electromagnetic and weak currents. The following discussion thus applies to both fractionally charged and integrally charged models.

5.4 The Colored Gluon Model

We now examine the three whys. In the colored quark description of hadrons the restriction that only color singlet states are observed immediately solves the triality why since only states of zero triality can be color singlets. But requiring all low-lying states to be color singlets is thus equivalent to requiring all low-lying states to have zero triality; it merely replaces one ad hoc assumption with another. What is needed is some dynamical description in which the color singlets turn out to be the low-lying states in a natural way. To attack this problem we return to the fictitious deuteron world where all low-lying states are isoscalar and which is the analog of the colored quark description of hadrons. We follow the treatment of ref. 44.

At first this isoscalar deuteron world seems very artificial. Why should all states with $I = 0$ be pushed down and all states with $I \neq 0$ be pushed up out of sight? But there turns out to be a very natural nuclear interaction which creates exactly this isoscalar deuteron world; namely nuclear two-body forces dominated by a very strong Yukawa interaction provided by ρ exchange. This interaction is attractive for isoscalar states and repulsive for isovector states, <u>in both nucleon-nucleon and nucleon-antinucleon systems</u>. It thus binds only isoscalar states. The ρ-exchange interaction between particles i and j can be expressed in the form

$$v_{ij} = V \vec{t}_i \cdot \vec{t}_j \quad , \tag{5.1a}$$

where $\vec{t}_i$ is the isospin of particle i and V contains the dependence on all other degrees of freedom except isospin. If we neglect these other degrees of freedom we can write for any n-particle system containing antinucleons and nucleons,

$$V(n) = \frac{1}{2}\sum_{i \neq j} v_{ij} = \frac{1}{2}\left[\sum_{\substack{\text{all}\\ ij}} v\vec{t}_i \cdot \vec{t}_j - \sum_{i} V\vec{t}_i \cdot \vec{t}_i\right] = \frac{V}{2}[I(I+1) - nt(t+1)] \quad (5.1$$

where I is the total isospin of the system and t is the isospin of one particle; i.e., 1/2 for a nucleon.

The interaction (5.1b) is seen to be repulsive for the two-body system with I = 1 and attractive for all isoscalar states. A pair of particles bound in the I=0 state is thus seen to behave like a neutral atom; it does not attract additional particles. Since the pair is "spherically symmetric" in isospace, a third particle brought near the pair sees each of the other particles with random isospin orientation, and its interaction with any member of the pair is described by the average of (5.1a) over a statistical mixture which is 3/4 isovector and 1/2 isoscalar. This average is exactly zero.

The neutral atom analogy is very appropriate for the description of the observed properties of hadrons. The forces between neutral atoms are not exactly zero, but are much weaker than the forces which bind the atom itself. These interatomic forces produce molecules which are much more weakly bound than atoms. Similarly the forces between hadrons do not vanish but are much weaker than the forces which bind the hadron itself. These interhadronic forces produce complex nuclei which are much more weakly bound than hadrons. In the approximation where we neglect energies much smaller than the quark mass these "molecular" effects are safely neglected.

We now generalize this picture for the colored quark description of hadrons. If there are n colors, the interaction (5.1) must be generalized from SU(2) to SU(n). The quark-antiquark system then still saturates at one pair, but the multiquark system can be seen to saturate at n quarks. A quark-antiquark system which is a singlet in SU(n) exists for all values of n. However, the existence of a singlet in the two-quark system is an accident which occurs only in SU(2) and is not generalizable to SU(n). However the I = 0 two-quark state is also characterized as antisymmetric under permutation of the two particles. This antisymmetry is generalized easily to SU(n) where totally antisymmetric states exist for a maximum of n particles, and the n particle antisymmetric state is a singlet in SU(n).

We now construct the analog of the interaction (5.1b) for a model with three triplets of different colors. Then the Yukawa interaction produced by the exchange of an octet of "colored gluons" has the form analogous to (5.1). For an n-particle system containing both quarks and antiquarks,

$$U(n) = \frac{1}{2} \sum_{i \neq j} u_{ij} \sum_{\sigma} g_{i\sigma} g_{j\sigma} \tag{5.2}$$

where u_{ij} depends on all the noncolor variables of particles i and j and $g_{i\sigma}$ (σ = 1, ..., 8) denote the eight generators of $SU(3)_{color}$ acting on a single quark or antiquark i.

If the dependence of u_{ij} on the individual particles i and j is neglected, the interaction energy of an n-particle system can be calculated by the same trick used in Eq. (5.1b) to give

$$V(n) = \frac{u}{2}(C - nc) \tag{5.3a}$$

where u is the expectation value of u_{ij}, integrated over the noncolor variables, C is the eigenvalue of the Casimir operator for $SU(3)_{color}$ for the n-particle system and c = 4/3 is the eigenvalue for a single quark or antiquark. These eigenvalues are directly analogous to the SU(2) Casimir operator eigenvalues I(I + 1) and t(t + 1) in Eq. (5.1b).

In the approximation where all energies small compared to the quark mass M_q are neglected, the interaction (5.3a) gives the mass formula

$$M(n) = nM_q + V(n) = n(M_q - \frac{cu}{2}) + Cu/2 \quad . \tag{5.3b}$$

The interaction (5.2) and the mass formula (5.3b) were first proposed by Nambu,[41] and the saturation properties of the interaction were considered by Greenberg and Zwanziger.[53] However, the remarkable properties of this interaction as demonstrated above in the simplified example of the analogous deuteron world have received little attention.

5.5 Answers to the Triality and Meson-Baryon Whys

The formula (5.3b) can test the triality why or the meson-baryon why by showing whether observable "zero mass" hadron states exist for a given number of quarks and antiquarks. However, it cannot test the exotics why, since it gives no information about the spatial properties of the states. It cannot distinguish between one-particle states and multiparticle scattering states and all zero-triality exotic states are allowed as multiparticle states.

Since $\bar{\bar{C}}$ is positive definite and has the eigenvalue zero only for a singlet[45] in $SU(3)_{color}$, and $u \geq 0$ as is evident from the two-body system, the state of the n-particle system with the strongest attractive interaction is a color singlet. Since the interaction is a linear function of n all such singlet states have zero mass if $cu/2 = M_q$. For this case

$$M(n) = (C/c)M_q \quad \text{if } cu/2 = M_q \quad . \tag{5.3c}$$

The model thus gives observable hadron states for all quark and antiquark configurations for which C = 0 states exist. Since C = 0 states exist only for configurations of triality zero, this answers the triality why.

The meson-baryon why is also answered by this interaction, since zero mass is attained both in two-body and three-body systems. To obtain C = 0, the two-body system must be a quark-antiquark pair, while the three-body system must be a three quark state, totally antisymmetric in color space. The approximation of neglecting the dependence of u_{ij} on i and j is justified in these two cases since there is only one pair in the two-body system, and a totally antisymmetric function has the same wave function for all pairs. The values[45] of the interaction parameter C-nc and the mass parameter C/c are listed in Table 5.1 for all states of the two-body system. These show that the quark-quark interaction in the baryon is exactly half of the quark-antiquark interaction in the meson, as required for the meson-baryon puzzle. The diquark mass is thus equal to one quark mass, since its interaction only cancels the mass of one of the two quarks.

Table 5.1 Values of the Interaction and Mass Parameters C-nc and C/c

System	$SU(3)_{color}$	Representation	C	C-nc	C/c
quark-quark	triplet	(antisymmetric)	4/3	-4/3	1
quark-quark	sextet	(symmetric)	10/3	+2/3	5/2
quark-antiquark	singlet		0	-8/3	0
quark-antiquark	octet		3	+1/3	9/4

The interaction averaged over all quark-quark states is seen to be zero and similarly for all quark-antiquark states. An antiquark or quark added to a meson or baryon thus has a zero net interaction, as there can be no color correlations between particles in a singlet state and an external particle, and each pair feels the average interaction over all color states. This suggests that the exotics puzzle is also answered, and that the states of zero mass obtained from the interaction (5.2) for exotic quantum numbers are multiparticle continuum states rather than bound states or resonances.

5.6 The Exotics Why--Spatial Properties of Wave Functions

To examine the exotics why in more detail we consider the spatial dependence of the interaction (5.2) for the specific case of the two-quark-two-antiquark system, with an interaction u_{ij} depending only on the positions of the particles and not on momenta, spin and unitary spin. In the representation with the corrdinates $\vec{r}_i$ of the four particles diagonal, the interactions u_{ij} are also diagonal and can be treated as c-numbers. In this representation the interaction (5.2) is a 2×2 matrix in color space as there are two independent couplings for four particles to a color singlet. We diagonalize this 2×2 matrix to obtain two functions of the coordinates $\vec{r}_i$ which describe the spatial dependence of the interaction in its two color eigenstates.

It is convenient to choose a nonorthogonal basis, related by permutations, which displays quark-antiquark couplings to C = 0,

$$|\alpha\rangle \equiv |(13)_1(24)_1\rangle \qquad (5.4a)$$

$$|\beta\rangle \equiv |(14)_1(23)_1\rangle \qquad (5.4b)$$

where particles 1 and 2 are quarks, 3 and 4 are antiquarks and $(ij)_1$ denotes that particles i and j are coupled to C = 0. Several useful identities follow from the properties of the C = 0 two-particle state.

$$<\alpha|\beta> = 1/3 \tag{5.5a}$$

$$\sum_\sigma g_{1\sigma}g_{3\sigma}|\alpha> = \sum_\sigma g_{2\sigma}g_{4\sigma}|\alpha> = -(8/3)|\alpha> \tag{5.5b}$$

$$\sum_\sigma g_{1\sigma}g_{4\sigma}|\beta> = \sum_\sigma g_{2\sigma}g_{3\sigma}|\beta> = -(8/3)|\beta> \tag{5.5c}$$

$$(g_{1\sigma} + g_{3\sigma})|\alpha> = (g_{2\sigma} + g_{4\sigma})|\alpha> = (g_{1\sigma} + g_{4\sigma})|\beta> = (g_{2\sigma} + g_{3\sigma})|\beta> = 0 \tag{5.5d}$$

$$<\alpha|g_{1\sigma}g_{4\sigma}|\alpha> = <\beta|g_{1\sigma}g_{3\sigma}|\beta> = 0 \tag{5.5e}$$

$$<\alpha|g_{1\sigma}g_{4\sigma}|\beta> = <\beta|g_{1\sigma}g_{3\sigma}|\alpha> = -(8/3)<\alpha|\beta> = -8/9 \tag{5.5f}$$

By operating with the interaction (5.2) on the wave functions (5.4) and eliminating the color variables with the aid of the identities (5.5) we obtain

$$-3U|\alpha> = (8u_\alpha - u_\beta + u_q)|\alpha> + 3(u_\beta - u_q)|\beta> \tag{5.6a}$$

and

$$-3U|\beta> = 3(u_\alpha - u_q)|\alpha> + (8u_\beta - u_\alpha + u_q)|\beta> \tag{5.6b}$$

where

$$u_\alpha = u_{13} + u_{24}; \quad u_\beta = u_{14} + u_{23}; \quad u_q = u_{12} + u_{34} \quad . \tag{5.7}$$

Solving the secular equation for Eqs. (5.6) gives the eigenvalues for U,

$$U' = -(7/6)(u_\alpha + u_\beta) - (1/3)u_q \pm (1/2)\sqrt{8(u_\alpha - u_\beta)^2 + (u_\alpha + u_\beta - 2u_q)^2} \quad . \tag{5.8}$$

If u_{ij} is a finite range potential which vanishes at large distances, the eigenvalues (5.8) reduce to those for two independent two-particle clusters for all values of the coordinates $\vec{r}_i$ which correspond to two pairs separated by a distance greater than the range of the potential. The case $u_\beta = u_q = 0$ describes such a separation between the pairs of particles (13) and (24). The corresponding eigenvalues from Eq. (5.8) are $U' = -(8/3)u_\alpha$ and $U' = +(2/3)u_\alpha$ exactly those of Table 5.1 for two separated quark-antiquark pairs in the singlet and octet states. The case $u_\alpha = u_\beta = 0$ describes separated pairs of like particles (12) and (34) and has eigenvalues $U' = -(3/4)u_q$ and $U' = +(2/3)u_q$ exactly those of Table 5.1 for two separated quark-quark and antiquark-antiquark systems in the triplet and sextet states.

To test the exotics puzzle we look for coordinate configurations where four-particle correlations may give stronger binding than in two noninteracting clusters. Since u_α and u_β appear symmetrically in (5.8), we need only consider values of $u_\beta \leq u_\alpha$. For any value of u_α the value of $u_\beta \leq u_\alpha$ which minimizes the interaction (5.8) is $u_\beta = u_\alpha$ with the negative sign for the square root. This gives

$$U' = -(8/3)u_\alpha - (2/3)(u_\alpha - u_q) \quad . \tag{5.9}$$

This expression is minimized by choosing the minimum values of u_q consistent with a given value of u_α. For monotonically decreasing potentials this is achieved by placing the four particles at the corners of a square with the like particles at opposite diagonals.

For a square well potential the particles can be arranged in a square with the diagonal greater than the range of the forces and the sides less than the range. This configuration has $u_q = 0$ and forms a stable four-particle state with a binding 25% greater than that of two quark-antiquark pairs. However, the sharp edge of the square well is essential for this binding and does not seem reasonable physically. For smooth potentials without sharp edges such as Coulomb, linear, Gaussian, Yukawa or harmonic oscillator potentials Eq. (5.9) shows that such a four-particle cluster is less strongly bound than two noninteracting quark-antiquark pairs, and the system simply breaks up into two clusters. This leads to a description in which all states having exotic quantum numbers are just scattering states of particles which individually have nonexotic quantum numbers, and answers the exotics why.

The presently accepted colored quark model with forces from exchange of an octet of colored gluons provides a saturation mechanism in which the $q\bar{q}$ and 3q states behave like neutral atoms.[41, 44, 45, 52] Different parts of the bound state wave function attract and repel an external particle and the net force exactly cancels. Thus theory and experiment now agree on the absence of naive exotics. But the possibility exists of higher exotics. Molecular-type exotics in which attraction results from spatial polarization of one hadron by another have been considered, but the results (5.9) indicate that the force is insufficient to produce binding. Rosner[54] has postulated the existence of exotics from the point of view of finite energy sum rules and duality. This approach has been carried further by other theorists and experiments have been suggested in a search for exotics by baryon exchange processes.

So far there is no evidence for exotic mesons with masses below 2 GeV. This has been taken as evidence against the $qq\overline{qq}$ configuration for low-lying states. Although $qq\overline{qq}$ states without exotic quantum numbers also exist, these were not taken seriously as possible configuations for the known states, because there was no good theoretical reason why such states should be present and their exotic partners should be absent. But now there seems to be evidence that the low-lying 0^{++} nonet is indeed such a $qq\overline{qq}$ state,[4] and there are new convincing theoretical reasons why only states with nonexotic quantum numbers are seen.[55]

5.7 A Simple Representation of Color Couplings

For a simplified version of how normal and exotic hadrons are constructed from coupling colored quarks together consider simple-minded vector couplings in a three dimensional color space. In the color SU(3), the quark is a complex vector in three-dimensional color space. If we simplify this description by considering real vectors, we use only the O(3) subgroup of SU(3) corresponding to real rotations and lose the distinction between quark and antiquark which are complex conjugates of one another. But enough of the basic physics remains to give an instructive pedagogical example. Let us therefore consider the quark as a vector Q in a three dimensional color space with red, blue, and green components denoted by

$$\vec{Q} \equiv (Q_R, Q_B, Q_G) \quad . \tag{5.10}$$

The color singlet meson state is the scalar product of quark and antiquark vectors

$$M = \vec{Q}_1 \cdot \overline{\vec{Q}}_2 = Q_R\overline{Q}_R + Q_B\overline{Q}_B + Q_G\overline{Q}_G \quad . \tag{5.11}$$

The color singlet baryon is the scalar product of three quark vectors.

$$B = \vec{Q}_1 \times \vec{Q}_2 \cdot \vec{Q}_3 = (Q_RQ_B - Q_BQ_R)Q_G + \text{cyclic permutations.} \tag{5.12}$$

Note that every quark pair in the baryon is in the antisymmetric diquark state which is a vector product of two quark vectors

$$\vec{D} = \vec{Q}_1 \times \vec{Q}_2; \quad D_G = Q_RQ_B - Q_BQ_R, \quad \text{etc.} \tag{5.13}$$

The antisymmetric diquark is seen to have the color quantum numbers of the antiquark. The baryon can thus be written as the scalar product of antisymmetric diquark vector and a quark vector

$$B = \vec{D} \cdot \vec{Q} \quad . \tag{5.14}$$

Let us now examine the states of a system containing two quarks denoted by Q_1 and Q_2 and two antiquarks denoted by $\overline{Q}_3$ and $\overline{Q}_4$. The four body system described by four vectors in the color space can be coupled to form a color singlet in several ways. For example, there is the two meson state formed by coupling two quark-antiquark pairs separately to color singlets

$$2M = (\vec{Q}_1 \cdot \vec{Q}_3)(\vec{Q}_2 \cdot \vec{Q}_4) \quad . \tag{5.15a}$$

There is also the state formed by coupling the two quarks and two antiquarks each to an antisymmetric vector and coupling the two vectors to a scalar

$$X = (\vec{Q}_1 \times \vec{Q}_2) \cdot (\overline{\vec{Q}}_3 \times \overline{\vec{Q}}_4) \quad . \tag{5.15b}$$

This state is a possible candidate for baryonium since it could be formed by annihilating a quark-antiquark pair in the baryon-antibaryon system without changing the states of the remaining quarks and antiquarks and requiring that the state remain a color singlet.

$$B\overline{B} \longleftrightarrow X \tag{5.16a}$$

$$(\vec{Q}_1 \times \vec{Q}_2 \cdot \vec{Q}_5)(\overline{\vec{Q}}_3 \times \overline{\vec{Q}}_4 \cdot \overline{\vec{Q}}_6) \longleftrightarrow (\vec{Q}_1 \times \vec{Q}_2) \cdot (\overline{\vec{Q}}_3 \times \overline{\vec{Q}}_4) \quad . \tag{5.16b}$$

One might imagine the situation where the baryonium state X created in some reaction would prefer to decay into the baryon-antibaryon state via the transition (5. 16) rather than to decay into two mesons by breaking up into two quark-antiquark pairs, because the latter transition involves changing the color couplings. In particular, this situation could arise if there is an appreciable spacial separation between the diquark and the antidiquark.

One can picture lines of force joining the quarks and antiquarks by analogy with electrodynamics but with essential modifications following from the non-abelian character. A color singlet quark-antiquark pair would have lines of force originating on the quark and ending on the antiquark as shown in Fig. 5. 1a. Fig. 5. 1b shows the two-meson system described by Eq. (5. 15a) as two such pairs with lines of force joining the members of each pair but no lines of force connecting the two pairs. Fig. 5. 2 shows the baryon described by Eq. (5. 12) as three quarks at the vertices of a triangle with lines of force between them. Here the non-abelian nature has new effects, with lines joining each quark and its neighbor rather than quark and antiquark, and with one line acting as a source for another, since the lines themselves carry color. The coupling of the baryon described in Eq. (5. 14) as the product of a diquark and a quark is seen by cutting the baryon diagram to separate a quark from a diquark and noting that the lines of force going from the quark to the diquark look the same as the lines of force going from a quark to an antiquark in a meson. This again shows us that the diquark has the same quantum numbers in color as the antiquark. The diquark is thus an unsaturated system with lines of force joining the two quarks but other lines of force left out and searching for a partner as shown in Fig. 5.3. However the number of lines of force originating from such a diquark are not twice the number originating from a quark but only the same as the number originating from a quark. Here we see again the essential difference between non-abelian and abelian vector theories. The lines of force for the electron-positron system are very much like the lines of force for quark-antiquark system. But the lines of force for the antisymmetrized diquark system are very different from the lines of force in the two electron system where there are no lines joining the two electrons and the number of lines which are unsaturated and looking for partners is exactly twice the number from one electron.

Let us now examine the lines of force in the two configurations (5.15a) and (5.15b) when the two quarks and two antiquarks are relatively close together in space, but the distance between the quark pair and the antiquark pair is much larger than the distance within the pairs. Such a situation could be produced by annihilating a quark-antiquark pair in an initial baryon-antibaryon state as shown in Fig. 5.4. We see that with the baryonium color coupling (5.15b) shown in Fig. 5.4b, the lines of force traversing the space between the quarks and the antiquarks are the same as the lines of force within a single quark-antiquark pair. However the two meson color coupling (5.15a) shown in Fig. 5.1b, has twice as many lines of force traversing this space. Thus it is plausible that a baryonium type state X created from a baryon-antibaryon system might prefer to decay by creating a quark-antiquark pair and breaking the lines of force to return to the configuration of Fig. 5.4a rather than changing the color couplings to the configuration (5.15a) shown in Fig. 5.1b which requires rearranging the lines of force to a state of higher energy for this particular spacial configuration. This argument is not intended to be rigorous but just to give an intuitive physical picture.

The two couplings (5.15a) and (5.15b) are not the most general couplings to construct a scalar from four vectors. Simple analysis shows that there are three independent couplings corresponding to coupling any two vectors to a scalar, vector, or tensor, coupling the other pair in the same way and coupling them both to a scalar. However when we return to the realistic case of complex vectors and SU(3), there are only two independent couplings. The two states (5.15a) and (5.15b) are linearly independent but not orthogonal and constitute a complete non-orthogonal basis for color singlet state of the two quark, two antiquark configuration.

The state orthogonal to the two meson state (5.15a), has the two quark-antiquark pairs (13) and (24) coupled to color octet states rather than color singlets and the two octets coupled to a color singlet. In our simplified model, with real vectors, this includes the two states obtained by coupling the two quark-antiquark pairs to vectors and tensors respectively. With complex vectors, only one linear combination of these two states is a color singlet.

Similarly, the color singlet state orthogonal to the baryonium state X is obtained by coupling the two quarks and two antiquarks to the symmetric sextet in SU(3) and coupling the two sextets to a singlet. In the O(3) subgroup of SU(3) corresponding to real vectors the symmetric sextet splits into two representations, a scalar and a symmetric tensor, and scalars under real rotations can be made either by taking the product of the two scalars or the scalar product of the two tensors. However only one linear combination of these two states is a color singlet in the SU(3) of complex vectors.

VI. COLOR SPIN (MAGNETIC) EXOTICS

6.1 Introduction

The question of exotic hadron states has been confused in the recent literature because some authors discover new things and confuse the public by giving them old names like molecules which really mean something else, while others rediscover old things and confuse the public by giving them new names like baryonium.

A more suitable analogy than a molecule for the 0^{++} states of two quarks and two antiquarks in the same spacial orbit is the α particle. The question of whether or not such bound four-quark states exist can be posed as follows: There are two analogs for the bound quark-antiquark meson state, the deuteron and positronium. If the meson is like the deuteron, then two mesons should form a bound four-quark system just as two deuterons bind together to form a much more strongly bound α particle. If the deuteron is like positronium, the forces saturate and the residual force between the two neutral systems is very small and does not produce a state more strongly bound than the original two particle states. From the experimental observation that there is no strongly bound doubly charged state of two positive pions, we conclude that the pion is more like positronium than like the deuteron.

However the positronium analogy is misleading because there is no bound state of three electrons while three quarks bind to make a baryon. The force between two positronium atoms is nearly zero because the repulsion between the electron pairs exactly cancels the attraction of the electron-positron pairs in the two positronium atoms. But in two positive pions the quark-quark force cannot be completely repulsive because the same quarks must have attractive forces to make baryons. Thus the quark-antiquark system has many of the features of positronium. But there is an essential new ingredient; namely non-abelian color and the color-exchange forces produced by the exchange of colored gluons.[8]

A red quark and a red antiquark can exchange a colored gluon and turn into a blue quark and a blue antiquark in the same way that a proton and an antiproton can turn into a neutron and an antineutron by exchanging charge or a charged meson. This does not occur in the abelian case, where an electron cannot change its other quantum numbers by emitting photons. The simplest example of a non-abelian interaction that we know is the model of nucleon-nucleon and nucleon-antinucleon forces produced by pion exchange or ρ exchange. In the ρ-exchange model of section 5.4 the nucleon-antinucleon interaction is attractive in the isoscalar state and repulsive in the isovector state. Thus there can be no bound state of a neutron and an antineutron or of a proton and an antiproton. The bound state is the isocolor eigenstate which is a linear combination of proton-antiproton and neutron-antineutron. This means that the proton-antiproton and neutron-antineutron states are continuously changing into one another by the exchange of charged ρ mesons.

This picture shows why states analogous to the hydrogen molecule are not easily constructed with non-abelian interactions. An attraction between two hydrogen atoms can be obtained by orienting the two states so that the proton in one is closer to the electron in the other than the two protons or two electrons are to one another. To see that this cannot be done in the model of two nucleon-antinucleon pairs bound by ρ-exchange, let us try to put two such nucleon-antinucleon bound pairs together so that a nucleon in one pair is much closer to an antinucleon from the other pair than any other pairs between the two bound states. The nucleon is changing rapidly from neutron to proton as it exchanges charged mesons with its partner and the antinucleon is also changing rapidly between antiproton and antineutron while it exchanges mesons with its partner. Thus the nucleon from one pair and the antinucleon from the other are part of the time in an isovector state where the interaction is repulsive and part of the time in an isoscalar state where the interaction is attractive. The net result with these isospin couplings is zero interaction because the attractions and repulsions exactly cancel as shown above in Eqs. (5.1 - 5.9). Attractive forces analogous to molecular forces between atoms cannot be obtained by introducing only spacial polarizations. Color couplings must also be changed and the result Eq. (5.9) is a much weaker force.

Thus the dominant forces binding quarks and antiquarks into hadrons saturate at the $q\bar{q}$ and 3q levels. The $qq\overline{qq}$ system behaves more like two positronium atoms than like an α particle. However one can ask whether residual forces much weaker than the color charge force might still produce binding of α-particle-like configurations. This would be analogous to having very strongly bound deuterons which bind together into comparatively weakly bound α particles.

In the early days of the quark model and SU(6) symmetry, it was not clear whether meson states with spin greater than one would be produced by adding more $q\bar{q}$ pairs to the qq system or by orbital excitation of a single $q\bar{q}$ pair.[56] The α-particle-like configurations of two quarks and two antiquarks were considered seriously and there were searches for states having the appropriate quantum numbers. States with two quarks and two antiquarks all in the lowest relative s-states have $J^P = 0^+, 1^+, 2^+$; exactly the same values obtained for a single quark-antiquark pair in a p-wave. Thus both models predict the same angular momentum and parity quantum numbers for the next set of excited states above the pseudoscalar and vector mesons. However two quarks and two antiquarks can give exotic isospin and strangeness quantum numbers not found in the $q\bar{q}$ system with orbital excitation. After several years of searches for exotics, more and more states of higher spins were found with non-exotic quantum numbers and none were found with exotics. The orbital excitation model gained in favor and the α-particle configurations were forgotten.[57]

Recently Jaffe has introduced a new idea.[55] He considers the binding of two quark-two antiquark states into α-particle-like configurations by the spin dependent force analogous to the magnetic or hyperfine interaction in atomic physics. In contrast to the atomic case where hyperfine splittings are very small compared to orbital splittings, the hyperfine splittings in hadron spectroscopy as indicated by the $\rho\pi$ and $N\Delta$ splittings are of the same order of magnitude as orbital splittings and could produce strong effects. Jaffe finds that the lowest lying states bound by these magnetic interactions should appear as 0^+ states with non-exotic values of isospin and hypercharge. This natural result of the model, obtained without any fudging or adjusting parameters, completely invalidates the argument that the failure to find low-lying states with exotic quantum numbers rules out α-particle-like configurations.

6.2 The Flavor Antisymmetry Principle

Jaffe[55] has suggested the existence of exotics bound by the "magnetic-type" spin dependent forces arising naturally in the colored-quark-gluon (QCD) models. The prediction rests on much more general ground than the specific M.I.T. bag model used in Jaffe's original derivation. The essential physical input is that the N-Δ mass difference is much larger than the binding energy of the deuteron:

$$M_\Delta - M_N \gg M_n + M_p - M_d \tag{6.1}$$

where n, p and d denote neutron, proton and deuteron, not quarks, and this equation shows that there are problems of ambiguities in both the pnλ and uds notations.

The physics of Eq. (6.1) is that the dominant spin-independent (color charge) forces which bind quarks into hadrons saturate at the $q\bar{q}$ and 3q states and the residual forces between color singlet hadrons is only of the order of 2 MeV like the deuteron binding energy. However, the spin dependent force responsible for the mass difference between the N and Δ is very much larger, of order 300 MeV. Thus if two hadrons are brought very close together so that the quarks in one can feel the interactions of the quarks in the other, there is only a very weak force if the wave functions of the individual hadrons are not changed. However, if the spins of the quarks are recoupled to optimize the spin dependent interactions between the quarks in different hadrons, binding energies of the order of 300 MeV are available and could give rise to bound exotics. In the quark-antiquark system, the ρ-π mass splitting shows that 600 MeV is gained by changing the spins from $S = 1$ to $S = 0$.

Jaffe has simply used the N-Δ and ρ-π mass splittings as input for the strength of the spin dependent interaction and calculated its effect in binding exotic configurations. Only one further ingredient is needed, the color dependence of the interaction. In color singlet $q\bar{q}$ and 3q systems, every $q\bar{q}$ pair is in a color singlet state and every qq pair is in the antisymmetric color triplet state. Exotic configurations, even if they are overall color singlets, can have some $q\bar{q}$ pairs in the color octet state and some qq pairs in the symmetric sextet state. The interactions in these states are not obtainable from observed masses, and are obtained by assuming that the color dependence of the interaction is that obtained from the spin-dependent part of the one-gluon exchange potential in QCD. Evidence supporting this interaction is the agreement with qualitative features of the low-lying hadron spectrum not obtained in any other way, in particular the sign of the N-Δ and Λ-Σ mass splittings.[58] With this form for the interaction, its contribution to the binding of exotic hadron states is easily calculated by the use of algebraic techniques.

One result of the algebraic derivation is simply expressed as the "flavor-antisymmetry principle."[59] <u>The binding force between two quarks of different flavors in the optimum color and spin state is</u>

<u>stronger than the binding force between two quarks of the same flavor.</u> Although the forces are assumed to be flavor-independent, their color and spin dependence appears as a flavor dependence because of the generalized Pauli principle. For maximum binding the state should be overall symmetric in color and spin together. Thus if the quarks are in the same orbit and therefore symmetric in space, they must be flavor antisymmetric. This is seen in the N-Δ example where the I = 1/2 state is lower than the I = 3/2 state even with isospin independent forces, because the Pauli principle requires the correlation between spin and isospin of (1/2, 1/2) and (3/2, 3/2) for a color singlet state.

The flavor antisymmetry principle requires the most strongly bound state of a system of quarks and antiquarks to have quarks and antiquarks separately in the most antisymmetric flavor state allowed by the quantum numbers. Thus for example the lowest state of the six quark system has the configuration (uuddss) with no more than two quarks of any one flavor.

The general question of dibaryon bound states and resonances as six quark systems has been considered by Jaffe,[50] with the prediction of a low-lying six quark state as a bound state or resonance of the $\Lambda\Lambda$ system. The exact values of the masses of these states calculated by Jaffe can be questioned because of uncertainties in parameters appearing in the bag model but certain qualitative features are reasonably clear. The spin-dependent force between quarks in the two baryons will be strongest in the $\Lambda\Lambda$ system because of the flavor-antisymmetry principle. The exact values of the masses depend not only on the strength of the spin-dependent interaction, but also on other effects not included in the model calculation and difficult to estimate. However, if these other effects do not depend strongly on flavor, dibaryon bound states or low-lying resonances are most likely to be found in the $\Lambda\Lambda$ system.

It is interesting to note that multiquark binding lies outside the conventional SU(6) classification of hadrons. In the SU(6) symmetry limit the nucleon and the Δ are degenerate and the color-magnetic forces responsible for multiquark binding are absent. The existence of magnetic multiquark exotics requires SU(6) symmetry breaking, and may be related to other SU(6)-breaking effects in addition to the mass differences. One possible effect is the finite neutron charge radius, which vanishes in the SU(6) symmetry limit. Carlitz et al.[61] have suggested that this results from the same spin-dependent interaction which gives rise to the mass splittings and have made a quantitative estimate which agrees with experiment. It is interesting to note that the sign of the neutron charge radius is seen immediately from the flavor antisymmetry principle. In the SU(6) symmetry limit the spatial separation between any quark pair in the neutron is the same as that of any other pair and there is no spatial charge distribution. Breaking SU(6) with the "flavor-antisymmetric" interaction provides a stronger attractive force between quarks of different flavors and distorts the SU(6) wave function to bring the ud pairs in the neutron closer together than the dd pair. Thus the negatively charged d quarks are farther out on the average than the odd u quark which likes to be closer to the differently flavored d quarks, and the charge distribution is negative at large radius and positive at smaller radius.

So far there is no experimental evidence for a strongly bound $\Lambda\Lambda$ state, and there is some evidence against it.[62] Hypernuclei with two Λ's have been observed,[63] and are bound by only about 5 MeV more than the binding of two single Λ's. A $\Lambda\Lambda$ bound state with a much stronger binding energy would be expected to be formed in such hypernuclei. The failure to observe this transition might be explained by selection rules or barrier penetration factors. But any such mechanism preventing formation of a bound state by two Λ's present in the same nucleus for a time equal to the Λ decay lifetime should produce even greater inhibition in any experiment where the two Λ's are produced in a strong interaction collision and are close together for a much shorter time. There may be many-body effects in the hypernucleus which invalidate this argument; e.g. repulsive cores in the Λ-nucleon interaction might prevent the two Λ's from coming too close together in the presence of a finite nucleon density. But except for such effects, the existence of the lightly bound $\Lambda\Lambda$ hypernuclei suggest that strongly bound $\Lambda\Lambda$ states are not easily produced even if they exist.

For the $qq\overline{qq}$ system flavor antisymmetry gives two very interesting qualitative predictions.[55,59]

1. The lowest states do not have exotic quantum numbers.
2. The lowest states which have both charm and strangeness include exotics.

These predictions are simply derived by noting that a four body system must have two bodies with the same flavor if there are only three flavors. Since the flavor-antisymmetry principle requires the flavors of the quark pair and of the antiquark pair to be different in the lowest states, the two bodies with the same flavor must be a quark-antiquark pair. The flavor quantum numbers of this pair cancel one another and the quantum numbers of the system are those of the remaining pair and therefore not exotic. Prediction 1 gives a natural explanation for the absence of low-lying states with exotic quantum numbers, while allowing low-lying four-quark states with nonexotic quantum numbers. Jaffe has called such states "cryptoexotic." Prediction 2 follows from the observation that the flavor antisymmetry principle is easily satisfied with exotic quantum numbers when there are four flavors. Thus exotic states with both charm and strangeness may be found in the same mass range as the lowest F and F^* mesons with both charm and strangeness.

We now examine some experimental implications of these two predictions.

6.3 Low-lying "Crypto-Exotics"

Experimental evidence seems to indicate that the lowest lying 0^{++} mesons are not the quark-antiquark p-states, as formerly believed, but are indeed four quark states, while the quark-antiquark 0^{++} states are up at higher mass together with the other p-wave excitations like the f^0 and A2 tensor mesons. It is significant that the lowest states predicted by the colored-gluon exchange model form precisely a nonet of 0^{++} states without exotic quantum numbers. Further experiments will tell whether these states are indeed four-quark states and will establish the existence of higher states.

The four-quark states constructed with flavor antisymmetry have very different properties from the quark-antiquark states with the same quantum numbers. An isovector non-exotic, for example, is required to have the quark constitution like $(us\overline{ds})$; it must have a strange quark-antiquark pair to avoid having two quarks or two antiquarks of the same flavor. Thus isovector four quark states will decay dominantly into modes containing strange quarks, KK, $\pi\phi$, etc. This is very different from the decays of conventional quark-antiquark isovector states, like the A2, which decay into nonstrange channels like $\rho\pi$ without any inhibition. This property is masked in the 0^{++} isovector, the δ, because it is below the $\rho\pi$ and the $K\overline{K}$ thresholds and its dominant decay mode $\eta\pi$ is ambiguous because of mixing in the η of both strange and nonstrange components. But striking features in decay rates should be seen in the first four-quark isovector state which is above the $\rho\pi$ and the $K\overline{K}$ thresholds. An unusual decay pattern is seen for the tensor meson T^{s}_{π} with the quantum numbers of the A2 but which does not decay into $\rho\pi$ but rather into $K\overline{K}$, $K\overline{K}^{*}$ and $\eta\pi$ and for the axial vector meson A^{s}_{π} with the quantum numbers of the B, but with the $\phi\pi$ decay dominant and $\omega\pi$ forbidden. The $\phi\pi$ decay mode is particularly interesting, since it is forbidden for all normal quark-antiquark mesons by the OZI rule, while perfectly allowed for four quark states. Thus a search for $\phi\pi$ resonances might be an interesting way to find four quark mesons.

The isovector non-exotic has a degenerate isoscalar companion formed by coupling the nonstrange quark-antiquark pair to isospin zero. This isoscalar state will also decay dominantly into modes containing strange quarks. This contrasts sharply with the behavior of the degenerate isoscalar isovector doublets of the quark-antiquark configuration like $\rho\omega$ and fA2 where both states are coupled more strongly to nonstrange than to strange channels, and another isoscalar state like the ϕ and the f' not degenerate with the isovector is coupled dominantly to strange channels. Thus the observation that the S^{*}, the isoscalar scalar meson nearly degenerate with the δ, couples dominantly to kaons supports the classification as a four quark state.[55]

The observation that the δ and the S^{*} are scalar mesons lying below the ϕ which is the lowest vector meson in the $(s\bar{s})$ configuration has interesting implications for the new particle spectrum. If we assume the charm-strange analogy and replace all strange quarks in the δ, S^* and ϕ by charmed quarks, we predict the existence of isoscalar and isovector scalar charmonium states denoted by δ_c and S^*_c with configurations like ($u\bar{u}$ $c\bar{c}$) which should lie below the lowest ($c\bar{c}$) vector meson state, namely the J/ψ. If we replace only one strange quark in the δ, S^* and ϕ by charmed quarks, we predict the existence of exotic charmed-strange states with configurations ($u\bar{d}c\bar{s}$) which should lie below the lowest ($c\bar{s}$) vector meson state, namely the F^*. We now consider these possibilities in more detail.

6.4 Charm-Strange Exotics

The four-quark states with four different flavors and the same color-spin couplings as the low-lying 0^{++} nonet constitute a set of charmed-strange scalar mesons which are expected to lie in the same mass range as the two-quark charmed-strange F mesons. These include exotic states whose quantum numbers differ from those of the F by having either the wrong sign of strangeness or the wrong isospin. The two types of states are denoted by $\tilde{F}_I$ (udcs, etc. - wrong isospin) and $\tilde{F}_S$ (udcs, etc. - wrong sign of strangeness). The "crypto-exotic" ($u\bar{u}c\bar{s}$) four-quark state with the same quantum numbers as the F is denoted by F_x. The $\tilde{F}_I$ can be considered as an $F\pi$ or DK resonance or bound state, the $\tilde{F}_S$ as a DK resonance or bound state, and the F_x as an excited F coupled to the DK channel. One way to see the relation of these exotics to the low-lying 0^+ nonet is to note that changing a charmed quark to a strange quark in the $\tilde{F}_I$ and F_X gives a state in the 0^+ nonet, while $\tilde{F}_S$ has no such charm-strange analog state. Rough estimates of their masses are near the DK threshold. If the $\tilde{F}_S$ and F_X are below the DK threshold, as appears likely, they would be stable against strong decays and decay only weakly or electromagnetically.

Table 6.1 lists these states with their quark structure, quantum numbers, dominant strongly coupled channels, possible weak and EM decay modes and their "charm-strange analog" states in the light quark spectrum, obtained by changing the charmed quark to a strange quark.

One way to see how spin-dependent forces can bind a charmed-strange exotic is by examining such an exotic configuration created by bringing together a D^+ and a K^- meson. The spin-dependent force between the $\bar{d}$ antiquark in the D^+ and the s quark in the K^- can be made stronger by recoupling the spins. In the D^+-K^- system these two spins are completely uncorrelated since both the D^+ and K^- have spin 0 and are spherically symmetric. Thus the $\bar{d}$ s system is a statistical mixture of triplet and singlet spin states, 75% S = 1 like the K^* and 25% S = 0 like the K. Modifying the wave function to give the spin coupling of the $\bar{d}$s system a larger S = 0 component produces additional binding on the mass scale of the 400 MeV K-K^* mass difference. A wave function 75% S = 0 and 25% S = 1 instead of vice versa would gain 200 MeV in binding. Since such recoupling of the $\bar{d}$ and s quark spins changes the spin couplings of each of these with the other quarks, the lowest configuration must

TABLE 6.1 PROPERTIES OF CHARMED-STRANGE FOUR-QUARK MESONS

State	Quark Structure	(I,S,C)	Resonance or Bound State of	Possible Weak or EM Decays	CS Anal
$\tilde{F}_S^{\,o}$	$cs\overline{ud}$	(0,-1,+1)	D^+K^-	$K^-K^-\pi^+\pi^+, K_SK_S$	None
$\overline{\tilde{F}}_S^{\,o}$	$\overline{cs}ud$	(0,+1,-1)	D^-K^+	$K^+K^+\pi^-\pi^-, K_SK_S$	None
$\tilde{F}_I^{\,\pm\pm}$	$cu\overline{sd}$ and c.c	(1,±1,±1)	$D^\pm K^\pm, F^\pm\pi^\pm$	$\pi^\pm\pi^\pm$	$\delta^\pm$
$\tilde{F}_I^{\,o}$	$(cd\overline{su})$	(1,+1,+1)	$D^oK^o, F^+\pi^-$	$\pi^+\pi^-, K^+K^-, K_SK_S$	δ^-
$\overline{\tilde{F}}_I^{\,o}$	$(\overline{cd}su)$	(1,-1,-1)	$\overline{D^oK^o}, F^-\pi^+$	$\pi^+\pi^-\pi^o, K^+K^-\pi^o$	δ^+
$\tilde{F}_I^{\,\pm}$	$c\bar{s}(q\bar{q})_{I=1}$ and c.c.	(1,±1,±1)	$D^\pm K_S, F^\pm\pi^o$	$F^\pm\gamma\gamma$	δ^o
$F_x^{\,\pm}$	$c\bar{s}(q\bar{q})_{I=0}$ and c.c.	(0,±1,±1)	$D^\pm K_S$	$F^\pm\gamma\gamma, F^\pm\pi$	S^*

minimize the total spin interaction energy of all pairs. The dependence of the interaction on color couplings must also be considered, and is treated by the use of the SU(6) color-spin algebra introduced by Jaffe.

The spin-dependent interactions of the charmed quark are much smaller than those of light quarks, as indicated by the small DD^* mass splitting relative to the $\rho\pi$ and KK^* splittings. This is also expected in QCD models, where the "magnetic" interaction of a quark is inversely proportional to its mass. This also suggests that the D^+K^- system will be bound, because recoupling the spin of the $\bar{d}$ anti-quark in a D^+ to a more favorable configuration with respect to the spins of the quark and antiquark in the K^- can only lose a small amount in the more unfavorable coupling of the $c\bar{d}$ system. The worst possible coupling can only lose the D-D^* mass difference.

Exact mass predictions for charm-strange exotics are difficult because of uncertainties in the model. Rough estimates are obtained by use of the charm-strange analogy, in which mass relations for systems involving strange quarks are assumed to hold when one strange quark is replaced by a charmed quark in each state. Examples of the success of this analogy are Eqs. (2.1 - 2.6). While the theoretical basis of these mass relations is still not understood, in particular why linear masses work in some cases and quadratic masses work in others, the observation that whatever works for strange quarks also works for charmed quarks suggests that the analogy may be used to extrapolate relations from the systems with two strange quarks to systems with one strange quark and one charmed quark. We assume that the $\delta(970)$ is a four-quark exotic 0^{+} state with the configuration $(q\bar{q}s\bar{s})$, where q denotes u or d light quarks, and note the inequalities

$$M(\eta) + M(\pi) < M(\delta) < 2M(K) \quad . \tag{6.2a}$$

Changing one strange quark to a charmed quark everywhere gives

$$M(F) + M(\pi) \overset{?}{<} M(\tilde{F}_I\, q\bar{q}c\bar{s}) < M(K) + M(D) \tag{6.2b}$$

where the question mark expresses the uncertainty due to mixing in the η, which is not a pure $s\bar{s}$ state and therefore not strictly the charm-strange analog of the F. Thus the statement that the δ is below the $K\bar{K}$ threshold and decays to $\eta\pi$ leads to the analog that the $\tilde{F}$ should be below the DK threshold and might decay to the $F\pi$, but it might also be below this threshold.

We now consider the most interesting possiblities for decay modes and signatures for the different mass ranges: Note that the $F\pi$ decay is forbidden by isospin for strong decays of the F_x, the $F2\pi$ decay is forbidden by angular momentum and parity for all strong and electromagnetic decays and the $F3\pi$ channel is probably well above DK threshold.

1. <u>All States Above the DK Threshold</u>: Strong decays would be recognized as resonances in mass plots of the DK, $D\bar{K}$, $\bar{D}K$ and $\overline{DK}$

systems. Decays in the $F\pi$ and $F3\pi$ mode would also be allowed for the $\tilde{F}_I$. Particularly striking signatures would be the double-strangeness decay modes

$$\tilde{F}_S \to D^+K^- \to K^-K^-\pi^+\pi^+ \tag{6.3a}$$

$$\overline{\tilde{F}}_S \to D^-K^+ \to K^+K^+\pi^-\pi^- \quad . \tag{6.3b}$$

2. <u>States below the DK Threshold but Above $F\pi$.</u> The $\tilde{F}_I$ would still decay strongly to final states containing an F, but the F_x would decay electromagnetically and the $\tilde{F}_S$ weakly. The $F_x \to F$ decay is a second order $0^+ \to 0^-$ transition with the emission of either two photons or no photon,

$$F_x^{\pm} \to F^{\pm} + 2\gamma \tag{6.4a}$$

$$F_x^{\pm} \to F^{\pm} + \pi^0 \quad . \tag{6.4b}$$

There is also the first order radiative decay

$$F_x^{\pm} \to F^{\pm}\pi^0\gamma \quad . \tag{6.4c}$$

An intermediate $F^*\gamma$ state could be present in the decay (6.4a).

Another possible decay for the F_x is into the $\tilde{F}_I$, if it is above the $\tilde{F}_I$. There would then be the cascade decay,

$$F_x^{\pm} \to \tilde{F}_I^{\pm} + (2\gamma \text{ or } e^+e^-) \to F^{\pm} + \pi^0 + (2\gamma \text{ or } e^+e^-) \quad . \tag{6.4d}$$

In this $0^{+} \rightarrow 0^{+}$ transition, the $e^{+}e^{-}$ decay can go via a single photon and be of second order in α like the 2γ decay.

The Cabibbo favored weak decays of the $\tilde{F}_S$ would be to states of strangeness -2. States with two charged kaons would provide the best signature for identifying these states, since neutral kaons lose the memory of their strangeness by decaying in the K_L and K_S modes,

$$\tilde{F}_S \rightarrow K^{-}K^{-}\pi^{+} + (\text{leptons and/or pions})^{+} \tag{6.5a}$$

$$\overline{\tilde{F}}_S \rightarrow K^{+}K^{+}\pi^{-} + (\text{leptons and/or pions})^{-} \quad . \tag{6.5b}$$

Strong K^{*} signals might be expected in the $K^{\pm}\pi^{\mp}$ combinations, and there should be no D present in the final state. Decays to the four-body final states $KK\pi\pi$ might be the best signature, analogous to the decays (6.3) but without the intermediate DK state and with the possibility of one or two K^{*}'s. Another possible signature is in the two-body neutral decays,

$$\tilde{F}_S \rightarrow \overline{K^{0}K^{0}} \rightarrow K_S K_S \tag{6.6a}$$

$$\overline{\tilde{F}}_S \rightarrow K^{0}K^{0} \rightarrow K_S K_S \quad . \tag{6.6b}$$

Note that the decay (6.6) can give only neutral kaons and not charged kaons because the final states have strangeness ±2 and zero electric charge. Thus although these final states have lost the memory of the double strangeness part of the memory remains in the absence of the charged two-body kaon decay modes. Since nearly all other non-leptonic decays into final states containing kaons tend to produce equal numbers of charged and neutral kaons, the presence of anomalously large numbers of neutral kaons without the corresponding number of charged kaons might be a good indicator for the $\tilde{F}_S$.

Because the same final state $K_S K_S$ is produced both in the decay of the $\tilde{F}_S$ and the $\bar{\tilde{F}}_S$, there can be mixing of the two states as in the neutral kaon system. In the approximation where CP violation is neglected the mixing will lead to eigenstates of the mass matrix which are CP eigenstates. The decay (6.6) will then be allowed only for the eigenstate which is even under CP. The state of odd CP will not be able to decay into two pseudoscalar mesons.

3. <u>States Below the DK and $F\pi$ Thresholds but Above the F.</u> The $\tilde{F}_I^{\pm}$ would decay electromagnetically by two photon emission

$$\tilde{F}_I^{\pm} \rightarrow F^{\pm} + 2\gamma \quad . \tag{6.7}$$

The other charge states of the $\tilde{F}_I$ would decay weakly. The multibody decays would resemble the expected decays of the F with an extra pion, and the Cabibbo favored decays would be into states of zero strangeness. In addition there would be the two-body decays

$$\tilde{F}_I^{\pm\pm} \rightarrow \pi^{\pm}\pi^{\pm} \tag{6.8a}$$

$$\tilde{F}_I^{0},\ \bar{\tilde{F}}_I^{0} \rightarrow \pi^{+}\pi^{-},\ \pi^{0}\pi^{0},\ K^{+}K^{-},\ K_S K_S \quad . \tag{6.8b}$$

The exotic double-charge signature for the decay (6.8a) might be a useful indicator for this state.

The two-pseudoscalar decay modes (6.8b) are all states of even CP for a J = 0 final state. The two states $\tilde{F}^o$ and $\bar{\tilde{F}}_I^{\,o}$ can be expected to mix like the neutral kaons. If CP violation is neglected then the eigenstates will be CP eigenstates and the even state will have the decay modes (6.8b) while the odd CP state will not decay into two pseudoscalars and will decay to three or more, in the nonleptonic modes and into semi-leptonic decay modes. Note that this $\tilde{F}_I^{\,o}$ - $\bar{\tilde{F}}_I^{\,o}$ mixing will be much stronger than K^o - $\bar{K}^o$ mixing in a gauge theory, because it can go via exchange of two intermediate W bosons, with all vertices Cabibbo favored and no cancellation of the GIM type.

4. States Below the F. This is highly improbable, but if the $\tilde{F}$ is below the F, the F would now decay into the $\tilde{F}_I$ and the roles of the F and $\tilde{F}_I$ would be reversed.

Note that if the F - $\tilde{F}_I$ mass difference is less than the pion mass in either direction there will be a particle whose dominant decay mode is electromagnetic with the emission of a low mass photon pair or electron pair. The mass spectrum of the pair will be continuous, but its maximum must be less than the pion mass.

6.5 Are There Low-Lying Charmonium Exotics?

We have seen that the classification of low-lying 0^{++} mesons as four quark states and the charm-strange analogy relating the qualitative systematics of systems containing charmed quarks to known systems containing strange quarks lead to the prediction that the charm-strange analogs of the δ and S^* denoted by δ_c and S^*_c with the configuration $(q\bar{q}c\bar{c})$ should be lower than the J/ψ. This suggests that perhaps the peculiar state at 2.8 GeV might not be the pseudoscalar η_c at all but rather a scalar δ_c. This would please theorists like 't Hooft who want this splitting between the pseudoscalar and vector charmonium states to be considerably less than 300 MeV.

There is therefore interest in investigating the possible existence and properties of the δ_c. How could it be made and how would it decay? Higher charmonium states might decay into δ_c and one or more pions, without violating the OZI rule. One might expect all charmonium states which can decay hadronically to δ_c plus pions to be very wide. However angular momentum and parity selection rules seriously restrict the states which are allowed to decay into δ_c and pions. The final state $\delta_c\pi$ has unnatural parity and even G. The I = 0 states produced from a strong decay of charmonium would then have positive C. Thus of all the low-lying s and p wave charmonium states, only the η_c, η'_c and the $\chi(1^+)$ would be allowed to decay into δ_c plus a pion.

The final state $\delta_c + 2\pi$ has I = 0 only if the two pions are in an I = 1, odd J state like the ρ. The final state, $\delta_c + \rho$ has odd G and could be produced in the decay of the ψ'. It would be interesting to see whether this decay exists in present data, whether it is ruled out and if so, what the upper limits are on this decay. The δ_c could also be produced in a radiative decay of the ψ.

The decay modes of the δ_c include

$$\delta_c \to \gamma\gamma \tag{6.10a}$$

$$\delta_c \to \omega\gamma \tag{6.10b}$$

$$\delta_c \to \eta\pi \tag{6.10c}$$

$$\delta_c \to \rho\omega \tag{6.10d}$$

$$\delta_c \to \rho\phi \tag{6.10e}$$

$$\delta_c \to K\overline{K}\ (K_S K_S) \tag{6.10f}$$

$$\delta_c \to K^* \overline{K^*} \quad . \tag{6.10g}$$

The possible production mechanisms for the δ_c thus include

$$\psi' \to \gamma + \chi(1^+) \sim 3.5 \to \gamma + \pi + \delta_c \tag{6.9a}$$

$$\psi' \to \gamma + \delta_c \tag{6.9b}$$

$$\psi' \to \rho + \delta_c \quad . \tag{6.9c}$$

To look for the δ_c, one of the decay modes (6.10) should be chosen to give a convenient signature and one of the production mechanisms (6.9) might be suitable. The decay modes (6.10c)-(6.10f) all have quantum numbers forbidden for the decay of the η_c and would distinguish between the two possibilities. The decay modes (6.10a), (6.10b) and (6.10g) are allowed for both states.

A discussion of the present experimental states of the suggestion that the state at 2.8 GeV might be the η_c is given by Gottfried.[64]

VII. CAN WE MEASURE THE CHARGE OF A QUARK?

7.1 Quark Charges, Coherence and Color Oscillations

The fractionally charged colored quark model has provided an adequate description of hadron spectroscopy,[8, 52, 65, 66] except for the failure to observe the quarks experimentally. Theoretical arguments are presently being developed to explain the unobservability of free fractionally charged particles as resulting from a fundamental confinement mechanism.[46] However, other models have been proposed, beginning with the Han-Nambu model,[67] which obtain all the conventional results of the fractionally charged quark model from a set of integrally charged constituents. The basic difference between the fractionally charged and integrally charged model is in the description of the internal color degree of freedom. Fractionally charged models assume that color is an exact symmetry of nature, that all observable hadron states are color singlets and that color is inherently unobservable. Integrally charged models assume that color symmetry is broken by the electromagnetic interaction in order to give different values of the electric charge to quarks differing only in color and having the same values of all other quantum numbers. To avoid conflict with existing experimental data on observed hadron states, all such states must be color singlets in the integrally charged models, but the possible existence of observable states at higher excitation is not ruled out. Experimental observation of such color-excited states would establish the validity of these integrally charged models. However, such states could lie very high in mass. The question arises whether it is possible to distinguish between fractionally charged and integrally charged models below the threshold for color excitation.[44, 65, 68]

One might think that the electric charge of the quark is observable directly from measurements of the electromagnetic couplings of hadrons as shown in Fig. 7.1 since these hadron couplings are commonly assumed to be given by the sums of the couplings of the constituent quarks. However, the coupling of the electromagnetic current to a color singlet hadron depends only on the color-averaged quark charge $\langle Q\rangle_c$; i.e. the average over color of the charges of quarks having identical values for all other quantum numbers except color. This can be seen because a color singlet state is completely symmetric in color space and can give no information about a particular preferred direction in this space; e.g. the charge of a red quark rather than that of a blue quark. The color-averaged quark charge does not contain information on whether quarks of all colors have the same charge or whether the charge depends upon color. A measurement of the color-averaged square of the quark charge

$\langle Q^2 \rangle$ should give this information. If $\langle Q^2 \rangle_c$ differs from $\langle Q \rangle_c^{\,2}$, there must be a color dependence of the quark charge. However, it is not simple to devise experiments on hadrons which measure $\langle Q^2 \rangle_c$ for a given quark in a hadron, as shown in specific examples below.

One of the difficulties in interpreting results of simple parton-type models in cases where color symmetry is important arises from peculiar quantum-mechanical coherence effects. As an example consider a model in which strong interactions exactly conserve color symmetry, but electromagnetic interactions break the symmetry by giving different electric charges to the red, white and blue quarks. The weak interaction can also break the color symmetry, but might be simply expressed in a different basis from the red, white and blue quarks which are eigenstates of electric charge. For example, there might be purple and lavender quarks, defined as two orthogonal linear combinations of red and blue quarks, rotated by a Colorbibbo[69] angle.

Suppose that a color singlet meson is given a high momentum transfer by a strong interaction which sends the quark in the meson to the moon, while the antiquark remains on earth. Since strong interactions conserve color, the system is still in a color singlet state. If an astronaut on the moon measures the electric charge of the quark and finds that it is red, then the antiquark on the earth must also be red, and similarly for blue or white. But if the astronaut does a weak interaction experiment and finds that the quark was purple or lavender, then the antiquark on the earth must also be purple or lavender. Thus whether the density matrix describing the antiquark on earth is diagonal in the red-blue or purple-lavender basis depends upon whether the astronaut on the moon chooses to measure an electromagnetic or a weak property of the quark on the moon. This is a manifestation of the famous paradox of Einstein, Podolsky and Rosen.

Such coherence properties arise in parton models where non-abelian symmetries are present. One generally draws diagrams like those of Fig. 7.1 in which one parton absorbs a photon and behaves as if it were free during the interaction. However, it is only as free as the quark on the moon in the previous example. If it comes from a color singlet hadron state, a measurement of its electric charge by a photon absorption process as in Fig. 7.1 affects the properties of the rest of the system, even though there is no interaction. This effect appears in the relative phases of the contributions of the three diagrams shown in Fig. 7.1, where the photon is absorbed by a red, white and blue quark respectively. If this phase information is ignored and the contributions of the diagrams are added incoherently, the results obtained can have serious errors.

An example of the importance of relative phases in cases where internal symmetries are present has been pointed out in the deep inelastic production of exclusive final states on a pion target by the isovector component of the photon.[65] Since the initial state has odd G-parity, only final states with odd numbers of pions can be produced.

However, a parton model in which individual quark partons absorb the photon and the amplitudes are added incoherently loses the G parity information and gives equal production of states with even and odd numbers of pions. The G parity information is contained in the relative phase of the contributions from pairs of diagrams which go into one another under the G transformation; i.e. those in which the current is absorbed by a quark and by its G-conjugate antiquark.

An important effect which must be understood to avoid pitfalls in intuitive treatments of color is the phenomenon of color oscillations. These are analogous to the strangeness oscillations in the neutral kaon system and the neutrino oscillations which have been suggested as possibly occurring if two neutrinos have different masses. Consider a colored quark-antiquark state of a red quark and a red antiquark. This is not a stationary state but oscillates between red-antired, white-antiwhite and blue-antiblue, just as the K^o is not a stationary state but oscillates between K^o and $\overline{K}^o$ as it decays and ν_e and ν_μ are not stationary states and oscillate in some models. The frequency of the color oscillation is determined, as in the neutral kaon and neutrino cases, by the mass difference between the true stationary states of the system. In the color case this mass difference is not very small, like the $K_L - K_S$ or neutrino mass difference, but is very large. It is the mass difference between the observed color singlet mesons and the as yet unobserved color octet states. Thus color oscillations occur at a very rapid rate.

In models where quarks are permanently confined, this mass difference is infinite and color oscillations occur with infinite frequency. The color of a quark is thus unobservable, and all properties of quarks measured in confined systems must be color-independent.

In models where quarks are not permanently confined and states which are not color singlets exist, the threshold for color-octet excitation defines a critical mass and a critical time at which drastic changes in particle physics can be expected. For times long compared with this time scale, the color oscillations are so rapid that all quark properties measured are averaged over color and there is no hope of distinguishing between fractionally charged and integrally charged models. To observe the difference in properties between quarks of different colors, an experiment must have a built-in time scale which is short in comparison with the color oscillations and which can measure the charge of a quark before it changes. This suggests that the experiment must have an energy or mass scale which is above the color threshold. Thus the question of distinguishing between models below the color threshold becomes one of measuring short-time behavior with lower energies.

With these difficulties in mind we examine the possibility of observing a color dependence of the electric charge.

Models like the Han-Nambu model[67] are constructed to make color averages of all matrix elements of the electromagnetic current exactly equal to these of the colored fractionally charged model. The difference between the currents of the two models has no color singlet component and its color average vanishes. Let us write

$$J_{em} = J_G(8_f, 1_c) + \Delta J, \tag{7.1a}$$

where J_G is the electromagntic current in Greenberg's colored quark model[40] with the fractional charges of the Gell-Mann-Zweig quark model, and the arguments $8_f, 1_c$ denote that this current transforms under flavor and color like an octet and singlet respectively. In the Han-Nambu model, ΔJ is a flavor singlet and color octet. Thus

$$J_{HN} = J_G(8_f, 1_c) + \Delta J(1_f, 8_c) \; . \tag{7.1b}$$

The matrix elements of ΔJ thus all vanish between color singlet states, and ΔJ is unobservable in any measurement described by such a matrix element.[8,52,65]

The electric charges of the quarks have the same structure as the current operators. Thus the charge of a quark of flavor f and color c in the Han-Nambu model is given by

$$Q_{HN}(f,c) = Q_G(f) + \Delta Q(c) \tag{7.1c}$$

where Q_G depends only on flavor and is independent of color, and ΔQ depends only on color. We thus obtain for the color averages of Q and Q^2,

$$\langle Q_{HN}(f) \rangle_c = Q_G(f) \tag{7.1d}$$

$$\langle Q_{HN}(f)^2 \rangle_c = Q_G(f)^2 + \langle \Delta Q(c)^2 \rangle_c = Q_G(f)^2 + (2/9) \tag{7.1e}$$

where the value 2/9 is obtained by substituting the numerical values of $Q_{HN}(f,c)$.

As long as ΔJ has no observable effects, it is impossible to distinguish between the integrally and fractionally charged models. There are two possible approaches to the observation of ΔJ: 1) by observing states which are not color singlets, 2) by observing matrix elements of operators which are quadratic in J_{em} between color singlet states. Since states which are not color singlets have a presumably high excitation threshold to explain the failure to observe them to date, we consider the possibility of detecting ΔJ below threshold by measurements on color singlet states of operators quadratic in ΔJ.

7.1 Two-Photon Decays of Pseudoscalar and Tensor Mesons

One important case where effects of color have been observed in a second order electromagnetic transition is in the decay $\pi^0 \to \gamma\gamma$. Decays of this type of meson into two photons are assumed to be described by a triangle diagram.[32] We consider all possible decays of common mesons which have allowed two-photon decays, namely the pseudoscalar and tensor mesons:

$$\pi^o \rightarrow 2\gamma \tag{7.2a}$$

$$\eta \rightarrow 2\gamma \tag{7.2b}$$

$$\eta' \rightarrow 2\gamma \tag{7.2c}$$

$$f^o \rightarrow 2\gamma \tag{7.2d}$$

$$A2 \rightarrow 2\gamma \tag{7.2e}$$

$$f' \rightarrow 2\gamma \tag{7.2f}$$

The contribution of the triangle diagram for each of these decays is obtained by summing the diagrams for different quark flavors with the appropriate weighting factors for each meson. The transition matrix element for this diagram with a quark of flavor f is proportional to the square of the quark charge and is given by

$$M_T(f) = M \sum_c [Q(f,c)]^2/\sqrt{N_c} = M \quad \langle Q(f)^2\rangle_c \ , \tag{7.3a}$$

where M is the reduced transition matrix element which contains the dependence on all degrees of freedom except color. Substituting from eq. (7.1e) into eq. (7.3a), we obtain the relation between the transition matrix elements in the Greenberg model and the Han-Nambu model, denoted by $M_G(f)$ and $M_{HN}(f)$.

$$M_{HN}(f) = \sqrt{N_c} M \quad [Q_G(f)^2 + \langle \Delta Q^2\rangle_c] \tag{7.3b}$$

$$M_{HN}(f) = M_G(f) \ [1 + (2/9)/ \ Q_G(f)^2] \quad . \tag{7.3c}$$

The expressions (7.3) can be written for the specific cases of u,d and s quarks in the following convenient form,

$$M(u) = 3M \ (4+2\kappa)/9 \tag{7.4a}$$

$$M(d) = M(s) = \sqrt{3} \ M(1+2\kappa)/9 \tag{7.4b}$$

where κ is a parameter describing the deviation from the fractionally charged colored quark model and we have set $N_c=3$. $M_G(f)$ is given by setting $\kappa = 0$ in eqs(7.4) and $M_{HN}(f)$ is given by setting $\kappa = 1$. Intermediate values of κ are also of interest as will be shown below.

For the decay of a π^o which is a coherent linear combination of a $u\bar{u}$ and $d\bar{d}$ state with equal magnitude and negative phase, the transition matrix element is proportional to the difference $M_T(u) - M_T(d)$. This difference is seen to have the same value in both models.

$$M_T(u)_G - M_T(d)_G = 1/\sqrt{3} = M_T(u)_{HN} - M_T(d)_{HN} \tag{7.5a}$$

A similar equality holds for the decay of the η_8 which is the eighth component of an octet and depends upon the linear combination

$$M_T(u)_G + M_T(d)_G - 2M_T(s)_G = 1/\sqrt{3} = M_T(u)_{HN} + M_T(d)_{HN} - 2M_T(s)_{HN}. \quad (7.5b)$$

The statistical factor $\sqrt{N}$ in eq. (3) has been used as evidence in favor of color in the experimental value of the $\pi^o \to \gamma\gamma$ decay rate. However, eqs. (7.5) show that it is impossible to distinguish between fractionally and integrally charged models with this decay or the decay of the isoscalar unitary octet meson. This is also evident from the form of ΔJ in eq. (7.1b), which is a flavor singlet. Squaring ΔJ gives an operator which has a color singlet component and is observable in the space of color singlet states. But because it is a flavor singlet, it cannot contribute to the decay of a flavor octet state.

For the decay of a flavor singlet meson, the two models give different results,

$$M(1_f) = (1/\sqrt{3}) \sum_f M(f) = M\cdot(2/3)\cdot(1+\kappa) \quad (7.6)$$

Unfortunately this difference is not easily checked experimentally. The physical η' meson is a mixture of singlet and octet and is at such a high mass that the PCAC derivation for the absolute rate of the π^o decay is unreliable. Kinematic factors resulting from the η-η' mass difference confuse any comparison of the two rates.

Thus, although $P \to \gamma\gamma$ decays appeared to have matrix elements quadratic in ΔJ which would distinguish between the two models, this is not feasible in practice. The situation looks somewhat better for an ideally mixed nonet, like the tensor mesons, where the mass degeneracy between nonstrange isoscalar and isovector states causes all kinematic factors to drop out in the ratio of the decay rates. For the ideally mixed f^o and f' decays.

$$M(f^o) = (1/\sqrt{2})\ [M(u)+M(d)] \quad (7.7a)$$

$$M(A2) = (1/\sqrt{2}\ \ [M(u)-M(d)] \quad (7.7b)$$

$$M(f') = \qquad M(s)\ . \quad (7.7c)$$

Substituting eqs. (7.4) into eqs. (7.7) gives

$$M(f^o)/M(A2)/M(f') = [(5/3)+(4/3)\kappa]/1/[(\sqrt{2}/3)+(2\sqrt{2}/3)\ \kappa] \quad (7.8)$$

Since the decay rates are proportional to the squares of the matrix elements, the ratio of the f^o to A2 decay rates is predicted to be 9 in the Han-Nambu model in comparison with 25/9 in the fractionally charged model. Furthermore, the f' decay rate is predicted to be larger than the A2 decay rate by a factor of 2 in the Han-Nambu model and lower by a factor of 2/9 in the fractionally charged model. These appear to be large observable effects.

Additional possibilities of observing the differences between the matrix elements (7.7a) and (7.7b) arise in coherent production of the f^o and A2 resonances by two photons in the reaction[56]

$$e^+e^- \to e^+e^- + \gamma\gamma \to e^+e^- + M^o \to e^+e^- + PP \qquad (7.9)$$

where M^o denotes a neutral nonstrange meson which is a coherent linear combination of f^o and A2 and PP denotes a state of two pseudoscalar mesons. In the approximation where the f^o and A2 are degenerate, SU(3) symmetry and the OZI Rule give the following results for the relative cross sections for the production of different PP states:

$$M(K^+K^-)/M(K^o\bar{K}^o)/\ M(\pi^+\pi^-) = (4+2\kappa)/(1+2\kappa)/(5+4\kappa) \qquad (7.10)$$

where the transition matrices M_T must be squared and multiplied by appropriate kinematic factors to obtain the observed cross sections. The kinematic factors should be identical for the charged and neutral kaon final states but may be somewhat different for the two-pion state. The results (7.10) are easily obtained by observing that the f^o and A2 are linear combinations of the $u\bar{u}$ and $d\bar{d}$ states and that charged kaon pairs are produced only via the $u\bar{u}$ state, neutral kaons only via the $d\bar{d}$ state and pion pairs only via the even G f^o state.

The relations (7.10) are derived under the assumption that the f^o and A2 are degenerate and have the same width. Calculations of the charged and neutral kaon pair mass spectra show that the qualitative features of eq. (7.10) remain when the masses and widths of the physical particles are introduced and each resonance decay is parametrized by a Breit-Wigner curve. In addition a strong interference effect appears in the region between the f^r and the A2 from the overlapping of the tails of the resonances. With the Han-Nambu model these interference effects should be somewhat different, and might be used to distinguish between the two models.

The relations (7.8) and (7.10) look very promising for distinguishing between the two models if the simple triangle diagram describes the decay and its transition matrix element is given by equation (7.3). However, there are doubts about the validity of this description for the tensor mesons.

Suppose the triangle diagram of fig. 7.2 is interpreted as the successive emission of two photons. Then an intermediate state exists of a quark-antiquark pair with the quantum numbers of the photon, a vector meson state which is either a color singlet and flavor octet or flavor singlet and color octet. The transition matrix element computed from this diagram must include a propagator for the intermediate state. From eqs. (7.1) it is apparent that J_G appears in diagrams with color singlet intermediate states and ΔJ appears in diagrams with color octet intermediate states. If color octet states have a high threshold, diagrams with color octet intermediate states will be suppressed by propagators relative to diagrams with color singlet intermediate states. Thus the effect of the color threshold will reduce the contribution of the terms depending upon ΔJ below the values given by eqs. (7.3), (7.4), (7.6) and (7.8). Since the contribution of ΔJ is seen from eq. (7.3b) to be positive definite and to be given in eqs. (7.4), (7.6) and (7.8) by the term proportional to κ, the reduction of the contributions from ΔJ are expressed quantitatively by reducing the value of κ from unity in these relations.

For the case of the pseudoscalar meson decays, the existence of the axial anomaly allows the transition matrix element to be expressed by a triangle diagram dominated by high momenta where color thresholds are hopefully no longer important. For other cases where there is no anomaly, there is no reason to expect this dominance by high momenta in intermediate states and color threshold effects can be important. Unfortunately, the large deviation from ideal mixing makes the use of pseudoscalar decay rates difficult for distinguishing between the two models. The ideal mixing of the tensor nonet gives simple predictions, but these may be rendered useless by color threshold effects.

The effects in eqs. (7.8) and 7.10) are so large that they may still be observable even with an appreciable reduction from the propagators of the color octet states. The parameter κ will have a value $(m_1/m_8)^2$, where m_1 and m_8 are the masses of the color singlet and color octet intermediate states which are dominant in the transitions. If κ is between 10^{-1} and 10^{-2}, there may still be a possibility of observing these effects. For example, if κ=2%, there will be an 8% increase in the ratio of the two-photon decay widths of the f' and A2, a 3% increase in the ratio of the widths of the f and A2, and a 3% decrease in the production ratio of charged to neutral kaon pairs over the predictions of the fractionally charged quark model. Thus even if the effects are small, they appear as uniquely related discrepancies from the predictions of the fractionally charged model in three different ways.

7.3 Deep Inelastic Processes

Another type of process where color effects might be observed is deep inelastic scattering described by the quark parton model.[68] Here again the simplest processes cannot distinguish between integrally and fractionally charged models because they are described by matrix elements of the current between color singlet states. We consider the possibilities of observing a color dependence of the electric charge in two ways: 1) Above the color excitation threshold; 2) By transitions depending quadratically on the electromagnetic current.

Once states which are not color singlets are produced operators which are not color singlets become observable. However the parton model sum rules do not necessarily hold immediately in this new domain with the integral quark charges of the Han-Nambu model. The basic incompatability between naive parton models and non-abelian internal symmetry must be carefully considered before drawing conclusions. We now spell out this incompatability explicitly and show the necessary conditions for validity of the naive parton model.

Consider the parton model description of the absorption of a current by a hadron. In a model with three colored quarks denoted by R, W and B for red, white and blue, the current can be absorbed by a quark parton of any color. There are three contributing diagrams in which exactly the same transition occurs on a red, white or blue quark as shown in Fig. 7.1. To calculate a partial or total cross section the transition amplitudes for the three processes shown in Fig. 7.1 must be added coherently and then squared. The naive parton model neglects the interference between different diagrams and adds them incoherently under the assumption that interference terms have random phases and average out. This is the source of error in calculations for processes invariant under non-abelian internal symmetries. The symmetry imposes conditions on the relative phases of different amplitudes so that the contribution of interference terms does not average out but is of the same order of magnitude as the direct terms. This has been pointed out in the G-parity example discussed above.[65]

We now demonstrate this interference effect for color symmetry in a simple example. Let us assume that the transition matrix element for the absorption of a current by a quark is proportional to the charge of the quark denoted by g_R, g_W and g_B respectively. The transition amplitude between a given initial and final state is then given by

$$\langle f|T|i\rangle = g_R\langle f|A_R|i\rangle + g_W\langle f|A_W|i\rangle + g_B\langle f|A_B|i\rangle \tag{7.11}$$

where A_R, A_W and A_B are reduced transition amplitudes for the red, white and blue quark transitions with the quark charge factored out. The values of these reduced matrix elements for different colors are related by the color symmetry and depend on the color quantum numbers of the states $|i\rangle$ and $|f\rangle$.

For the case where both $|i\rangle$ and $|f\rangle$ are color singlets

$$\langle f_1|A_R|i_1\rangle = \langle f_1|A_W|i_1\rangle = \langle f_1|A_B|i_1\rangle \equiv A/\sqrt{3}. \tag{7.12a}$$

The contributions of the three diagrams of Fig. 7.1 are all equal and have the same phase and the invariant amplitude A is defined for convenience with the normalization indicated.

For a transition between a color singlet initial state and color octet final states, two linearly independent color octet states occur. For convenience we choose as basic states, denoted by $|f_3\rangle$ and $|f_8\rangle$, the vector and scalar states under the SU(2) subgroup of the color SU(3) which acts only in the space of white and blue quarks. For these transitions we find

$$\langle f_3|A_R|i_1\rangle = 0 \qquad \text{(7.12b}$$

$$\langle f_3|A_W|i_1\rangle = -\langle f_3|A_B|i_1\rangle \equiv C/\sqrt{2} \qquad \text{(7.12c}$$

$$\tfrac{1}{2}\langle f_8|A_R|i_1\rangle = -\langle f_8|A_W|i_1\rangle = -\langle f_8|A_B|i_1\rangle \equiv C/\sqrt{6} \quad , \qquad \text{(7.12d}$$

where the SU(3) color symmetry relates the two transitions (7.12c) and (7.12d), and the invariant amplitude C is normalized for convenience.

Substituting equations (7.12) in equations (7.11) we obtain

$$\langle f_1|T|i_1\rangle = (g_R + g_W + g_B)\,A/\sqrt{3} \qquad \text{(7.13a}$$

$$\langle f_3|T|i_1\rangle = (g_W - g_B)\,C/\sqrt{2} \qquad \text{(7.13b}$$

$$\langle f_8|T|i_1\rangle = (2g_R - g_W - g_B)\,C/\sqrt{6} \quad . \qquad \text{(7.13c}$$

The corresponding transition probabilities are given by

$$|\langle f_1|T|i_1\rangle|^2 = [(g_R{}^2 + g_W{}^2 + g_B{}^2) + 2(g_Rg_W + g_Wg_B + g_Bg_R)]\,A^2/3 \qquad \text{(7.14a}$$

$$|\langle f_3|T|i_1\rangle|^2 = [(g_W^2 + g_B^2) - 2g_Wg_B]C^2/2 \qquad (7.14b$$

$$|\langle f_8|T|i_1\rangle|^2 = [(4g_R^2 + g_W^2 + g_B^2) + 2(g_Wg_B - 2g_Rg_W - 2g_Rg_B)]C^2/6 \quad (7.14$$

where each expression is split into the direct terms considered in the naive parton model and the interference terms normally neglected. Note that when the charges are independent of color the expressions (7.13b), (7.13c), (7.14b) and (7.14c) vanish and color octet states cannot be excited from a color singlet state, as is expected.

Equation (7.14a) shows that when only color singlet final states are excited the interference terms for all final states have the same sign and cannot be neglected without causing a serious error. Thus, the naive parton model which neglects interference terms cannot be used when only color singlet states are excited.

Combining equations (7.14b) and (7.14c) gives the total transition probability for color octet transitions

$$|\langle f_3|T|i_1\rangle|^2 + |\langle f_8|T|i_1\rangle|^2 = 2[(g_R^2+g_W^2+g_B^2) - (g_Rg_W+g_Wg_B+g_Bg_R)]C^2/3 . \qquad (7.14d$$

The interference terms for transitions to color octet final states are seen to also all have the same sign and give a non-negligible coherent contribution. However, the phase of these interference terms is opposite to that of the transitions for the color singlet final states. Thus the condition for the naive parton model to hold is that the interference terms from transitions to color octet final states must exactly cancel those from color singlet final states. From Eq. (7.14) this condition can be expressed

$$\sum_1 A^2 = \sum_8 C^2 \tag{7.15}$$

where the summation on the left-hand side is over all transitions to color singlet final states and on the right-hand side over all transitions to color octet final states.

Eq. (7.15) shows that the naive parton model which neglects coherence between the three diagrams of Fig. 7.1 is valid only when there is a definite relation between the total cross sections for producing color singlet and color octet final states. The exact value of the ratio of octet to singlet production depends upon the values of the coupling constants, but is always of order unity. For the Han-Nambu model this ratio can be seen to be exactly unity by noting that the expressions (7.14a) and (7.14d) become equal when the Han-Nambu coupling constants and the condition (7.15) are used. Thus if the Han-Nambu model is correct, the naive parton model predictions become valid only when the total cross sections for color singlet and color octet production become equal.

We now consider the possibility of observing a color octet component in the electromagnetic current in a second order electromagnetic deep inelastic process whose matrix element depends quadratically on the current. However, as we have seen in the meson decay example, the validity of any second order treatment depends upon a model dependent factor often overlooked. If the underlying model for the second order transition involves successive emission and/or absorption of photons the intermediate state between the two electromagnetic transitions must have an appreciable color octet component if the effects of the color octet component of the current are to be detected. Dynamical suppression factors for this color octet component may prevent the observation of such effects, as shown above in the effect of intermediate state propagators in the meson decays discussed in section 7.2.

As an example of such a suppression we note that the total cross section for photon absorption considered above is related by the optical theorem to the imaginary part of the forward Compton amplitude. A calculation of this second order amplitude without consideration of the above arguments would include quadratic contributions from the color octet component of the current, which had a color singlet component and could give a nonvanishing matrix element for the elastic scattering process. However the discussion of total absorption cross sections shows that the contribution of such color octet contributions to the absorptive part of the amplitude must vanish as long as the energy is below the color threshold. A translation of equations (7.11) to (7.15) into the language of forward Compton scattering shows that the propagator of the intermediate state in the scattering process must be considered very carefully and this propagator violates the conditions of the naive parton model.

The essential features of the properties of the propagator can be seen by noting that the optical theorem represents the absorption process shown in Fig. 7.1 by squaring the amplitude. This square includes not only the diagonal terms, like that shown in Fig. 7.3a, in which Figs. 7.1a, 7.1b and 7.1c are individually squared but also the off diagonal terms, like the one shown in Fig. 7.3b, in which the diagram of Fig. 7.1a is joined to the conjugate of the diagram of Fig. 7.1b or Fig. 7.1c. When this is expressed as a diagram for elastic Compton scattering it shows an intermediate state undergoing a color change. Even though the conditions of the naive quark model are assumed to hold and the same quark which has absorbed the initial photon emits the final photon, the color of this quark can change during the intermediate state as a result of the color oscillations mentioned above. These oscillations can be studied in detail by examining the properties of the propagator.

We denote the three states produced by the diagrams of Fig. 7.1 as $|R\rangle$, $|W\rangle$ and $|B\rangle$ respectively, corresponding to transitions in which a red, white and blue quark absorbs the photon. These states are linear combinations of the color eigenstates $|f_1\rangle$, $|f_3\rangle$ and $|f_8\rangle$,

$$|R> = (1/\sqrt{3})|f_1> + (\sqrt{2}/\sqrt{3})|f_8> \tag{7.16a}$$

$$|W> = (1/\sqrt{3})|f_1> + (1/\sqrt{2})|f_3> - (1/\sqrt{6})|f_8> \tag{7.16b}$$

$$|B> = (1/\sqrt{3})|f_1> - (1/\sqrt{2})|f_3> - (1/\sqrt{6})|f_8> \quad . \tag{7.16c}$$

The color eigenstates $|f_1>$ and $|f_8>$ have different energies E_1 and E_8 because of the energy required for excitation of color octet states. Thus if the state $|R>$ is created at a time $t = 0$, the relative phase of the components $|f_1>$ and $|f_8>$ change with time and introduce admixtures of the other states. For example,

$$<W|e^{-iHt}|R> = (1/3)\left(e^{-iE_1t} - e^{-iE_8t}\right) \tag{7.17a}$$

$$|<W|e^{-iHt}|R>|^2 = (4/9)\sin^2[(E_8 - E_1)t/2] \quad . \tag{7.17b}$$

The color change of a quark in the intermediate state thus takes place at a frequency $(E_8 - E_1)/2$.

Thus the color excitation threshold defines a time or energy scale which determines whether a given process measures the charge of a Han-Nambu quark or the average charge over the color degree of freedom. The color is seen to change in the intermediate state at a rate determined by eq. (7.17). If the transition takes place in a time short compared to this charge fluctuation time, then the naive parton model result should be valid and give the charge of the quark. If however the intermediate state lives a long time compared to this fluctuation time, the charge is averaged over color, and the results are the same as that given by the fractionally charged model.

The lifetime of the intermediate state is short if it is dominated by high momenta; i.e. by states which are high above the color threshold. Thus we see again that the relevant parameter is the ratio of the energy of a typical intermediate or final state to the threshold for color excitations.

VIII. MIXING AND PSEUDOSCALAR MESONS

8.1 Introduction

Why is SU(3) such a good symmetry in some places and so badly broken in others? Why are some hadrons good SU(3) eigenstates and others badly mixed? The quark model seems to give part of the answer. Mesons are quark-antiquark states and baryons are three quark states. This plus isospin and hypercharge conservation automatically force most of the hadron states to be good SU(3) eigenstates. Consider the π^+, for example. This is the ^1S $(u\bar{d})$ configuration. There is no other $(q\bar{q})$ state available with which it can mix without violating isospin or hypercharge conservation or introducing larger numbers of quarks. The π^+ is thus a pure SU(3) octet state, even if there is a large SU(3) violation in quark-quark interactions. The same is true for all states in the lowest meson and baryon multiplets (0^-, 1^-, $1/2^+$ and $3/2^+$) except for the $I = Y = 0$ mesons, where we find η - η' mixing and ω - ϕ mixing.

The general conclusion is that SU(3) symmetry breaking is strong enough to mix any states which are allowed to mix. But the quark model, which restricts hadrons to $q\bar{q}$ and 3q, and isospin and hypercharge conservation leave very few states which can mix.

Mixing can be described by perturbation theory in most cases. If ψ_0 is the unperturbed SU(3) eigenstate and ψ_i denotes the states with which it can mix, the physical eigenstate in broken SU(3) is

$$|\psi\rangle = |\psi_0\rangle + \sum_i \frac{\langle i|V|0\rangle}{E_i - E_0} |\psi_i\rangle$$

where V is the interaction responsible for the SU(3) symmetry breaking and E_i and E_0 are the energies of the unperturbed states.

If the energy denominator is very small compared with V, then degenerate perturbation theory must be used, and the interaction V is diagonalized in the subspace of nearly degenerate states. This occurs in the standard treatment of ω - ϕ mixing, for example.

Two kinds of symmetry breaking terms are generally considered:

1. Mass terms. A flavor-dependent mass term for quarks seems to be the dominant symmetry breaking mechanism for the vector and tensor mesons. Diagonalizing the mass term gives a good approximation to the physical eigenstates.

2. Loops. Hadron states can be mixed by transitions via intermediate two-particle or multiparticle states. The loop diagrams describing

these transitions are of two types, depending upon the nature of the intermediate state.

a. Gluon loops. Since gluons are assumed to be flavor singlets, all gluon intermediate states are flavor singlets, and they are connected only to flavor singlet hadron states by the conventional gluon emission and absorption interactions. Thus gluon loops do not break SU(3) in this approximation. However, they must be considered in cases where there are other interactions which break SU(3), because they affect the amount of breaking by the other interaction.

b. Hadron loops. Even if the three point functions for coupling a hadron to a two-hadron intermediate state is assumed to be SU(3) invariant, these loop diagrams break SU(3) when the physical masses are introduced for the propagators of the intermediate states.

8.2 The Axial Vector (Q) Mesons

As an example of mixing by loops, let us consider the strange axial vector mesons 1^+ classified as $^3P_1$ and $^1P_1$ in the quark model. We denote the strange members of the A_1 and B octets by Q_A and Q_B respectively. The dominant decay modes $K^*\pi$ and ρK are allowed for both Q_A and Q_B states. In the limit of SU(3) symmetry, conserved "parities" G_u and G_v analogous to G parity can be defined by replacing isospin by U spin or V spin in the definition of G parity. The neutral and charged Q's are eigenstates of G_u and G_v respectively. However, the charged ρ and π mesons are not eigenstates of either of these parities, just as the K mesons are not eigenstates of G parity. Thus there is no selection rule forbidding $K^*\pi$ and ρK final states for either of these decays. If the Q_A and Q_B are produced coherently in some experiment, they contribute coherently to the ρK and $K^*\pi$ final states.[70]

If SU(3) is broken, G_u and G_v parities are not conserved. There can then be mixing, analogous to $\omega\phi$ mixing, between the Q_A and Q_B states, even though G parity remains conserved and prevents mixing of the corresponding non-strange states. However, there is no ideal mixing angle determined by quark masses, as in the $\omega\phi$ case, because the Q_A and Q_B have the same quark constituents and are not mixed by a mass term. Some other SU(3) breaking mechanism is needed to produce the observed mixing.

Consider the decay of the mixed states

$$|Q_1\rangle = \cos\theta|Q_A\rangle + \sin\theta|Q_B\rangle \tag{8.1a}$$

$$|Q_2\rangle = -\sin\theta|Q_A\rangle + \cos\theta|Q_B\rangle \quad , \tag{8.1b}$$

where θ is the mixing angle.

For the $K^*\pi$ and ρK decay modes the branching ratio is unity in the SU(3) limit except for differences in kinematic (phase space) factors for the two final states. However, because the two octets have opposite charge conjugation behavior, the A_1-octet decay is described with F-coupling and the B-octet decay with D-coupling. The relative phases of the $K\rho$ and $K^*\pi$ decay amplitudes are thus opposite for the two cases

$$\langle K\rho | Q_A \rangle = - \langle K^* \pi | Q_A \rangle \tag{8.2a}$$

$$\langle K\rho | Q_B \rangle = \langle K^* \pi | Q_B \rangle \qquad , \tag{8.2b}$$

the decay amplitudes for the mixed states (8.1) are then

$$\langle K^* \pi | Q_1 \rangle = \cos\theta \langle K^* \pi | Q_A \rangle + \sin\theta \langle K^* \pi | Q_B \rangle \tag{8.3a}$$

$$\langle K\rho | Q_1 \rangle = -\cos\theta \langle K^* \pi | Q_A \rangle + \sin\theta \langle K^* \pi | Q_B \rangle \tag{8.3b}$$

$$\langle K^* \pi | Q_2 \rangle = -\sin\theta \langle K^* \pi | Q_A \rangle + \cos\theta \langle K^* \pi | Q_B \rangle \tag{8.3c}$$

$$\langle K\rho | Q_2 \rangle = \sin\theta \langle K^* \pi | Q_A \rangle + \cos\theta \langle K^* \pi | Q_B \rangle \quad . \tag{8.3d}$$

Eqs. (8.3) show that for any mixing with a real phase, the effect for one eigenstate is to enhance the $K^* \pi$ decay mode and suppress the $K\rho$, and vice versa for the orthogonal eigenstate. For $\theta = 45^\circ$, we obtain

$$\frac{|\langle K\rho | Q_1 \rangle|^2}{|\langle K^* \pi | Q_1 \rangle|^2} = \frac{|\langle K^* \pi | Q_2 \rangle|^2}{|\langle K\rho | Q_2 \rangle|^2} = \frac{|\langle K^* \pi | Q_A \rangle - \langle K^* \pi | Q_B \rangle|^2}{|\langle K^* \pi | Q_A \rangle + \langle K^* \pi | Q_B \rangle|^2} \tag{8.4a}$$

Thus Q_1 is decoupled from $K\rho$ and Q_2 is decoupled from $K^* \pi$. The decoupling is exact for the case where the Q_A and Q_B states are equally coupled to the $K^* \pi$ mode and is still a good approximation over a wide range of couplings. For example, as long as

$$\frac{1}{4} \le \frac{|\langle K^* \pi | Q_A \rangle|^2}{|\langle K^* \pi | Q_B \rangle|^2} \le 4 \qquad , \tag{8.4b}$$

we still have

$$\frac{|\langle K\rho | Q_1 \rangle|^2}{|\langle K^* \pi | Q_1 \rangle|^2} = \frac{|\langle K^* \pi | Q_2 \rangle|^2}{|\langle K\rho | Q_2 \rangle|^2} \leq \frac{1}{9} \quad . \tag{8.4c}$$

A dynamical mechanism which naturally leads to this mixing is the SU(3) breaking in decay channels originally introduced to explain[71] $\omega\phi$ mixing before SU(6) and the quark model. The states Q_A and Q_B are coupled to one another via their decay channels $K^*\pi$ and $K\rho$.

$$|Q_A\rangle \longleftrightarrow |K^*\pi\rangle \longleftrightarrow |Q_B\rangle \tag{8.5a}$$

$$|Q_A\rangle \longleftrightarrow |K\rho\rangle \longleftrightarrow |Q_B\rangle \quad . \tag{8.5b}$$

In the SU(3) symmetry limit, the two transitions (8.5a) and (8.5b) exactly cancel one another and produce no mixing. This cancellation no longer occurs when SU(3) breaking introduces kinematic factors arising from the mass difference between the two intermediate states. These suppress the strength of the transition (8.5b) via the higher mass $K\rho$ intermediate state relative to the transition (8.5a) via $K^*\pi$.

The simple analysis of the transitions (8.5a) and (8.5b) gives 45° mixing for the eigenstates if $\langle K^*\pi | Q_A\rangle = \langle K^*\pi | Q_B\rangle$. This decouples the two states from $K^*\pi$ and $K\rho$ respectively. However, a more careful analysis shows that two partial waves are present in the decay, s-wave and d-wave, and the result is very sensitive to the relative amplitudes and phases of the s and d waves. In particular, for the ratio of s to d wave amplitudes predicted by the naive $SU(6)_W$ quark model, the transitions (8.5) vanish and cannot produce mixing, because the Q_A is coupled only to vector meson states with transverse polarization and the Q_B is coupled only to longitudinally polarized states.[70] For this reason the mechanism (8.5) for mixing was dropped.

A recent analysis of the experimental properties of the Q mesons suggests a mixing of SU(3) eigenstates with a 45° mixing angle with one of the eigenstates decaying only to $K^{*}\pi$ and not to $K\rho$ and vice versa for the other state.[72]

Now that the $SU(6)_W$ predictions are known not to agree with experiment,[73] particularly in the closely related polarization predictions for B and A_1 decays, and the experimental data are consistent with pure s-wave for the Q decays, the mixing mechanism (8.5) should perhaps again be considered. However, a more realistic calculation would consider the coupled channels $K^{*}\pi$ and $K\rho$ through the resonance region, with phase space factors changing within the resonances because of the proximity to threshold.

8.3 Troubles With Pseudoscalar Mesons

The vector and tensor meson nonets are well-described by attributing all the SU(3) symmetry breaking to a flavor-dependent quark mass term, and assuming nonet degeneracy except for this mass term. Many experimental predictions of this description have been successfully tested. However, the analogous predictions do not work for the pseudoscalar mesons. At first it was assumed that some additional interaction could change the mixing angle from the so-called ideal mixing produced by the quark mass term, and phenomenological predictions were made in which the mixing angle was left as a parameter to be determined from experimental data. However, these are also in disagreement with experiment.

The conventional mixing description seems to be in both experimental and theoretical trouble for the pseudoscalar mesons. The η and η' do not behave like orthogonal mixtures of a single SU(3) singlet and a single SU(3) octet. More complicated mixing is indicated perhaps requiring inclusion of radially excited states as well as ground state configurations.[23,74]

The use of the quark model to determine mixing angles of neutral mesons from experimental data on neutral meson production processes was first suggested by G. Alexander.[75] This work, based on the Levin-Frankfurt additive quark model[52,76] in which every hadron transition is assumed to involve only one active quark with all remaining quarks behaving as spectators, presented a number of predictions which have since been shown to be in very good agreement with experiment. These include the first derivation of the A...Z rule for four point functions, as the prediction that ϕ production is forbidden in πN reactions since the process requires two active quarks in the same hadron. Also obtained were the prediction of no exotic t-channel exchanges and some sum rules and equalities which are listed below. Analysis of a decade of experimental data show a consistent pattern of good agreement with all predicted relations for processes of vector meson production and strong disagreement with relations for processes of pseudoscalar meson production, particularly for relations involving η' production. We suggest that an appropriate conclusion from these results is that the quark model description indeed holds for these processes, but that something is wrong with the pseudoscalars, particularly the η'.

The relevant sum rules are the charge exchange sum rule (CHEX)

$$\sigma(\pi^-p \to \pi^0 n) + \sigma(\pi^-p \to \eta n) + \sigma(\pi^-p \to \eta' n)$$
$$= \sigma(K^+n \to K^0 p) + \sigma(K^-p \to \overline{K}^0 n) \quad , \tag{8.6a}$$

and the strangeness exchange sum rule (SEX)

$$\sigma(K^-p \to \eta Y) + \sigma(K^-p \to \eta' Y) = \sigma(K^-p \to \pi^0 Y) + \sigma(\pi^-p \to K^0 Y) \tag{8.6b}$$

These sum rules hold for any meson nonet and do not make any assumption about the mixing angle, except for the conventional description of the η and η' as two orthogonal linear combinations of pure SU(3) singlet and octet states defined in terms of a single mixing angle. For the case of ideal mixing, as in the vector mesons the two sum rules each split into two equalities, CHEX becoms

$$\sigma(\pi^- p \to \phi n) = 0 \tag{8.7a}$$

which is just the A...Z rule, and substituting (8.7a) into (8.6a) gives

$$\sigma(\pi^- p \to \rho^o n) + \sigma(\pi^- p \to \omega n) = \sigma(K^+ n \to K^{*o} p) + \sigma(K^- p \to \bar{K}^{*o} n). \tag{8.7b}$$

With ideal mixing SEX becomes

$$\sigma(K^- p \to \omega Y) = \sigma(K^- p \to \rho^o Y) \ , \tag{8.8a}$$

$$\sigma(K^- p \to \phi Y) = \sigma(\pi^- p \to K^{*o} Y) \ . \tag{8.8b}$$

The relation (8.8a) is seen also to be a consequence of the A...Z rule for the meson vertex. The incident K^- contains no d for d quarks or antiquarks and therefore produced via the $u\bar{u}$ component which is a linear combination of the two with equal weight.

If there is no mixing, which is a rough approximation for the pseudoscalar mesons, the charge exchange sum rule simplifies to

$$\sigma(\pi^- p \to \pi^o n) + 3\sigma(\pi^- p \to \eta_8 n) = \sigma(K^+ n \to K^o p) + \sigma(K^- p \to \bar{K}^o n) \ , \tag{8.9a}$$

$$\sigma(\pi^- p \to \eta_8 n) = 2\sigma(\pi^- p \to \eta_1 n) \ . \tag{8.9b}$$

All the vector meson relations (8.7) and (8.8) are in excellent agreement with experiment. However, the pseudoscalar meson relations (8.6) and (8.7b) are in strong disagreement. The relation (8.7a) agrees with experiment if the η is assumed to be pure octet. This suggests that the conventional picture in which there is small mixing may be valid for the η, but that something is wrong with the η', and it is wrong in the direction that the η' has an inert piece in the wave function which does not contribute to the sum rules (8.6) and (8.7)

More recent evidence of trouble in the $\eta - \eta'$ system comes from data presented at this conference on neutral meson production in $K^- p$ reactions at 4.2 GeV/c. The previously observed trouble with the SEX sum rule (8.1b) is confirmed with higher statistics. In addition there may be difficulty with backward production. Okubo[77] has pointed out that conventional mixing predicts that the ratio of η' to η production must be a universal constant in all processes where there are no active strange quarks,

$$\frac{\sigma(A+B\to\eta'+X)}{\sigma(A+B\to\eta+X)} = K = \frac{|\langle\eta'|\eta_{ns}\rangle|^2}{|\langle\eta|\eta_{ns}\rangle|^2} \tag{8.10}$$

where A, B and S do not contain strange quarks and η_{ns} denotes the particular linear combination of η and η' which contains only non-strange quarks, the analog of the physical ω. Okubo finds a value of K=0.5±0.25 by analysis of a large number of processes. But recent experiments[78] give

$$K = (2.0\pm0.68)/(1.27\pm0.39) \tag{8.11}$$

for the ratio of η' to η production in the backward direction in $K^-p\to\Lambda\eta$ or $\Lambda\eta'$. If this is a baryon exchange process, the coupling of the η and η' to nonstrange baryons should also go via the η_{ns} component and the processes should satisfy Okubo's universality relation (8.10). The fact that $K > 1$ for this case whereas $K < 1$ for all meson exchange processes investigated by Okubo might suggest a difference between meson exchange and baryon exchange, if the discrepancy of less than two standard deviations proves to be statistically significant. This is again consistent with the description of the η as having a piece in the wave function which does not contribute to the meson exchange sum rules because of poor overlap with the wave function of the incident meson. For baryon exchange there is no such overlap integral and the additional piece could have a sizeable contribution, giving a higher η' production cross section relative to the η than in meson exchange reactions.

We suggest that there is indeed an additional piece in the η' wave function and that it is a radially excited configuration. This leads to a re-examination of the standard mixing folklore and the discovery that it is completely unjustified.[74] In a formulation which begins with unperturbed singlet and octet states in the SU(3) symmetry limit, there is no reason to assume that SU(3) symmetry breaking should admix only the lowest ground states of the singlet and octet spectra. This may work for the tensor and vector mesons, where the entire nonet seems to be degenerate in the SU(3) symmetry limit and the dominant breaking of nonet symmetry is by a quark mass term. The degeneracy suggests the use of degenerate perturbation theory which diagonalizes the symmetry breaking interaction in the space of the degenerate unperturbed states. The mass term has no radial dependence and would not mix ground state and radially excited wave functions which are orthogonal and would have a zero overlap integral.

For the pseudoscalars where there is a large singlet-octet splitting in the SU(3) symmetry limit there is no reason to use degenerate perturbation theory and mix only ground state wave functions. Furthermore, the singlet-octet splitting can only be produced by an interaction which violates the A...Z rule because it is not diagonal in the quark basis and mixes $s\bar{s}$ with $u\bar{u}$ and $d\bar{d}$. The accepted mechanism for such A..Z violation in the pseudoscalars is annihilation of the quark-antiquark pair into gluons and the creation of another pair. Here there is no reason to restrict the pair creation to the ground state configuration. There is no overlap integral between the two $q\bar{q}$ states, as the intermediate gluon state does not remember which radial configuration it came from. If the annihilation process depends primarily on the value of the $q\bar{q}$ wave function at the origin, then all radially excited configurations couple with equal strength for wave functions from a c confining linear potential.

Thus there is considerable reason to suspect that the trouble with pseudoscalar meson sum rules is in admixture of a radially excited wave function into the η'. One might expect the η to be purer because the SU(3) flavor octet state does not couple to gluons which are singlets and because it is the lowest state, far in mass from the nearest SU(3) singlet radial excitation. The η', on the other hand is sitting in between the ground state and first radially excited octet states and would be expected to mix with both. Note that mixing of the octet ground state and first radially excited octet state by an SU(3)-symmetric potential need not be considered because it is merely a change in the radial wave function. This mixing can be transformed away by choosing a new radial basis (i.e. a slightly different potential) for which the modified ground wave function in the original basis is the exact ground state in the new basis.

IX. WHY ARE THERE MYSTERIOUS REGULARITIES IN HADRON TOTAL CROSS SECTIONS?

9.1 Flavor Dependence of Hadron Total Cross Sections

The very precise experimental data[79] now available on pion, kaon and nucleon total cross sections give us some information about the difference between the interactions of strange and nonstrange particles with matter. Careful examination of the data show very clearly that there is a difference between strange and nonstrange particles and that there are puzzles not explained by the quark model. This is strikingly shown in linear combinations of cross sections which have no Regge component and are therefore conventionally assumed to be pure pomeron. The K^+p and pp channels are exotic and have no contribution from the leading Regge exchanges under the common assumption of exchange degeneracy. The following linear combinations of meson-nucleon cross sections are constructed to cancel the contributions of the leading Regge trajectories

$$\sigma(\phi p) = \sigma(K^+p) + \sigma(K^-p) - \sigma(\pi^-p) \tag{9.1a}$$

$$\Delta(\pi K) = \sigma(\pi^-p) - \sigma(K^-p). \tag{9.1b}$$

Figure 9.1 shows these two quantities on the conventional plot of cross section versus P_{lab} on a log scale.

$\bar{\sigma}(\phi p)$ as defined by Eq. (9.1a) is the quark model expression for $\sigma(\phi p)$; i.e., the cross section for the scattering of a strange quark-antiquark pair on a proton. The very simple energy behavior of this quantity as seen in Fig. 9.1 is striking. It shows a monotonic rise beginning already at 2 GeV/c.

The quantity $\Delta(\pi K)$ defined by Eq. (9.1b) represents the difference in the scattering of a strange particle and a nonstrange particle on a proton target. In the quark model this is the difference between the scattering of a strange quark and a nonstrange quark on a proton target after the leading Regge contributions have been removed. This difference between strange and nonstrange also has a very simple energy behavior, decreasing constantly and very slowly (less than a factor of 2 over a range P_{lab} of two orders of magnitude). So far there is no good explanation for why strange and nonstrange mesons behave differently in just this way.

Since the two quantities (9.1) have no contribution from the leading Regge trajectories they represent something loosely called the pomeron. However, their energy behaviors are different from one another and also from that of the quantities $\sigma(K^+p)$ and $\sigma(pp)$ which should also be "pure pomeron." However the following linear combinations of $\sigma(K^+p)$ and $\sigma(pp)$ have exactly the same energy behavior as the meson-baryon linear combinations (9.1)

$$\sigma_1(pK) = \frac{3}{2}\,\sigma(K^+p) - \frac{1}{3}\,\sigma(pp) \tag{9.2a}$$

$$\Delta(MB) = \frac{1}{3}\,\sigma(pp) - \frac{1}{2}\,\sigma(K^+p). \tag{9.2b}$$

These quantities are also plotted in Fig. 9.1.

The equality of the quantities (9.2) and the corresponding quantities (9.1) suggest that the pomeron, defined as what is left in the total cross sections after the leading Regge contributions are removed by the standard prescription, consists of two components, one rising slowly with energy and the other decreasing slowly. The coefficients in Eq. (9.2) were not picked arbitrarily but were chosen by a particular model. In this model the rising component of the total cross section is assumed to satisfy the standard quark model recipe exactly.

$$\sigma_1(Kp) = \sigma_1(\pi p) = \frac{2}{3}\,\sigma_1(pp) = \frac{2}{3}\,\sigma_1(Yp) = \frac{2}{3}\,\sigma_1(\Xi p), \tag{9.3a}$$

where Y denotes a Λ or Σ hyperon. The falling component has been assumed to satisfy the following relation

$$\sigma_2(Kp) = \frac{1}{2}\,\sigma_2(\pi p) = \frac{2}{9}\,\sigma_2(pp) = \frac{1}{3}\,\sigma_2(Yp) = \frac{2}{3}\,\sigma_2(\Xi p). \tag{9.3b}$$

This particular behavior is suggested by a model in which the correction to a simple quark-counting recipe comes from a double exchange diagram involving a pomeron and an f coupled to the incident particle.[80]

We thus see unresolved problems in the total cross-section data associated with the questions of what is the difference between strange and nonstrange particles and what is the nature of the pomeron. Note that Eq. (9.1b) defines the difference between the scattering of a nonstrange quark and a strange quark while Eq. (9.2b) can be interpreted as the difference between the scattering of a quark in a baryon and a quark in a meson. The fact that the strange-nonstrange difference and the meson-baryon difference are equal and have the same energy behavior over such a wide range is a puzzle which may be explained by pomeron-f double exchange but may also indicate something deeper.

9.2 The Two Component Pomeron Formula

A very good fit to the experimental total cross section data up to 200 GeV/c has been obtained with the two components (9.2) and (9.3) parametrized by simple power behavior. This gives a formula with five parameters which were adjusted to fit the data,[80]

$$\sigma_{tot}(Hp) = C_1\sigma_1(Hp) + C_2\sigma_2(Hp) + C_R\sigma_R(Hp) \tag{9.4}$$

where $C_1 = 6.5$ mb., $C_2 = 2.2$ mb., $C_R = 1.75$ mb.,

$$\sigma_1(Hp) = N_q^H(P_{lab}/20)^{\epsilon} \tag{9.5a}$$

$$\sigma_2(Hp) = N_q^H N_{ns}^H(P_{lab}/20)^{-\delta} \tag{9.5b}$$

$$\sigma_R(Hp) = (N_{\overline{n}}^H + 2N_{\overline{p}}^H)(P_{lab}/20)^{-\frac{1}{2}} \quad , \tag{9.5c}$$

N_q^H is the total number of quarks and antiquarks in hadron H ($N_q^H = 2$ for mesons and 3 for baryons), N_{ns}^H is the total number of non-strange quarks and antiquarks in hadron H and $N_{\overline{n}}^H$ and $N_{\overline{p}}^H$ are the total number of $\overline{n}$ and $\overline{p}$ antiquarks in hadron H, $\epsilon = 0.13$ and $\delta = 0.2$.

The dependence of the individual terms in Eqs. (9.5a) and (9.5b) on the quantum numbers of H are determined by the model and discussed in ref. 80. The explicit form for the energy dependence is chosen to minimize the number of free parameters. Thus power behavior is chosen rather than logarithmic for the two components of the Pomeron, because two parameters are sufficient to describe a power and at least three are needed to describe logarithmic behavior. The Regge term was chosen to minimize the number of free parameters by assuming exact duality and exchange degeneracy for the leading trajectories with the conventional intercept of one-half.

The formula (9.4) predicts that plots of $\sigma_{tot}(Hp) \cdot (P_{lab}/20)^{\delta}$ vs. $P_{lab}^{(\epsilon + \delta)}$ should show straight lines for all cross sections and linear combinations of cross sections which have no Regge contribution. This is strikingly verified in Figs. 9.2a and 9.2b, which show straight lines for $\sigma_{tot}(K^+p)$, $\sigma_{tot}(pp)$, and for the linear combinations (9.1) and (9.2). A straight line is not obtained for $\sigma_{tot}(\pi^-p)$, which has a Regge component. However, when this Regge contribution is removed by plotting $\sigma_{tot}(\pi^-p) - \sigma_R(\pi^-p)$, as defined by eq. (9.5c) another straight line appears.

The formula (9.4) predicts that the straight lines for $(2/3)\sigma_{tot}(pp)$, $\sigma_{tot}(\pi^-p) - \sigma_R(\pi^-p)$, $\sigma_{tot}(K^+p)$, and $\sigma(\phi p) = \sigma_1(pK)$ should have the same slope and be equally spaced. This is clearly shown also in Fig. 9.2a. The straight lines for $\Delta(\pi K)$ and $\Delta(MB)$ are predicted to have zero slope and a value equal to the spacing between the equally spaced parallel lines. This is in qualitative agreement with Fig. (9.2a), although there is a slight rise, suggesting that the value of 0.2 for the parameter δ is a bit too high. Similar straight lines are obtained with slight variations of the parameters. Changing δ to 0.185 gives a better fit to the data.

The extension of the formula (9.4) to the real part of the amplitude is a straightforward application of analyticity and crossing, which is particularly simple for terms with power behavior[81] and gives the following expression for the ratio of the real to imaginary parts of the Hp amplitude

$$\rho(Hp) = \frac{C_1\sigma_1(Hp)\tan(\pi\epsilon/2) - C_2\sigma_2(Hp)\tan(\pi\delta/2) - C_R\sigma_R(\overline{H}p)}{\sigma_{tot}(Hp)} . \qquad (9.6)$$

9.3 Fits to Higher Energy of the Two Component Pomeron Formula

The total proton-proton cross section and the real part of the forward scattering amplitude have been recently measured[82] at ISR. Table 9.1 shows that the new data in the energy range equivalent to P_{lab} = 500 to 2000 GeV/c are in excellent agreement with predictions from the five parameter formula (9.4) - (9.6) with no adjustment of the values of these parameters from already published values fixed by fits to data below 200 GeV/c. Table 9.1 also lists predictions for higher energies and shows remarkable agreement with results from Cosmic Ray experiments[83] up to P_{lab} = 40,000 GeV/c. The plots of fig. 9.2a are extended to these higher energies in fig. 9.2b and show a good straight line with the same parameters. An equally good straight line is obtained if δ is changed to 0.185. Whether these agreements confirm the validity of the oversimplified two-component model is unclear. However, the formula can certainly be used as a simple parametrization of the data and a guide to the physics of further experiments. The ISR group fit their data with a seven parameter formula.[82]

The good fits obtained to very high energy data indicate that these rather crude approximations are nevertheless adequate up to these energies. As long as this reasonable fit continues models containing more detailed assumptions will not be easily tested by the available data. For example, as long as a good fit is obtained with power behavior for the first component the necessity for logarithmic terms will be difficult to demonstrate since a considerably better fit is required to justify the use of additional parameters. The same is true for more detailed or realistic descriptions of the Regge component, since breaking exchange degeneracy or choosing a value different from one-half for the intercept necessarily requires more parameters. However, as soon as data appear which fail to fit this formula, the underlying assumptions are so simple that the physics of the disagreement should be readily apparent. The nature of the disagreement might suggest, for example that the rise of the cross sections is logarithmic rather than a power, that exchange degeneracy is breaking down, or that the Regge intercept is not one-half. There may also be a breakdown of the two-component pomeron picture if the dependence on the quantum numbers of hadron H no longer satisfies the simple relations of the model. Thus, regardless of the validity of the two component pomeron description, the formula (9.4) should be a valuable guide to the analysis of data on high energy total cross sections and real parts of scattering amplitudes.

TABLE 9.1 Theoretical Predictions
and experimental data for $\sigma_{tot}(pp)$ and $\rho(pp)$

P_{lab}	$\sqrt{s}$	$\sigma_{tot}(p\bar{p})$	$\sigma_{tot}(pp)$		$\rho(pp)$	
		Theory	Theory	Experiment	Theory	Experiment
(GeV/c)	(GeV)	(mb)	(mb)	(mb)		
498	30.6	41.8	40.0	40.1 ± 0.4	.025	.042 ± .011
1064	44.7	42.8	41.6	41.7 ± 0.4	.064	.062 ± .011
1491	52.9	43.5	42.5	42.4 ± 0.4	.079	.078 ± .010
2075	62.4	44.3	43.5	43.1 ± 0.4	.092	.095 ± .011
4600	92.9	46.8	46.2	47.0 ± 0.8	.118	
10000	137.	49.8	49.5	50.6 ± 1.2	.138	
25000	217.	54.3	54.0	53.8 ± 2.2	.156	
40000	274.	56.9	56.7	55.0 ± 3.0	.163	
100000	433.	62.7	62.6		.174	

9.4 Lipkin's Crazy Parton Model

Another puzzle is suggested by the fit to the data with $\delta = 0.185$ and $\epsilon = 0.13$, which satisfy the condition $\delta = (0.5 - \epsilon)/2$. This condition suggests that the total cross section is the square of an amplitude with two components, one varying as $P_{lab}^{\epsilon/2}$ and one varying as $P_{lab}^{-1/4}$. Then σ_1 and σ_R represent the squares of these components and σ_2 is the interference term. It is tempting to try to fit this regularity with a parton model in which the total cross section is assumed to come from two contributions: 1) a "Pomeron diagram" in which a quark in the beam exchanges a "Pomeron" with the proton target, and then fragments into the final state; 2) a "Reggeon diagram," in which a Reggeon is exchanged before fragmentation into a different final state. The new decreasing component $\sigma_2(Hp)$ might arise from interference between Pomeron and Reggeon amplitudes. One might even explain this interference by invoking "f-dominance of the Pomeron" to show that the same complicated final states produced by fragmentation after Pomeron exchange can also be produced after f exchange, and therefore the two must be coherent and interfere.

The dependence of the Pomeron and Regge diagrams on the quantum numbers of the beam particle are exactly those required by eqs. (9.5a) and (9.5c), while the interference term naturally has the flavor dependence of the Pomeron-f double exchange of eq. (9.3b) which leads to eq. (9.5b).

STUDENTS: SOMETHING IS OBVIOUSLY WRONG WITH THIS MODEL. SEE IF YOU CAN GUESS IT BEFORE READING FURTHER!

Unfortunately, the "interference term" $\sigma_2(Hp)$ also exists in channels like pp and K^+p which have no Regge term. Thus the interference model is in contradiction with elementary quantum mechanics. We are left with the puzzle:

Why can hadron total cross sections be fit with three components having the energy dependence of a slowly rising Pomeron, a decreasing Regge exchange and Pomeron-Regge interference, when a non-vanishing interference term is present in some cases where the direct Regge term vanishes?

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FIGURE CAPTIONS

Fig. 5.1a: Meson

Fig. 5.1b: Two Mesons

Fig. 5.2: Baryon

Fig. 5.3: Diquark

Fig. 5.4a: Baryon-Antibaryon Pair

Fig. 5.4b: Baryonium

Fig. 7.1: Photon Absorption by Colored Quark Parton

(a) by red quark

(b) by white quark

(c) by blue quark

Fig. 7.2: Triangle Diagram for Two Photon Decay of a Meson

Fig. 7.3: Compton Scattering in Quark Parton Model

Fig. 9.1: Plots of Eqs. (9.1) and (9.2).

Fig. 9.2a: $\sigma_{tot}(Hp) \times (P_{lab}/20)^{0.2}$ plotted against $(P_{lab})^{0.33}$.

$\triangle$ $\sigma_{tot}(\pi^- p)$

$\bigcirc$ $(2/3)\sigma_{tot}(pp)$

$\bullet$ $\sigma_{tot}(\pi^- p) - \sigma_R(\pi^- p)$

$\blacktriangle$ $\sigma_{tot}(K^+ p)$

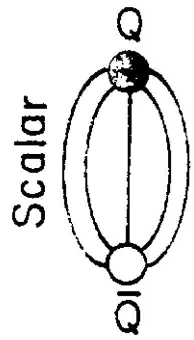
$\times$ $\sigma(\phi p)$

∇ $\sigma_1(pK)$

$\square$ $\Delta(MB)$

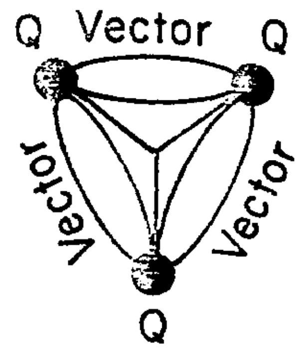
$+$ $\Delta(\pi K)$

Fig. 9.2b: $\sigma_{tot}(pp) \times (P_{lab}/20)^{0.2}$ plotted against $(P_{lab})^{0.33}$.



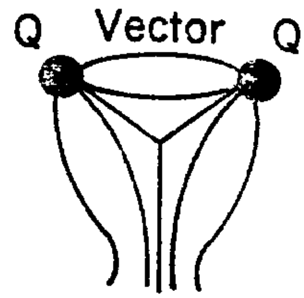
Meson

Fig. 5.1a



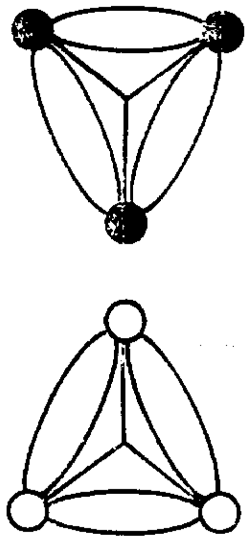
Baryon

Fig. 5.2



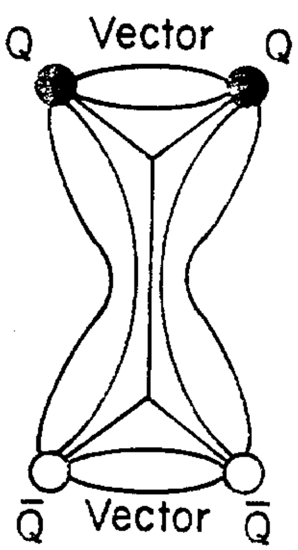
Diquark

Fig. 5.3



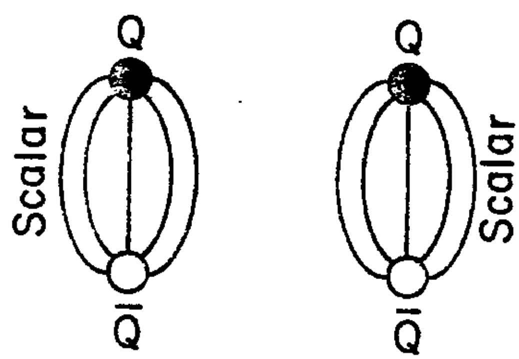
Baryon-Antibaryon Pair

Fig. 5.4a



Baryonium

Fig. 5.4b



Two Mesons

Fig. 5.1b

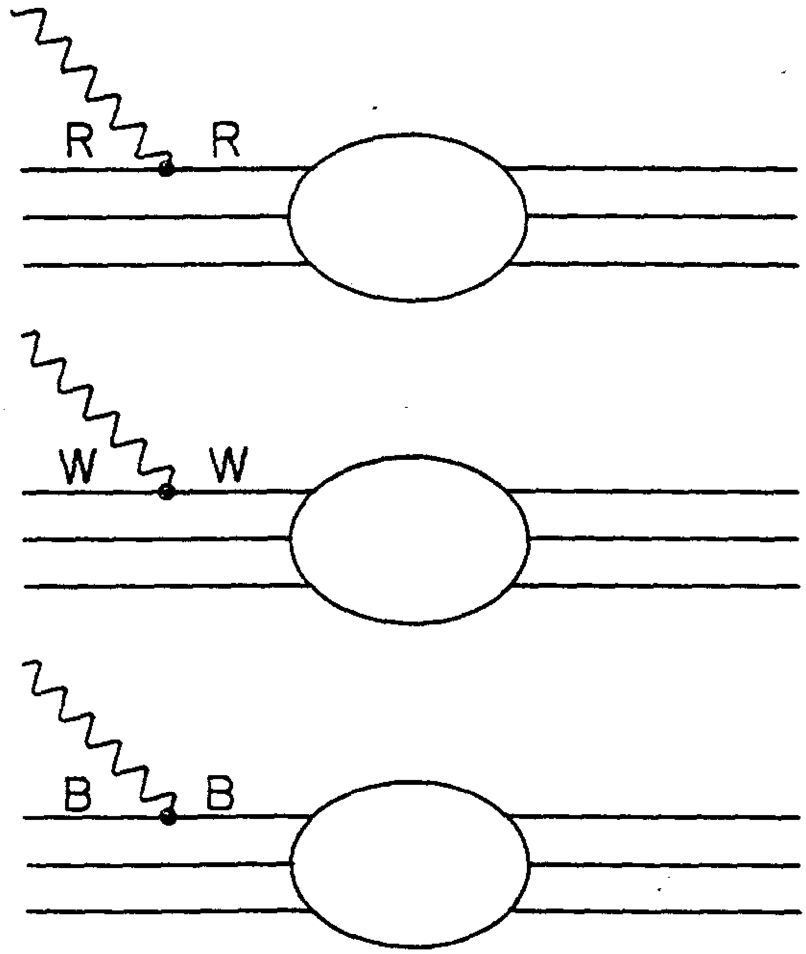


Fig. 7.1

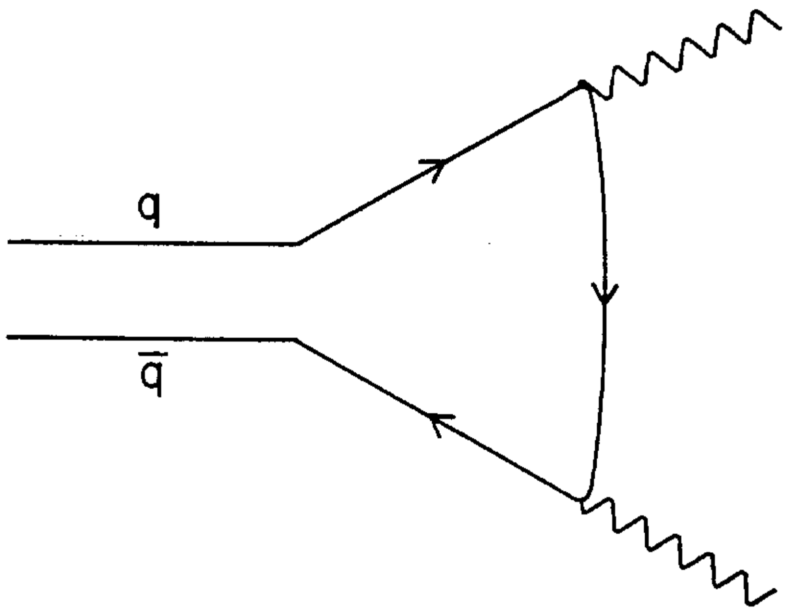


Fig. 7.2

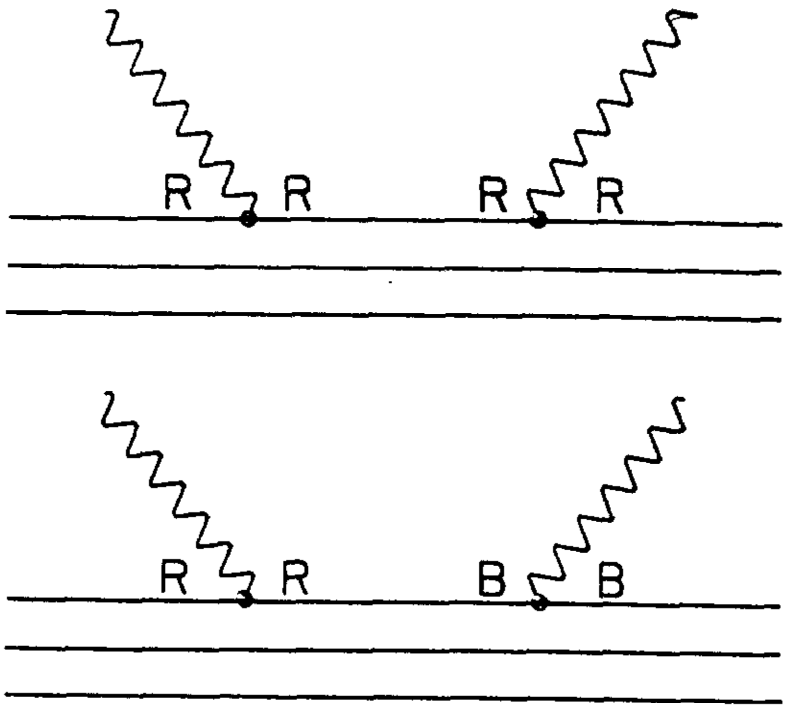


Fig. 7.3

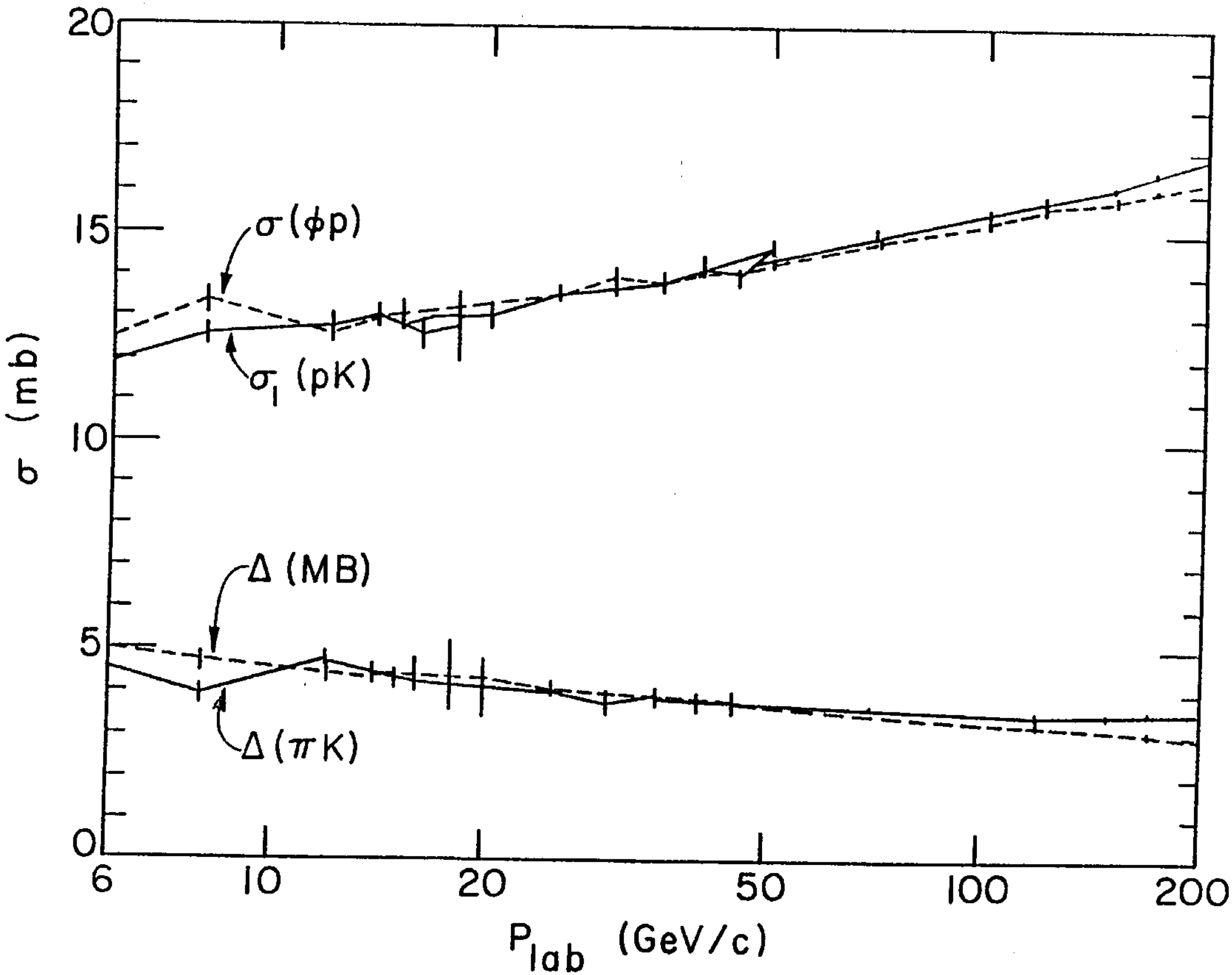
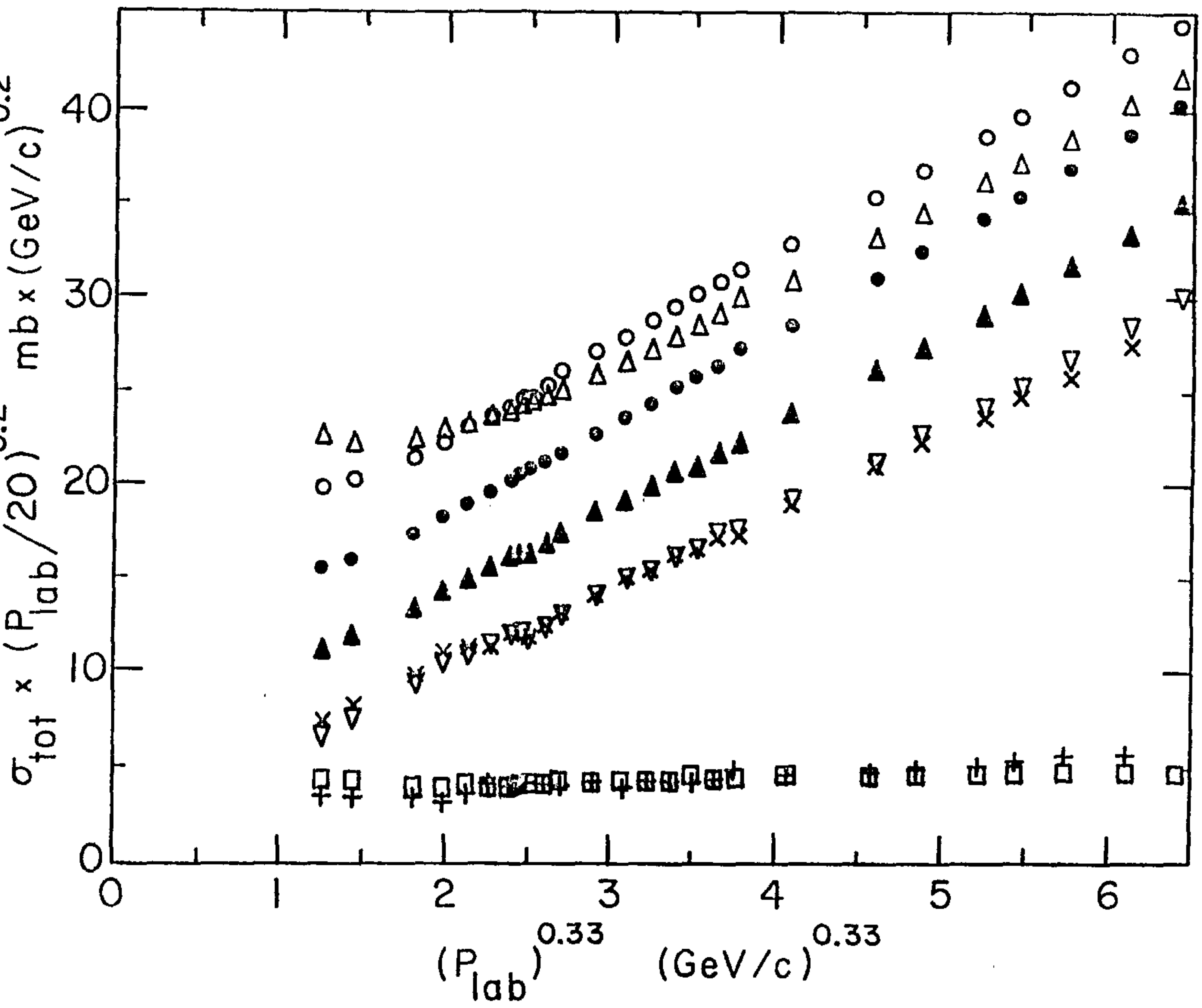
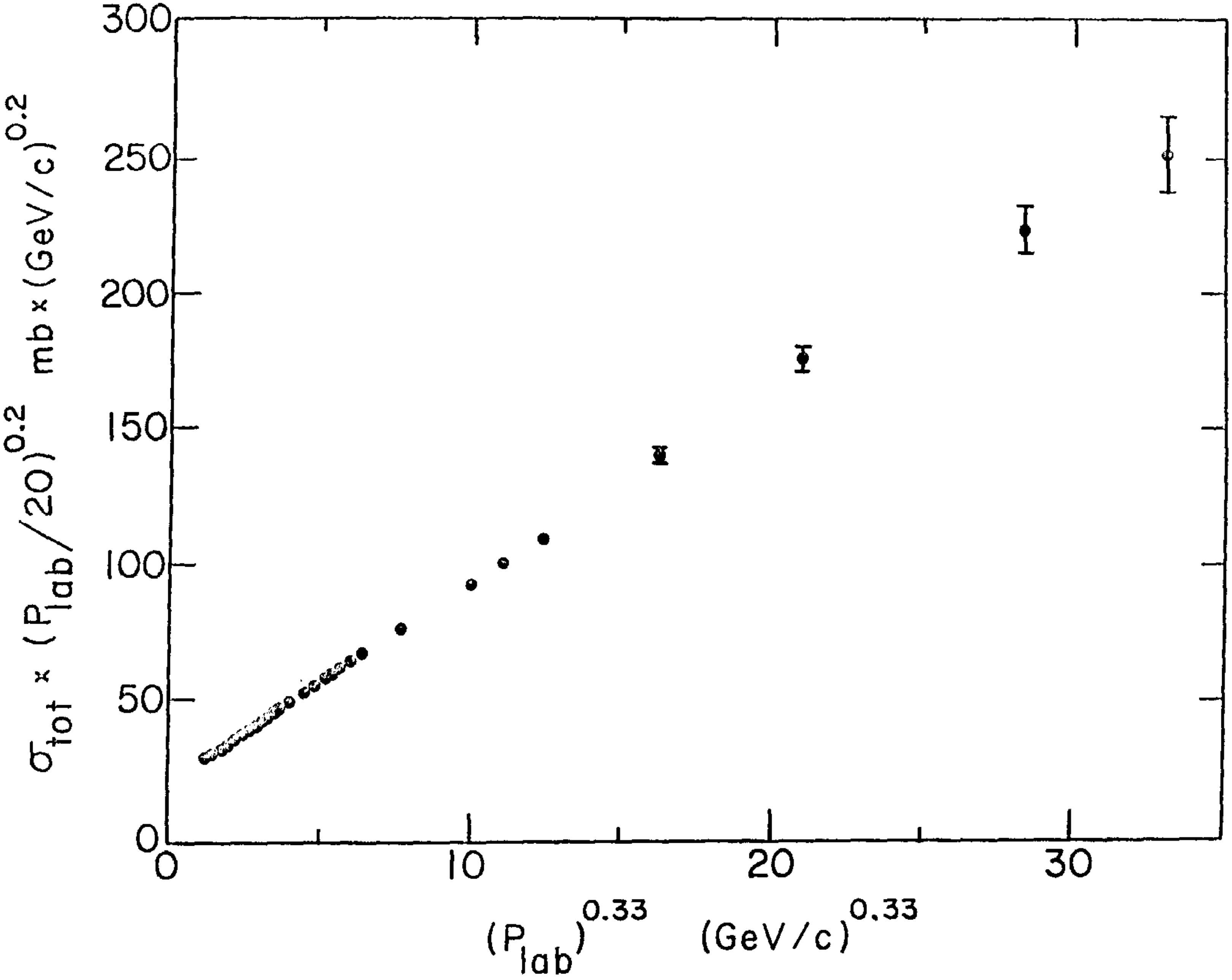


Fig. 9.1



0
10
20
30
40
$\sigma_{tot} \times (P_{lab}/20)^{0.2}$ mb $\times$ (GeV/c)$^{0.2}$
0
1
2
3
4
5
6
$(P_{lab})^{0.33}$ (GeV/c)$^{0.33}$

Fig. 9.2a



300
250
200
150
100
50
0
0
10
20
30
$\sigma_{tot} \times (P_{lab}/20)^{0.2}$ mb $\times$ $(GeV/c)^{0.2}$
$(P_{lab})^{0.33}$ $(GeV/c)^{0.33}$

Fig. 9.2b