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Gauge Theories of Microweak CP Violation

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ABSTRACT

We investigate systematically the condition that CP violation in $|\Delta S| = 1$ processes is "microweak" [i.e., of order of $G_F \varepsilon (m/m_W)^2$ where m is a typical hadronic mass] naturally (i.e., for all values of complex parameters of the theory) in $SU(2) \times U(1)$ gauge theories of weak and electromagnetic interactions. We consider only those models in which CP violation occurs in the quark mass terms. The conditions for microweak CP violation in $|\Delta S| = 1$ processes are that (1) quarks of charge - 1/3 and of a given chirality have the same weak isospin I and I_3, and (2) quarks of charge 2/3 (-4/3, if they exist) and quarks of charge - 1/3 do not belong to the same weak isomultiplets for at least one chirality.

Special attention is given to a more restricted class of models in which (1) quarks of a given charge and chirality have the same weak isospin I and I_3, and (2) quarks of charge q and quarks of charge q+1 do not belong to the same isomultiplets for at least one chirality. In such models, the electric dipole moment of a quark arises only in second-order weak interactions, and is estimated to be of order of 10^{-30} cm. Several examples of this class of models are given, one of which is the six quark model of Kobayashi and Maskawa.

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I. INTRODUCTION

Since the discovery of CP violation in two pion decays of K_L in 1964[1] all attempts to detect CP violation effects elsewhere have been futile. Phenomenological success of superweak theory[2] in accounting for the known CP violation effects in the neutral K meson system[3] suggests that in the regime of known particles (but excluding charmed particles and particles in the psion family) observable CP violation is confined to $|\Delta S| = 2$ processes.

In the meantime, the theory of weak interactions has experienced spectacular advances in the development of unified gauge theory of weak and electromagnetic interactions.[4] We propose in this paper to investigate systematically a class of gauge theories which describe the observed weakness of CP violation in $|\Delta S| = 1$ processes naturally, that is, for any values of parameters of the theory.[3] More precisely, we will investigate the conditions that have to be met in order for a renormalizable gauge model to be a theory of microweak CP violation, the precise meaning of which will be given in Proposition IV below, and investigate the consequences of such models. To be systematic, it is necessary to spell out the basic assumptions explicitly and in detail.

Proposition I. The theory of weak and electromagnetic interactions is a gauge theory based on $SU(2) \times U(1)$.[6] CP violation is to be described in this framework. The theory of strong interactions is a gauge theory based on color $SU(3)$.[7]

The important point of this proposition is that, contrary to what we assume, CP violation may not be associated with the ordinary weak interactions after all. It may be due to a new interaction not at all envisaged within the present framework,[3] or it may arise in a grand unification of all interactions through the mismatch of CP properties of known interactions with unknown.[8] The reason CP violation has only been observed in a weak process may be that only in this process backgrounds are low enough.

Proposition II. The theory is maximally CP violating, in the sense that any parameter that can be complex is complex. In a renormalizable theory, once CP is broken, any parameters of the theory that can be complex has to be complex to ensure renormalizability. We reject the possibility, in this paper, that the observed CP violation arises spontaneously. It is not that there is something wrong with spontaneous CP violation; on the contrary, it is an attractive possibility.[9] Rather, it is outside the scope of the present study.

Proposition III. The Lagrangian for Higgs scalars is CP conserving, not because CP invariance is imposed on it, but because it is impossible to write down a renormalizable Lagrangian which violates CP. This again is an ad hoc assumption. This proposition sets a restriction on the representation contents of the Higgs mesons. It is possible to write down a theory in which CP violation arises only in the Higgs Lagrangian and not anywhere else due to the representation contents of fermions, as indeed Weinberg has done.[10] Either possibility simplifies the problem and is therefore esthetically preferable to the general case in which CP violation arises everywhere.

Proposition IV. CP violation does occur in lowest-order weak interactions. However, it is suppressed by a factor $(m/m_W)^2$ in $|\Delta S| = 1$ processes for arbitrary values of parameters of the theory. Here m is a typical hadronic (or quark) mass, assumed to be at most several GeV. This proposition asserts that there is no CP violation of order $G_F\varepsilon \sim 10^{-3}\ G_F$ in $|\Delta S| = 1$ processes; in these processes CP violation has to be at most of order $G_F\varepsilon(m/m_W)^2 \sim 10^{-6}\ G_F$. Here ε is the usual parameter $\sim 10^{-3}$ which measures CP admixing in the $K_L K_S$ system.[11] (It is in principle possible that two different CP violating phases are involved in $|\Delta S| = 1$ and $|\Delta S| = 2$ processes, in which case CP violation in $|\Delta S| = 1$ processes need not be proportional to ε. We need not worry about this point, however, if we are interested in models where there is only one CP violating phase.) This, together with Proposition II, is what we mean by a "natural gauge theory of microweak CP violation." The alternative, nanoweak proposition that "CP violation in any process is at most of order $10^{-9}\ G_F$" is hard to satisfy within the present framework.[12] It must be said that experimental evidence for nanoweak (usually called superweak) proposition, or for that matter, a microweak condition, is not that compelling. A milliweak theory with proper selection rules can fake a nanoweak theory as far as what is experimentally known today.[13] A test for milliweak theories is the electric dipole moment of the neutron for which the present experimental upper limit[14] $\sim 4 \times 5^{-25}$ cm is close to the expectations in most of these theories.[15] We also require

Proposition V. There is no CP violation in semileptonic and leptonic processes for all values of parameters in the theory in first-order weak interactions. This has to be achieved naturally, by the number and representation contents of leptons.

Proposition VI. Neutral current conserves strangeness to order $G_F\alpha$ naturally. Evidence for this is so strong, especially with the almost certain discoveries of charmed particles,[16] that I take it as aximatic. The analysis of microweak CP violation can be carried out without it; in fact we can deduce Proposition VI from Proposition III. However we might as well assume it, if only to save verbiage, since the observational foundation for Proposition VI is immeasurably stronger than that for Proposition III. As Glashow and Weinberg[17] have shown, this proposition requires that all quarks of charge -1/3 and a given chirality have the same weak I^2 and I_3.

Sometimes, we will assume a stronger condition:

Proposition VI'. Neutral current conserves all flavors as well as strangeness to order $G_F\alpha$ naturally. There is no experimental evidence for or against this proposition. However, theoretical inferences one can draw from this proposition is so far-reaching that it is worthwhile to entertain this possibility. This implies that all quarks of any given charge and a given chirality must also have the same weak I^2 and I_3.

We find that these propositions taken together are restrictive enough to single out a class of models admissible. In these models the quarks of charge 2/3 and the quarks of charge -1/3 should not belong to the same weak isomultiplets for at least one chirality, and the quarks of charge -4/3 (if they exist) and the quarks of charge -1/3 should not belong to the same weak isomultiplets for at least one chirality in order to contain CP violation to the microweak level in $|\Delta S| = 1$ processes. We shall emphasize a more restrictive class of models in which the quarks of charge q and the quarks of charge $q + 1$ do not belong to the same isomultiplets for at least one chirality, for any q. In such models, the electric dipole moment of any quark appears only in second-order weak interactions, and is estimated to be of order of 10^{-30} cm.

When is a natural theory CP-violating? Often, complex parameters in a given theory do not give rise to CP violation because of the possibility of changing the definition of CP transformations on fields. In Section II, we give the necessary and sufficient conditions for CP violation in theories satisfying our propositions. More precisely, we give a formula for the number of CP violating phases that a natural gauge theory can have. In Section III, we explore the consequences of Proposition IV and arrive at the characterization of admissible models given in the last paragraph. Section IV is a general discussion of the process $s + s \to d + d$ in these models, as the prototype of $|\Delta S| = 2$ processes. In Section V, we discuss the electric dipole moment of quarks in the more restrictive models mentioned. In Section VI, we give three examples of natural models of microweak CP violation. In Appendix A, we discuss the condition for CP conservation in single exchange of a physical Higgs meson in $|\Delta S| = 1$ processes.

II. CP VIOLATION - CRITERION

We shall denote by ξ and η the left- and right-chiral quark fields. The components of ξ and η are labeled by I, Y, α and I_3, where α distinguished different multiplets of the same I and Y. The couplings of the fermions to gauge bosons are

$$\begin{aligned} & g(\xi^+\gamma_\mu\, T_+^L\, \xi + \eta^+\gamma_\mu\, T_+^R\, \eta)W^{-\mu} + \text{h.c.} \\ & + \sqrt{g^2 + g'^2}\left[\xi^+\gamma_\mu(T_3^L - \sin^2\theta_W Q^L)\xi \right. \\ & \qquad \left. + \eta^+\gamma_\mu(T_3^R - \sin^2\theta_W Q^R)\eta\right] Z^\mu\ , \end{aligned} \tag{2.1}$$

where $Q = T_3 + Y/2$ is the electric charge operator. The mass term of quarks is of the form

$$\xi^{+} M \eta + \text{h.c.} \; , \tag{2.2}$$

where M is a general matrix[18]:it need not be real. It commutes with the electric charge:

$$Q^{L} M - M Q^{R} = 0 \; . \tag{2.3}$$

Since $MM^{\dagger}$ and $M^{\dagger}M$ are hermitian and have the same eigenvalues, it is possible to write the matrix M in the form

$$M = U_L^{+} \, M_D \, U_R \; , \tag{2.4}$$

where $U_{L,R}$ are unitary, and M_D is nonvanishing only along the diagonal with nonnegative real elements. It follows from Eq.(2.3) that

$$\left[Q^{L}, U_L\right] = \left[Q^{R}, U_R\right] = 0 \; . \tag{2.5}$$

The physical fermion fields ψ_L and ψ_R are defined by

$$\psi_L = U_L \xi, \quad \psi_R = U_R \eta \; ,$$

$$\psi = \left(\frac{1-\gamma_5}{2}\right)\psi_L + \left(\frac{1+\gamma_5}{2}\right)\psi_R \; . \tag{2.6}$$

The components of ψ are labeled by the electric charge q and the flavor a. The mass term (2.2) can be written as

$$\bar{\psi}\, M_D\, \psi \quad . \tag{2.7}$$

The couplings of the fermions to gauge bosons can be written as

$$\begin{aligned} & g\left[\bar{\psi}\,\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)\ \mathcal{T}_+^L\,\psi + \bar{\psi}\,\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\ \mathcal{T}_+^R\,\psi\right] W^{-\mu} + \text{h.c.} \\ & + \sqrt{g^2 + g'^2}\left[\bar{\psi}\,\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)\left(\mathcal{T}_3^L - \sin^2\theta_W Q\right)\psi\right. \\ & \qquad \left. + \bar{\psi}\,\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\left(\mathcal{T}_3^R - \sin^2\theta_W Q\right)\psi\right] Z^\mu \ , \end{aligned} \tag{2.8}$$

where

$$\mathcal{T}_i = U\, T_i U^+ \ , \tag{2.9}$$

for both left-and right-handed ones.

We ask under what circumstances the expressions (2.7) and (2.8) are CP invariant. We define CP transformation on ψ as

$$\begin{aligned} \text{CP:}\quad & \psi \to S\beta\ \mathcal{C}\,\bar{\psi}^T \ , \\ & \bar{\psi} \to -\,\psi^T\ \mathcal{C}^{-1}\ \beta\ S^+ \ , \\ & \beta = \gamma_0, \quad \mathcal{C}\,\gamma_\mu\ \mathcal{C}^{-1} = -\,\gamma_\mu^T \ , \end{aligned} \tag{2.10}$$

where S is a diagonal unitary matrix of the form

$$\langle a|S|b\rangle = e^{i\sigma_a}\delta_{ab}, \quad \sigma_a = 0.$$

We insist on the unimodularity of S for later convenience; the overall phase of ψ is associated with quark number conservation, and is not physically observable. We assume that there is no symmetry -- discrete or otherwise -- of the mass matrix other than that implied by charge conservation, Eq.(2.5). In particcular, these are no degenerate eigenvalues of the matrix M_D. It then follows that Eq.(2.10) is the most general form of the definition of CP transformation which leaves the mass term (2.7) invariant. The condition of CP invariance of Eq.(2.8) is

$$(S^\dagger U_L T_i^L U_L^\dagger S)^T = U_L T_i^L U_L^i ,$$

$$(S^\dagger U_R T_i^R U_R^\dagger S)^T = U_R T_i^R U_R^\dagger .$$

for i=1,2,3, or

$$\left[U_L^T S^* U_L, T_i^L\right] = 0 \qquad (2.11L)$$

and

$$\left[U_R^T S^* U_R, T_i^R\right] = 0 . \qquad (2.11R)$$

These conditions imply that, by Schur's lemma,

$$U_{L,R}^T S^* U_{L,R} = A_{L,R} \qquad (2.12)$$

where

$$\langle I'Y'\beta;\ I'|A_{L,R}|IY\ ;\ I\ \rangle$$
$$= \delta_{II'}\ \delta_{YY'}\ \delta_{I_Z I'_Z}\ A^{L,R}_{\beta\alpha}(IY)\ , \tag{2.13}$$

and the matrices $A^{L,R}(IY)$ are unitary and symmetric. Since $A_{L,R}$ are symmetric and unitary they can be written as

$$A_{L,R} = O_{L,R}(e^{i2\Phi})_{L,R}(O^T)_{L,R} \tag{2.14}$$

where $O_{L,R}$ are real orthogonal:

$$\langle I'Y'\beta;I'_3|O_{L,R}|IY\alpha;I_3\rangle = \delta_{II'}\delta_{YY'}\delta_{I_3I'_3}O^{L,R}_{\beta\alpha}(IY) \tag{2.15}$$

and $\Phi_{L,R}$ are real diagonal

$$\langle I'Y'\beta;I'_3\ |\Phi_{L,R}|IY\alpha;I_3\rangle = \delta_{II'}\delta_{YY'}\delta_{I_3I'_3}\delta_{\alpha\beta}\phi^{L,R}_{\alpha}(IY)\ . \tag{2.16}$$

Equation (2.12) can be solved. We obtain

$$U_{L,R} = S^{\frac{1}{2}}X_{L,R}(e^{-i\Phi})_{L,R}O^T_{L,R} \tag{2.17}$$

where $X_{L,R}$ are real orthogonal matrices which commute with Q. That is, if and only if $U_{L,R}$ have the representations of Eq. (2.17), there is a definition of CP transformation which leaves the interaction (2.8) and the mass term (2.7) invariant. We have chosen S to be unimodular, since then the phases of det $U_{L,R}$ are uniquely given by $-\mathrm{Tr}\Phi_{L,R}$. Let $N_{L(R)}(q)$ be the number of left (right)-handed chiral fermions of charge q, $N_{L(R)}(IY)$ the number of left (right) chiral fermion multiplets of isospin I and hypercharge Y, and N_f the number of flavors. The number of real parameters associated with the matrix S is N_f-1, since S is diagonal and unitary unimodular. The number N_c of CP conserving parameters associated with U_L and U_R is, from Eq. (2.17),

$$N_c = N_f - 1 + \sum_{i=L,R} \left\{ \sum_q \tfrac{1}{2} N_i(q) \left[N_i(q) - 1 \right] + \sum_{I,Y} \tfrac{1}{2} N_i(IY) \left[N_i(IY) + 1 \right] \right\} . \tag{2.18}$$

In the general case, U_L and U_R are arbitrary unitary matrices (since M is arbitrary) which commute with Q. Thus the number of real parameters associated with U_L and U_R, $N_c + N_v$, is

$$N_c + N_v = \sum_{i=L,R} \sum_q N_i^2(q) \quad .$$

Thus the maximum number of CP-violating parameters is given by

$$N_v = \sum_{i=L,R} \left\{ \sum_q \tfrac{1}{2} N_i(q) \left[N_i(q) + 1 \right] - \sum_{I,Y} \tfrac{1}{2} N_i(IY) \left[N_i(IY) + 1 \right] \right\} - N_f + 1 \quad . \tag{2.19}$$

In using this formula, the color degrees of freedom are to be completely ignored. Thus, in the minimal model, the pair (u,d_c) must be counted as one isodoublet, not three. The reason is, of course, that we assume complete degeneracy of color multiplets, so that the color degrees of freedom do not increase the number of parameters.

Let us check how the formula (2.19) works. For the minimal model for hadrons,[6] we have $N_L(q=2/3) = N_L(q=-1/3) = N_R(q=2/3) = N_R(q=-1/3) = 2$; $N_L(I=1/2, Y=1/3) = N_R(I=0, Y=4/3) = N_R(I=0, Y=-2/3) = 2$, and $N_f = 4$, so $N_v = 0$. Therefore, there is no possibility of CP violation in the minimal model, through the couplings of quarks to gauge bosons.

The condition that the couplings of quarks to gauge bosons are CP violating is clearly $N_V > 0$. If coupling constants are not restrained, then such a theory with $N_V > 0$ is CP non-invariant.

The formula (2.19) does not apply to leptons. This has to do with the fact that when there are several massless neutrinos, the definition of CP transformation, Eq. (2.10), can be generalized to include S which is not diagonal, but which mixes degenerate neutrinos. In that case, our argument fails, which depends critically on S being symmetric. In most cases, however, this is not a drawback. The fact that massless neutrinos have only one chiral component, and therefore their phases may be varied at will, simplifies the determination of CP violating phases, which can be done in most cases by inspection.

It is possible for Higgs scalar fields to violate CP invariance through their self-interactions, if the theory is not constrained to be CP invariant. However, there are theories in which the Higgs mesons cannot violate CP invariance. For example, the potential for the doublet Higgs fields in the minimal model is always CP conserving. In this paper, we shall only consider theories in which the Higgs potential is necessarily CP conserving by the representation contents of the Higgs fields.

It is also possible that the Higgs-meson couplings to fermions are CP violating. In Appendix A, we discuss the condition that physical Higgs meson exchanges conserves CP in $|\Delta S| = 1$ processes in a natural way.

III. MICROWEAK CP VIOLATION

The condition that there be no CP violating term in the neutral current, and therefore in Z-boson exchange, is equivalent to the condition that the neutral current is diagonal in flavor , since terms off-diagonal in flavor are in general CP violating. The latter condition has been investigated by Glashow and Weinberg.[17] The condition that strangeness is conserved naturally to order $G_F\alpha$ in neutral current transitions requres that all quarks of charge -1/3 and a given chiralty have the same values of I^2 and I_3.

Effects of W-boson exchange may be expressed by a phenomenological interaction

$$H_W = g^2 \int d^4x \ \Delta_F(x;m_W^2) j_\mu(x) j^{+\mu}(0) \tag{3.1}$$

where j_μ is the charged current:

$$j_\mu = \bar{\psi}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right) T_+^L\psi + \bar{\psi}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) T_+^R\psi \ . \tag{3.2}$$

Since the relevant distance in the operator product of two currents is a short one of order $1/m_W$, we may expand H_W of Eq. (3.1) in a series of local operators in ascending dimensions.[19] Relevant operators of dimension less than seven are listed below:

$$D=3 \ : \ \bar{q}X^{(3)}q \quad ,$$
$$D=4 \ : \ \bar{q}\gamma\cdot DX^{(4)}q \quad ,$$
$$D=5 \ : \ \bar{q}\sigma_{\mu\nu}F^{\mu\nu}X_1^{(5)}q \quad , \quad \bar{q}D^2X_2^{(5)}q \quad ,$$
$$D=6 \ : \ \bar{q}\gamma\cdot DD^2X_1^{(6)}q \quad , \quad \bar{q}\gamma\cdot D\sigma_{\mu\nu}F^{\mu\nu}X_2^{(6)}q \text{ and}$$
similar terms;
$$j_\mu(0)j^{\dagger\mu}(0) \quad ,$$

where $X_i^{(D)}$ is a matrix, which may include the γ_5 matrix, in the flavor space, and D_μ is the covariant derivative in chromodynamics of strong interactions. We have suppressed inessential color indices. In this asymptotically free theory the coefficient $C_i^{(D)}$ of the operator $\bar{q}\cdots X_i^{(D)}q$ is of order of

$$\begin{aligned} g^2m, &\quad \text{for } D=3 \\ g^2 \ , &\quad \text{for } D=4 \\ g^2(m/m_W^2), &\quad \text{for } D=5 \\ g^2m_W^2 \sim G_F \ , &\quad \text{for } D=6 \quad . \end{aligned}$$

ignoring logarithmic factors. Terms of dimension higher than six are suppressed by at least one additional factor of $(m/m_W)^2$ and need not be considered in our discussion. Here m is a typical hadronic (or quark) mass, assumed to be at most a few GeV.

Operators of dimension three and four are eliminated by renormalization of the quark fields q and the mass matrix of quarks (this may entail renormalization of the quark-Higgs scalar couplings). Consider now operators of dimension five. The leading term in $(m/m_W)^2$ of $X_i^{(5)}$ has the structure

$$X_i^{(5)} = (T_+^R M_D T_-^L + T_-^R M_D T_+^L)\left(\frac{1-\gamma_5}{2}\right) + (T_+^L M_D T_-^R + T_-^L M_D T_+^R)\left(\frac{1+\gamma_5}{2}\right) \quad . \tag{3.3}$$

In a natural theory of microweak CP violation, in which the mass matrix $M = U_L^+ M_D U_R$ is arbitrary, the matrix element between the s- and d-quarks of each term on the right hand side of Eq. (3.3) must vanish separately. Thus we must have

$$\sum_k \langle d_j | T_+^R | x_k \rangle \langle x_k | M_D | x_k \rangle \langle x_k | T_-^L | d_i \rangle = 0$$

and

$$\sum_k \langle d_j | T_-^R | u_K \rangle \langle u_k | M_0 | u_k \rangle \langle u_k | T_+^L | d_i \rangle = 0 \quad ,$$

where u_i, d_j, and x_k are quarks of charge 2/3, -1/3 and -4/3, respectively. In other words, we must have

$$\langle x_k | T_-^R | d_i \rangle = 0 \quad \text{or} \quad \langle x_k | T_-^L | d_i \rangle = 0 \ ,$$

and

$$\langle u_j | T_+^R | d_i \rangle = 0 \quad \text{or} \quad \langle u_j | T_+^L | d_i \rangle = 0 \quad . \tag{3.4}$$

This requires that the quarks of charge 2/3 and the quarks of charge -1/3 do not belong to the same weak isomultiplets for at least one chirality, and the quarks of charge -4/3 (if they exist), and the quarks of charge -1/3 do not belong to the same weak isomultiplets for at least one chirality. In particular the right-handed d- and s-quarks must be the states of highest I_3 in their respective multiplets, and the right-handed quarks of charge 2/3 must be linear superpositions (in general) of states of lowest I_3, since transitions from the left-handed u-quarks to the left-handed d-and s-quarks must be allowed on phenomenological grounds. This means that the u-d and u-s charged currents must be pure V-A.

Next consider single-quark operators of dimension six. The leading term in $(m/m_W)^2$ of $X_i^{(6)}$ has the structure

$$X_i^{(6)} = (T_+^L T_-^L + T_-^L T_+^L)\left(\frac{1-\gamma_5}{2}\right) + (T_+^R T_-^R + T_-^R T_+^R)\left(\frac{1+\gamma_5}{2}\right) \quad . \tag{3.5}$$

Since $T_+T_- + T_-T_+ = T^2 - T_3^2$, it follows that the leading terms of matrix elements of $X_i^{(6)}$ between the s- and d-quarks vanish if strangeness is naturally conserved in the neutral current to order $G_F\alpha$.

Finally, we consider the two-quark operator $j_\mu(0)j^{\dagger\mu}(0)$. We need consider only the term $\left[\bar{s}\gamma_\mu(a+b\gamma_5)u\right]\left[\bar{u}\gamma^\mu(c+d\gamma_5)d\right]$. In a natural theory of CP violation, a,b,c and d are in general complex. If both currents are pure V-A, i.e., a+b = c+d = 0, as we have required to suppress CP violation by local operators of dimension five, there cannot be CP violation by the two quark local operator $j_\mu(0)\ j^{\mu\dagger}(0)$ in $|\Delta S| = 1$ processes, [the constants a and c may be complex; however, they cannot cause CP violation since they are overall factors].

In conclusion, two possible patterns of quark multiplets emerge for natural models of microweak CP violation. One is that the right-handed quarks of charge -1/3 are singlets. In this case, there is no further restriction on quark multiplets other than that implied by $N_V>0$. The other possible pattern is that all right-handed quarks of charge-1/3 are the states of highest $I_3(=I)$, of multiplets of the same weak isospin $I\neq 0$. It is then necessary that all left-handed quarks of charge -1/3 are the states of lowest I_3 of multiplets of the same weak isospin.

The condition of microweak CP violation is patently not met by vector models.[20]

In natural gauge theories with microweak CP violation, operators of dimension five are all suppressed, so that the $\Delta I=\frac{1}{2}$ rule must arise from selective enhancement of the $\Delta I=\frac{1}{2}$ (or octet) channel of the two-quark operator $j_\mu(0)j^{\mu\dagger}(0)$. The short distance enhancement discussed by Gaillard and Lee,[19] and Altarelli and Maiani[19] may be much bigger if there are more than four quarks, as noted by Kingsley et al.[20] The magnitude of hadronic matrix elements of the two-quark operator is being estimated by Kluberg-Stern (private communication).

IV. $|\Delta S| = 2$ PROCESSES

In order to investigate the size of $|\Delta S| = 2$ processes in natural gauge theories of microweak CP violation, we study the quark process $s + s \to d + d$ in the general context of such theories in the free quark approximation.

To fourth order in semiweak coupling this process is described by four Feynman diagrams, shown in Fig. 1. Their contributions may be summed, with the approximation of Ref. 21, into the form

$$
\begin{aligned}
T(s+s\to d+d) &= \frac{g^4}{2}\int \frac{d^4k}{(2\pi)^4}\,\frac{1}{(k^2-m_W{}^2)^2} \\
&\times\Biggl\{\gamma_\mu\left[T_+{}^L\left(\frac{1-\gamma_5}{2}\right)+T_+{}^R\left(\frac{1+\gamma_5}{2}\right)\right]\frac{\gamma\cdot k+M_D}{k^2-M_D{}^2}\,\gamma_\nu\left[T_-{}^L\left(\frac{1-\gamma_5}{2}\right)+T_-{}^R\left(\frac{1-\gamma_5}{2}\right)\right] \\
&\quad+\gamma_\nu\left[T_-{}^L\left(\frac{1-\gamma_5}{2}\right)+T_-{}^R\left(\frac{1+\gamma_5}{2}\right)\right]\frac{-\gamma\cdot k+M_D}{k^2-M_D{}^2}\,\gamma_\mu\left[T_+{}^L\left(\frac{1-\gamma_5}{2}\right)+T_+{}^R\left(\frac{1+\gamma_5}{2}\right)\right]\Biggr\} \\
&\times\Biggl\{\gamma_\mu\left[T_-{}^L\left(\frac{1-\gamma_5}{2}\right)+T_-{}^R\left(\frac{1+\gamma_5}{2}\right)\right]\frac{-\gamma\cdot k+M_D}{k^2-M_D{}^2}\,\gamma_\nu\left[T_+{}^L\left(\frac{1-\gamma_5}{2}\right)+T_+{}^R\left(\frac{1+\gamma_5}{2}\right)\right] \\
&\quad+\gamma_\nu\left[T_+{}^L\left(\frac{1-\gamma_5}{2}\right)+T_+{}^R\left(\frac{1+\gamma_5}{2}\right)\right]\frac{\gamma\cdot k+M_D}{k^2-M_D{}^2}\,\gamma_\mu\left[T_-{}^L\left(\frac{1-\gamma_5}{2}\right)+T_+{}^R\left(\frac{1+\gamma_5}{2}\right)\right]\Biggr\}. \qquad (4.1)
\end{aligned}
$$

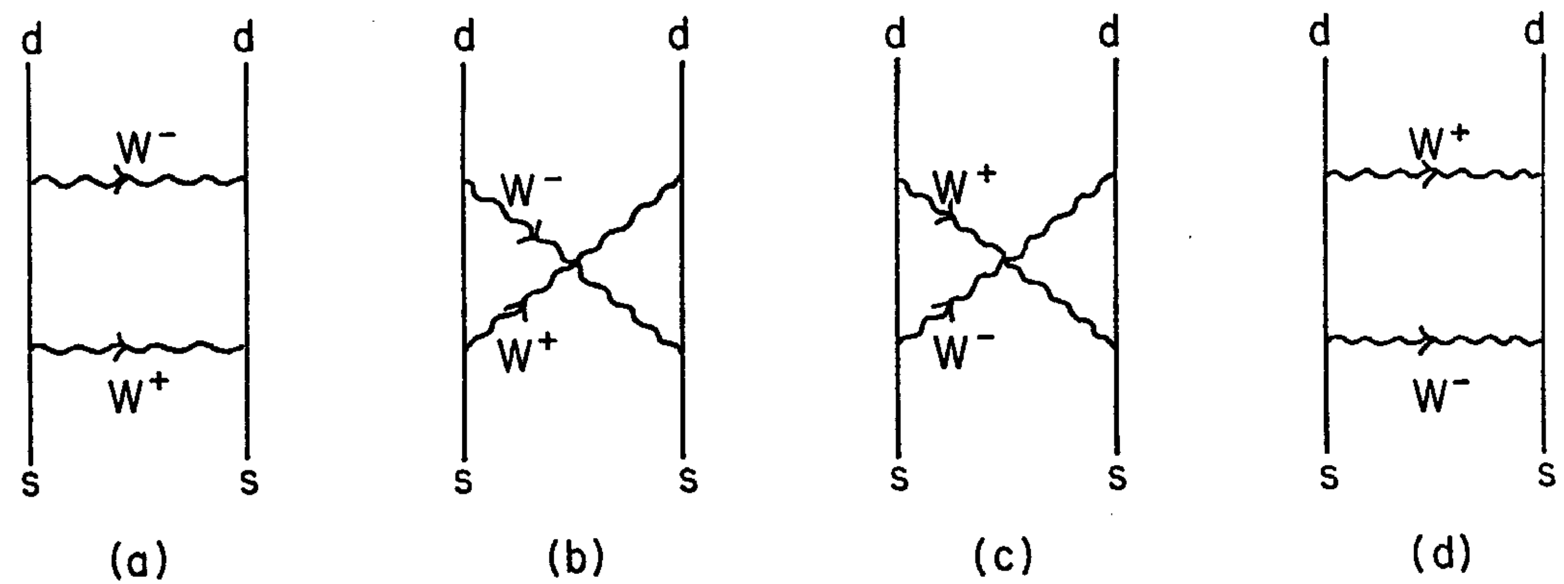


FIG. 1

Four diagrams which contribute to the process $s + s \to d + d$ to fourth in semiweak coupling.

There is an important consequence of the natural microweak condition (3.4). It is

$$T_{\pm}^{L} \frac{M_D}{k^2-M_D^2} T_{\mp}^{R} = T_{\pm}^{R} \frac{M_D}{k^2-M_D^2} T_{\mp}^{L} = 0 \quad . \tag{4.2}$$

After some manipulations, Eq. (4.1) can be simplified to

$$\begin{aligned} T(s+s \to dd) \simeq g^4 \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k^2-\bar{m}^2}\right)^4 \left(\frac{1}{k^2-m_W^2}\right)^2 k^2 \\ \times \Bigg\{ 5\left[\bar{d}L_-\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)s + \bar{d}R_-\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)s\right]^2 \\ -3\left[\bar{d}L_+\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)s - \bar{d}R_+\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)s\right]^2 \Bigg\} \end{aligned} \tag{4.3}$$

where $\bar{m}$ is the average quark mass and

$$L_{\pm} = T_{+}^{L} M_D^{\,2} T_{-}^{L} \pm T_{-}^{L} M_D^{\,2} T_{+}^{L}$$

and

$$R_{\pm} = T_{+}^{R} M_D^{\,2} T_{-}^{R} \pm T_{-}^{R} M_D^{\,2} T_{+}^{R} \quad . \tag{4.4}$$

The magnitude of the matrix elements of $L_{\pm}$ and $R_{\pm}$ between the d- and s-quarks is of order of $x\Delta m^2$, where x is a complex constant modulus of order less than one and which depends on mixing angles and the CP violating phase, and Δm^2 is some typical mass difference of quarks of charge 2/3 or -4/3, since if the quarks of charge 2/3 were degenerate and the quarks of charge -4/3 were degenerate, then these matrix elements would vanish. The integral on the right-hand side of Eq. (4.3) is

$$\int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2-\bar{m}^2)^4(k^2-m_W^2)} \sim \frac{i}{16\pi^2\bar{m}^2 m_W^2} \quad ,$$

so that

$$|T(s+s \to d+d)| \lesssim \frac{G_F}{\pi}\,\alpha\left(\frac{\Delta_1 m^2 \Delta_2 m^2}{\bar{m}^2 m_W^{\,2}}\right) x_1 x_2 \tag{4.5}$$

where $\Delta_{1,2}m^2$ refer to certain mass differences among quarks of given charges, and $x_{1,2}$ are complex constants of modulus of order less than one. As has been shown elsewhere,[21] the magnitude of Eq. (4.5) is adequate to explain the observed $K_L K_S$ mass difference, for certain ranges of quark mass differences and mixing angles.

In a naturally CP violating theory, $T_\pm^{\;L}$ and $T_\pm^{\;R}$ are arbitrary, aside from the hermiticity and the commutation relations which they must satisfy. The matrix elements of $T_\pm^{\;L}$ and $T_\pm^{\;R}$ are in general complex and in magnitude of order one. Models of microweak CP violation do not predict the size of ε. Rather, it is possible in these theories to choose the CP violating phase so that ε is what it is, $\sim 2\times 10^{-3}$.

V. ELECTRIC DIPOLE MOMENT

We can generalize the conditions for microweak CP violation for $|\Delta S| = 1$ processes deduced in Section III, and consider a restricted class of theories in the following way. We demand that (1) <u>all quarks of a given charge and a given chirality have the same I^2 and I_3</u>, and (2) <u>quarks of charge q and quarks of charge q+1 do not belong to the same weak isomultiplets for at least one chirality, for any q</u>. The first of these conditions is equivalent to natural conservation of all flavors by the neutral current to order $G_F\alpha$. Both conditions are straightforward extentions of the conditions derived in Section III; these extended conditions are even less directly motivated by experimental observations than the microweak condition for CP violation in $|\Delta S| = 1$ processes. We entertain these conditions because in theories which satisfy these conditions, the electric dipole moment of any quarks vanish to order G_F . The present upper limit[14] on the electric dipole moment of the neutron is barely compatible with theoretical expectations based on some milliweak theories.[15]

To lowest order in weak coupling, there are three classes of diagrams, shown in Fig. 2, contributing to the electromagnetic form factor of a quark.[22] The contribution of Fig. 2a is

$$F_\lambda{}^{(a)}(p,q) = -i\,g^2 \lim_{\xi\to 0}\int \frac{d^4k}{(2\pi)^4}\,\gamma_\alpha\left[T_\pm{}^L\left(\frac{1-\gamma_5}{2}\right) + T_\pm{}^R\left(\frac{1+\gamma_5}{2}\right)\right]$$

$$\times\,\frac{\not p+\not q/2+\not k+M_D}{(p+q/2+k)^2-M_D{}^2}\,\gamma_\lambda Q\,\frac{\not p-\not q/2+\not k+M_D}{(p-q/2+k)^2-M_D{}^2}\,\gamma_\beta\left[T_\mp{}^L\left(\frac{1-\gamma_5}{2}\right) + T_\mp{}^R\left(\frac{1+\gamma_5}{2}\right)\right]$$

$$\times\left(g^{\alpha\beta} + \frac{k^\alpha k^\beta(1-\xi)}{k^2\xi-M_W{}^2}\right)\frac{1}{k^2-M_W{}^2} \quad . \tag{5.1}$$

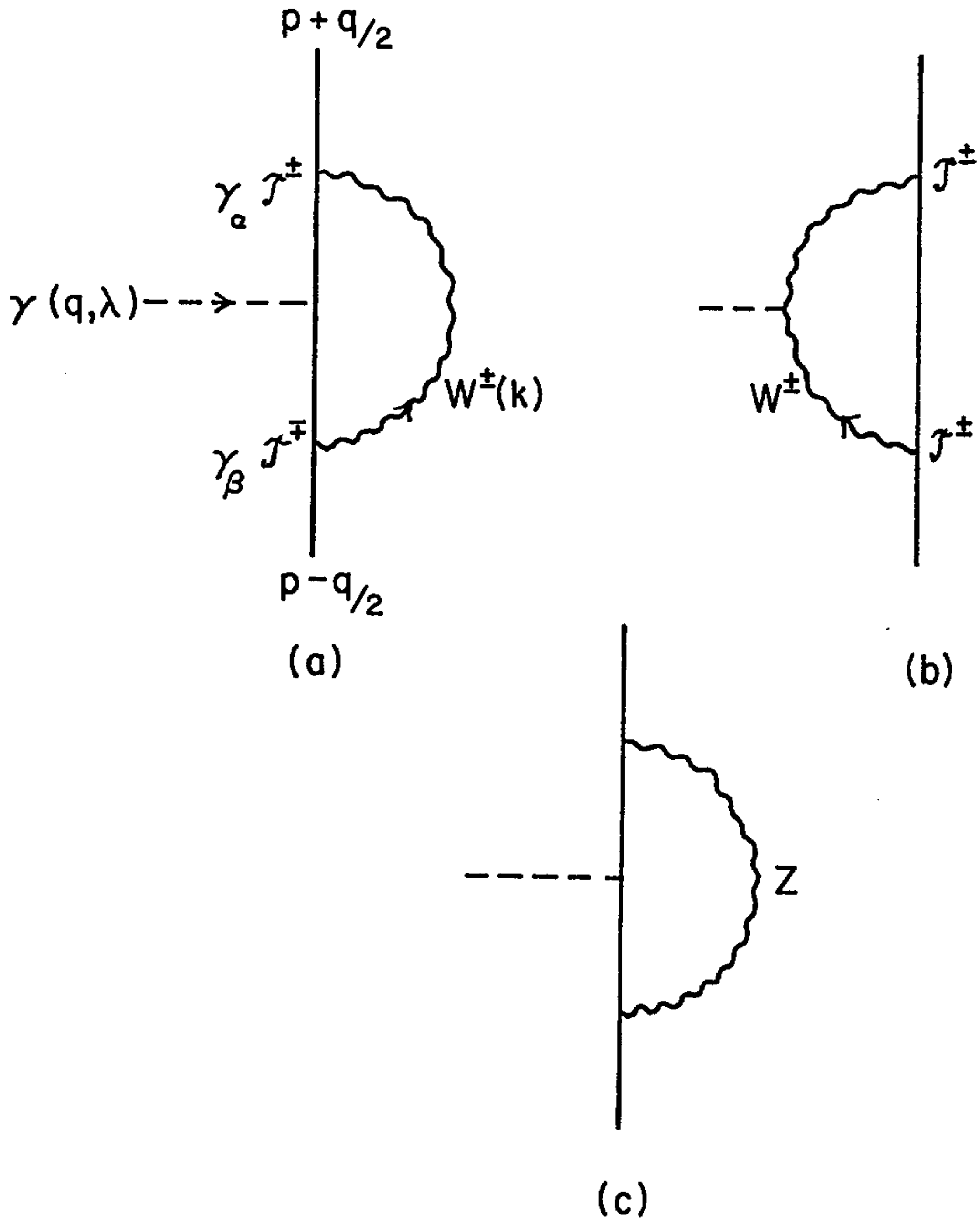


FIG. 2

Three classes of diagrams which contribute to the electromagnetic vertex of a quark in order g^2. The straight, wavy, and dotted lines represent the quarks, gauge boson and photon, respectively.

We are using the R_ξ-gauge[23] in the limit $\xi \to 0$ (this limit is to be taken after integrations, and not in the integrand); in this limit, the contributions from unphysical Higgs mesons vanish.[24]

The conditions (1) and (2) above imply that

$$T_\pm^L X T_\mp^R = T_\pm^R X T_\mp^L = 0 \tag{5.2}$$

where X is any matrix which commutes with Q. Thus, Eq. (5.1) simplifies to

$$\begin{aligned}
F_\lambda^{(a)} = & -ig^2 \lim_{\xi\to 0} \int \frac{d^4k}{(2\pi)^4} \left(g^{\alpha\beta} \to \frac{k^\alpha k^\beta}{\xi k^2 - m_W^2}\right) \frac{1}{k^2-m_W^2} \\
& \times \Bigg\{ \gamma_\alpha (\not{p} + \frac{\not{q}}{2} + \not{k}) \gamma_\lambda (\not{p} - \frac{\not{q}}{2} + \not{k}) \gamma_\beta \\
& \times \Bigg[T_\pm^L \frac{1}{(p+k+q/2)^2 - M_D^2} Q \frac{1}{(p+k-q/2)^2 - M_D^2} T_\mp^L \left(\frac{1-\gamma_5}{2}\right) \\
& + T_\pm^R \frac{1}{(p+k+q/2)^2 - M_D^2} Q \frac{1}{(p+k-q/2)^2 - M_D^2} T_\mp^R \left(\frac{1+\gamma_5}{2}\right) \Bigg] \\
& + \gamma_\alpha \gamma_\lambda \gamma_\beta \Bigg[T_\pm^L \frac{M_D}{(p+k+q/2)^2 - M_D^2} Q \frac{M_D}{(p+k-q/2)^2 - M_D^2} T_\mp^L \left(\frac{1-\gamma_5}{2}\right) \\
& + T_\pm^R \frac{M_D}{(p+k+q/2)^2 - M_D^2} Q \frac{M_D}{(p+k-q/2)^2 - M_D^2} T_\mp^R \left(\frac{1+\gamma_5}{2}\right) \Bigg] \Bigg\} .
\end{aligned} \tag{5.3}$$

We use the exponential parametrization of the propagator:

$$\frac{i}{k^2 - \mu^2 + i\varepsilon} = \int_0^\infty d\alpha \, e^{i\alpha(k^2 - \mu^2 + i\varepsilon)}$$

and perform the momentum space integration. Rotating the paths of integration over α by $+90^\circ$, we obtain terms of the form, for example,

$$F_\lambda(p,q) \sim \not{p}\gamma_\mu\not{q} \int_0^\infty d\alpha_1 d\alpha_2 \cdots \frac{P(\alpha)}{Q(\alpha)} e^{-f(p^2,q^2,p\cdot q,m_W{}^2,\xi;\alpha)}$$

$$\mathcal{T}_\pm{}^L \, e^{-\alpha_1 M_D{}^2} \, Q \, e^{-\alpha_2 M_D{}^2} \, \mathcal{T}_\mp{}^L \quad . \tag{5.4}$$

The functions f, P and Q are real for real values of the arguments, and homogeneous in α's.

The electromagnetic form factor $F_\lambda(p,q)$ conserves time-reversal invariance (thus CP invariance, by the TCP theorem) if it satisfies, for $q^2 \leq 0$ (see Appendix B):

$$T\, F_\lambda{}^*(p,-q)T^{-1} = g_{\lambda\lambda}F_\lambda(p,q) \tag{5.5}$$

where $T \sim i\gamma_1\gamma_3$ satisfies

$$T\gamma_\mu{}^* T^{-1} = g_{\mu\mu}\gamma_\mu \tag{5.6}$$

so that

$$T\gamma^*\cdot pT^{-1} = \gamma\cdot p \; ; \; T\gamma^*\cdot(-q)T^{-1} = \gamma\cdot q \quad . \tag{5.7}$$

Thus we see that the term of Eq. (5.4) is CP conserving if

$$\mathcal{T}_\pm{}^L e^{-\alpha_1 M_D{}^2} \, Q \, e^{-\alpha_2 M_D{}^2} \, \mathcal{T}_\mp{}^L \tag{5.8}$$

is real. Diagonal elements of such a matrix are real because $M_D{}^2$ and Q commute, and

$$\left[\mathcal{T}_\pm{}^{L,R}\right]^* = \left[\mathcal{T}_\mp{}^{L,R}\right]^T \quad . \tag{5.9}$$

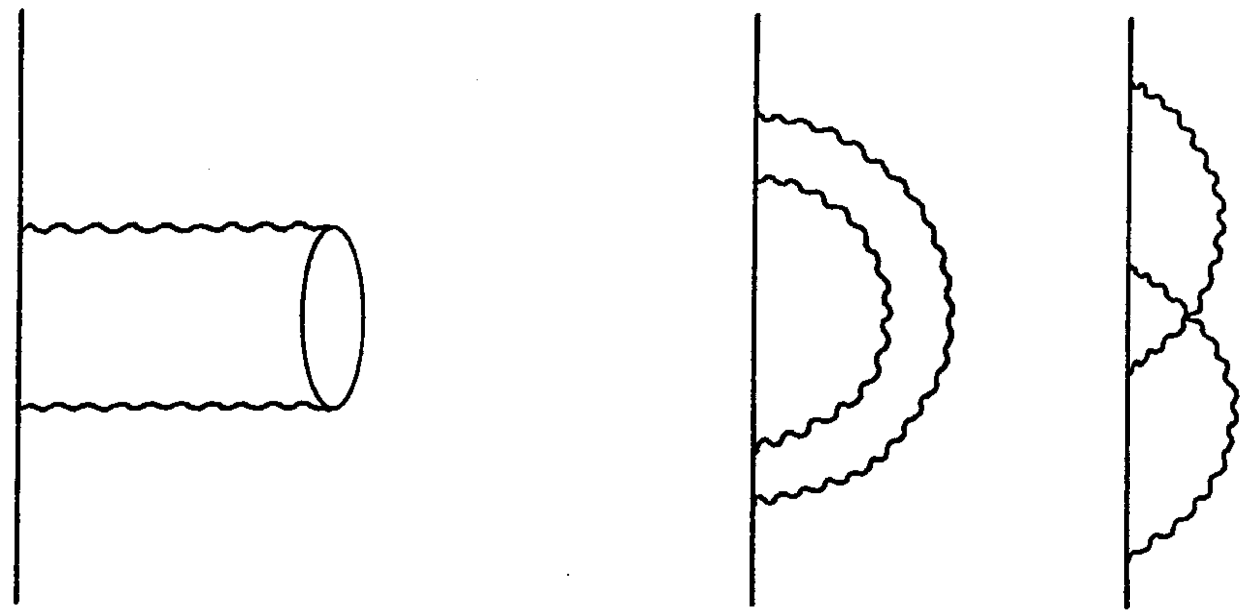
This argument may be applied to each term arising from the expression (5.3) to show that diagrams in Fig. 2a do not cause CP violation. The argument is unaffected by emission and absorption of color gluons by the quark line.

Similar arguments can be given for diagrams of Figs. 2b,c, to show that the electromagnetic vertex diagonal in flavor conserves CP to order g^2. The crux of the argument is again Eq. (5.2) for Fig. 2b; for Fig. 2c, it is the fact that the Z-boson couplings to quarks are diagonal in flavor and therefore real and CP-conserving.

In fourth order in g, there are two classes of diagrams to be considered separately, which are shown in Fig. 3. The photon line is to be attached to each line carrying a charge in all possible ways. Other diagrams not in these classes are easily seen not to cause CP violation by extention of arguments used for order g^2. It is easy to show that diagrams of Fig. 3a cannot violate CP. In the flavor space the corresponding amplitudes have the form

$$T_{\pm}^{(L,R)} Z \, T_{\mp}^{(L,R)} \mathrm{Tr}\left[T_{\pm}^{(L,R)} X T_{\mp}^{(L,R)} Y\right] \tag{5.10}$$

after momentum-space integrations, where X,Y and Z are diagonal in flavors and real. Diagonal elements of the above expression are real, due to the property (5.9).



(a) (b)

FIG. 3

Two classes of diagrams which may cause CP violation in order g^4. The photon line is to be attached to charged lines in all possible ways. The above are "skeletal" grapsh, from which all internal gluon lines have been removed.

In the flavor space, the amplitudes corresponding to the class of diagrams in Fig. 3b have the form

$$\int d\alpha_1 d\alpha_2 d\alpha_3 \cdots \frac{P(\alpha)}{Q(\alpha)}\, e^{-f(p^2,q^2,p\cdot q,m_W{}^2,\xi;\alpha)} M \tag{5.11}$$

where P, Q, and f are real and homogeneous in α's. To compute static moments it suffices to put $q^2=p\cdot q=0$. The matrix M must have one of the following forms:

$$\begin{array}{ll} \text{a.} & T_-{}^L X_1 T_+{}^L X_2 T_-{}^L X_3 T_+{}^L \quad , \\ \text{b.} & T_-{}^L X_1 T_-{}^L X_2 T_+{}^L X_3 T_+{}^L \quad , \\ \text{c.} & T_-{}^L X_1 M_D T_-{}^R X_2 T_+{}^R X_3 M_D T_+{}^L \quad , \\ \text{d.} & T_+{}^L X_1 T_-{}^L X_2 T_-{}^L X_3 T_+{}^L \quad , \\ \text{e.} & T_+{}^R X_1 T_-{}^R X_2 M_D T_-{}^L X_3 T_+{}^L \quad , \end{array} \tag{5.12}$$

and those obtained from the above by exchanges of L and R, and + and -, where X_i has one of the forms:

$$X_i = \begin{pmatrix} 1 \\ Q \\ M_D Q M_D \end{pmatrix} e^{-\alpha_i M_D{}^2}$$

and is real and diagonal in flavor. The other possible structures vanish due to Eq. (5.2).

Diagonal elements of the matrix M are in general complex and the electromagnetic vertex F_λ violates CP in this order.

However, to order $M_D{}^2$, these diagonal elements persist to be real. This assertion can be verified for all twenty cases, by the use of the relations (5.9) and

$$\left[Q, T_\pm{}^{L,R}\right] = \pm T_\pm{}^{L,R} \quad , \tag{5.13}$$

$$\left[Q, M_D\right] = 0 \quad , \tag{5.14}$$

and

$$T_\pm T_\mp = (I^2 - I_3{}^2 \pm I_3)/2 \quad , \tag{5.15}$$

and by noting that Q, M_D and $T_\pm T_\mp$ are real diagonal. Thus, CP violation arises only in order $g^4(m^2/m_W{}^2)$.

Therefore, we conclude that the electromagnetic moment of a quark in theories specified at the beginning of this section is of order, ignoring possible logarithmic factors, of

$$d_q \sim \frac{1}{\pi^4}\, g\left(\frac{m_q}{m_W{}^2}\right)\left(\frac{m_q{}^2}{m_W{}^2}\right)^2 \varepsilon$$

where m_q is the generic quark mass of order of a few GeV. Thus

$$d_q \sim \frac{1}{\pi^4}\, G_F{}^2\left(\frac{m_q{}^5}{m_W{}^2}\right)\varepsilon \sim 10^{-30}\mathrm{cm} \quad ,$$

with $m_W \approx 60$ GeV, $m_q \approx 3$ GeV.

VI. MODELS

In this section we shall give several examples of models in which the conditions for microweak CP violation are naturally satisfied. The examples we will discuss have only singlets and doublets of quarks under weak $SU(2)\times U(1)$, and only one CP violating phase.

The first example is the six-quark model of Kobayashi and Maskawa,[25] which assigns six quarks to

$$\begin{pmatrix} u_1 \\ d_1 \end{pmatrix}_L \quad \begin{pmatrix} u_2 \\ d_2 \end{pmatrix}_L \quad \begin{pmatrix} u_3 \\ d_3 \end{pmatrix}_L \; ; \quad \begin{matrix} u_R, & c_R, & t_R, \\ d_R, & s_R, & t_R \,. \end{matrix} \tag{6.1}$$

where the u and d are quarks of charge 2/3 and -1/3, respectively. As the authors have shown, this system, with arbitrary values of parameters, has one CP violating phase; this can be readily verified from Eq. (2.19):

$$N_L(Q=2/3) = N_R(Q=2/3) =$$

$$= N_L(Q=-1/3) = N_R(Q = -1/3) = 3 \;,$$

$$N_L(1/2,1/3) = N_R(0,4/3) = N_R(0,-2/3) = 3 \;.$$

The leptons are placed in the following multiplets:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_L \; ; \quad e_R, \quad \mu_R, \quad \ell_R.$$

Since all three neutrinos are massless, we can always call the neutrino associated with the electron the electron-neutrino, etc. It is then clear that there is no CP violation in the lepton sector. One complex Higgs doublet, as in the original Weinberg-Salam proposal, is sufficient to generate masses for the quarks and leptons (e, μ and ℓ). The Higgs potential for this system is necessarily CP conserving. This model has been examined in great detail by Pakvasa and Sugawara,[26] Maiani,[22] and Ellis, Gaillard and Nanopoulos.[22]

The second model we shall consider assigns six quarks to the following multiplets:

$$\begin{pmatrix} u_1 \\ d_1 \end{pmatrix}_L, \quad \begin{pmatrix} u_2 \\ d_2 \end{pmatrix}_L; \quad \begin{matrix} u_R \\ \begin{pmatrix} d_1 \\ x_1 \end{pmatrix}_R \end{matrix} \quad \begin{matrix} c_R \\ \begin{pmatrix} d_2 \\ x_2 \end{pmatrix} \end{matrix}$$
$$x_L \quad , \quad y_L \tag{6.2}$$

where x and y are quarks of charge -4/3. According to Eq. (2.19), this model contains one CP violating phase. It is convenient to express the quark multiplet structure in terms of mass eigenstates, u, c, d, s, x and y. One possible representation is

$$\begin{bmatrix} u\cos\theta - c\sin\theta \\ d \end{bmatrix}_L \begin{bmatrix} u\sin\theta + c\cos\theta \\ s \end{bmatrix}_L; \begin{matrix} u_R \\ \begin{bmatrix} e^{i\alpha}d \\ x\cos\beta - y\sin\beta \end{bmatrix}_R \end{matrix} \begin{matrix} c_R \\ \begin{bmatrix} e^{-i\alpha}s \\ x\sin\beta + y\cos\beta \end{bmatrix}_R \end{matrix},$$
$$x_L \qquad y_L \tag{6.3}$$

where θ is the Cabibbo angle, and β is a new mixing angle. α is the CP violating phase.

This model has been discussed in other contexts by Glashow and Weinberg,[17] Barnett,[27] and more recently by Albright, Quigg and Shrock,[28] without, however, the CP violating phase α.

Barnett postulates the lepton family consisting of doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L; \begin{pmatrix} E^{+++} \\ E^{++}\cos\gamma - M^{++}\sin\gamma \end{pmatrix}_R, \begin{pmatrix} M^{+++} \\ E^{++}\sin\gamma + M^{++}\cos\gamma \end{pmatrix}_R. \tag{6.4}$$

and six singlets, e_R, μ_R, $(E^{+++})_R$, etc. The leptonic sector does not accomodate a CP violating phase.

To generate fermion masses, we require at least two Higgs multiplets -- a complex doublet:

$$h = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}, \qquad \tilde{h} = i\sigma_2 h^* = \begin{pmatrix} \bar{h}^0 \\ -h^- \end{pmatrix}, \tag{6.5}$$

and a complex triplet:

$$H = \begin{pmatrix} H^+/\sqrt{2} & H^{++} \\ H^0 & -H^+/\sqrt{2} \end{pmatrix} . \tag{6.6}$$

The Higgs potential

$$\begin{aligned} V(H,h) &= \alpha(h^\dagger h)^2 + \beta(\mathrm{Tr}\, H^\dagger H)^2 + \gamma(\mathrm{Tr}H^\dagger H)(h^\dagger h) + \varepsilon\mathrm{Tr}(H^\dagger H)^2 \\ &\quad + \delta(h^\dagger H\tilde{h}) + \delta^*(\tilde{h}^\dagger H^\dagger h) \\ &\quad + \lambda(h^\dagger h) + \mu\mathrm{Tr}HH^\dagger \end{aligned} \tag{6.7}$$

where α, β, γ, ε, λ and μ are real, is CP conserving, with the definition

$$\begin{aligned} \mathrm{CP} \qquad & H \to \frac{\delta^*}{\delta} H^*(-\underset{\sim}{x}, x_0), \\ & h \to e^{i\phi} h^*(-\underset{\sim}{x}, x_0). \end{aligned} \tag{6.8}$$

ϕ being arbitrary. As discussed in Appendix, with one doublet h and one triplet H, single exchange of a physical Higgs meson between two quarks is CP conserving in $|\Delta S| = 1$ processes.

Current interest in this model is due to the possibility that the high y anomaly observed in inclusive antineutrino interactions,[29] and the increase in the ratio of neutrino and antineutrino charged-current cross sections above certain energy[29] may be due to the excitation of a quark of charge -4/3, for example,

$$\bar{\nu} + d \rightarrow \mu^+ + x \quad .$$

This circumstance has been analyzed by Barnett,[27] and more recently by Albright, Quigg and Shrock;[28] as emphasized by the latter authors, the high y anomaly effect should be more pronounced with the neutron target than with the proton, if this model is right. In any SU(2) × U(1) model the mass ratio of the Z boson and the W boson is given by[30]

$$\left(\frac{m_Z}{m_W}\right)^2 = \left(\frac{g^2+g'^2}{g^2}\right)\frac{2\sum I_3{}^2\lambda^2_{I,I_3}}{\sum(I^2-I_3{}^2+I)\lambda^2_{I,I_3}} \tag{6.9}$$

where λ_{I,I_3} is the vacuum expectation value of the neutral member with I_3 of a Higgs multiplet with isospin I. In this model with one Higgs doublet h and one triplet H, we have

$$\frac{m_Z}{m_W} = \frac{1}{\cos\theta_W}\left[\frac{1+4(\lambda_{1,-1}/\lambda_{\frac{1}{2},-\frac{1}{2}})^2}{1+2(\lambda_{1,-1}/\lambda_{\frac{1}{2},-\frac{1}{2}})^2}\right]^{\frac{1}{2}}$$

or

$$1 \leq \frac{m_Z}{m_W}\cos\theta_W \leq 2 \quad . \tag{6.10}$$

In this model, as well as in the model of Kabayashi and Maskawa,[25] the stronger conditions of Sec. V hold so that the electric dipole moment of the neutron is expected to be of order of 10^{-30}cm. In this model there are processes in which CP violation is milliweak, rather than microweak. They are

$$(x,y) \to d + s + (\bar{c},\bar{u}) \tag{6.11}$$

where $(x,y)_R \to d_R + s_L + (\overline{c_L},\overline{u_L})$ and $(x,y)_R \to s_R + d_L + (\overline{c_L},\overline{u_L})$ interfere with a CP-violating relative phase $e^{2i\alpha}$. CP violation in charmed particle decays in this model is expected to be very similar to that in the model of Kobayashi and Maskawa,[25] which has been discussed by Ellis, et al.[22]

Lastly, a trivial modification on Eq. (6.2) gives the third example. We write

$$\begin{matrix} x_L & y_L \\ \begin{pmatrix} u_1 \\ d_1 \end{pmatrix}_L & \begin{pmatrix} u_2 \\ d_2 \end{pmatrix}_L \end{matrix} \quad ; \quad \begin{matrix} \begin{pmatrix} x_1 \\ u_1 \end{pmatrix}_R & \begin{pmatrix} x_2 \\ u_2 \end{pmatrix}_R \\ d_R & s_R \end{matrix}$$

where, this time, x and y are quarks of charge +5/3. As far as CP violation in $|\Delta S| = 1$ processes go, this example is very similar to the second one.

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APPENDIX A - HIGGS MESONS

Single exchange of physical Higgs meson can in general cause CP violation of order $G_F(m/m_H)^2\varepsilon$ where m_H is the typical mass scale of physical Higgs mesons. The lower limit that can be deduced theoretically on m_H is of order of several GeV.[31] If $m_H \gtrsim m_W$, then CP violation caused by Higgs meson exchange is microweak automatically.

We shall investigate in this Appendix the condition that single exchange of a physical Higgs meson is CP conserving in $|\Delta S| = 1$ processes, with the additional assumption that all quarks of charge 2/3 have the same I_L^2, I_R^2, $(I_L)_3$ and $(I_R)_3$. (This assumption is stronger than the natural conservation of strangeness by the neutral current, and presupposes natural conservation by the neutral current of flavors associated with quarks of charge 2/3 -- charm, for example.).

From the requirements we have derived for a natural theory of microweak CP violation, we find that masses of the u-, d- and s-quarks must arise entirely from vacuum expectations values of neutral Higgs fields, because it is not possible to form invariant bilinear couplings involving the right-handed u-quark and the left-handed u-quark, etc. We shall denote by U_L and U_R the multiplets to which the left-handed u-quark and right-handed u-quarks belong, respectively. We shall denote Higgs multiplets by H^k and their vacuum expectation values by λ_k. We choose the phases of H^k so that all λ_k are real nonnegative. We can write U_L and U_R in the form

$$U_L = \begin{bmatrix} u \\ xd+ys+\cdots \end{bmatrix}_L , \qquad U_R = \begin{bmatrix} \\ u \end{bmatrix}_R .$$

where we can always arrange the phases of the d- and s-quarks so that the coefficients x and y are real, and

$$x^2 + y^2 \leq 1$$

The u-quark mass arises from terms of the form

$$\sum_k U_L^\dagger a_k U_R H^k + \text{h.c.}$$

where the a_k are in general complex matrices, subject to the condition

$$\sum_k u_L^\dagger a_k u_R \lambda_k = m_u u_L^\dagger u_R$$

where m_u is real positive. In order that the coupling of neutral Higgs mesons to $u_L^\dagger u_R$ by CP conserving, it is necessary that all a_k be real. This requires that there be one and only one Higgs multiplet whose neutral member couples to the quarks of charge 2/3. In this case, $a_1 = m_u/\lambda$, which is positive. Once a_1 is positive, we see that the coupling of members of the Higgs multiplet H^1 to $d_L^\dagger u_R$ and $s_L^\dagger u_R$ are automatically CP conserving.

Similar arguments can be extended to couplings involving

$d_L^\dagger d_R$, $s_L^\dagger s_R$, $d_L^\dagger s_R$ and $s_L^\dagger d_R$. In conclusion, we find that the conditions that single Higgs exchange conserves CP in $|\Delta S| = 1$ processes is that <u>the quarks of charge 2/3 receive their masses through the couplings to one and only one Higgs multiplet, and the quarks of charge -1/3 receive their masses through the couplings to one and only one Higgs multiplet</u>. The two Higgs multiplets may or may not be identical. Further, there should be no Higgs multiplet which does not contribute to quark masses.

APPENDIX B - ELECTROMAGNETIC VERTEX

The electromagnetic vertex of a quark $F_\lambda(p,q)$ is defined as

$$\left[F_\lambda(p,q)\right]_{\alpha\beta} = (\not p + \frac{\not q}{2} - M_D)_{\alpha\rho} \int d^4x d^4y \; e^{i(p+q/2)\cdot x} e^{-i(p-q/2)\cdot y}$$

$$<0|(\psi^\rho(x) j_\mu(0) \bar\psi^\sigma(y))_+|0>(\not p - \frac{\not q}{2} - M_D)_{\sigma\beta}.$$

where j_μ is the electromagnetic moment, and the subscript + denotes chronological ordering. The relation (5.5) follows from this definition and the assumption that

$$<0|T(\psi^\rho(x) j_\mu(0) \bar\psi^\sigma(y))|0>^*$$

$$= <0|\mathcal{T}^\dagger \mathcal{T} T(\psi^\rho(x) j_\mu(0) \bar\psi^\sigma(y))|0>$$

where $\mathcal{T}$ is the time-reversal operator, which amounts to time-reversal invariance of the vacuum.

Thus,

$$F_\mu{}^*(p,-q) = T^{-1}(\not p + \frac{\not q}{2} - M_D) \int d^4x d^4y \; e^{i(p+q/2)\; x} e^{-i(p-q/2)\; y}$$

$$g_{\mu\mu} <0|(\psi^\rho(x) j_\mu(0) \bar\psi^\sigma(y))_-|0>(\not p - \not q/2 - M_D)T$$

where the subscript - denotes antichronological ordering. We have used the fact that

$$\begin{aligned} \mathcal{T} \;:\; & j_\mu(0) \rightarrow g_{\mu\mu} j_\mu(0) \;, \\ & \psi(x) \rightarrow T^{-1} \psi(\underset{\sim}{x}, -x_0) \\ & \bar\psi(x) \rightarrow \bar\psi(\underset{\sim}{x}, -x_0) T \end{aligned}$$

where $T \sim i\gamma_1\gamma_3$ satisfies

$$T\gamma_\mu T^{-1} = g_{\mu\mu}\gamma_\mu^{*} \quad .$$

For $q^2<0$, the Fourier transforms of the chronological and antichronological orderings are identical.

FIGURE CAPTIONS

Fig. 1 Four diagrams which contribute to the process $s + s \to d + d$ to fourth order in semiweak coupling.

Fig. 2 Three classes of diagrams which contribute to the electromagnetic vertex of a quark in order g^2. The straight, wavy, and dotted lines represent the quarks, gauge boson and photon, respectively.

Fig. 3 Two classes of diagrams which may cause CP violation in order g^4. The photon line is to be attached to charged lines in all possible ways. The above are "skeletal" graphs, from which all internal gluon lines have been removed.

REFERENCES

[1]J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay, Phys. Rev. Lett. 13, 138 (1964).

[2]L. Wolfenstein, Phys. Rev. Lett. 13, 562 (1964).

[3]For the latest review on this subject, see K. Kleinknecht, in Proceedings of the XVII International Conference on High Energy Physics, London, 1974, edited by J.R. Smith (Rutherford Laboratory, Chilton, England, 1974), p. III-23.

[4]This subject has been reported on since 1972 at all major international conferences on high energy physics: (a) B.W. Lee, in Proceedings of the XVI International Conferences on High Energy Physics, ed. J.D. Jackson and A. Roberts (Fermilab, Batavia, Illinois, 1972) Vol. IV, p. 249; (b) M. Veltman, in Proceedings of the 6th International Symposium on Electron and Photon Interactions at High Energies, ed. H. Rollnik and W. Pfeil (North Holland, Amsterdam and London, 1973, p. 429; (c) C. H. Llewelyn-Smith, in the Bonn Conference, loc. cit., p. 469; (d) S. Weinberg, in "11^{e} Conference Internationale sur les Particules Elementaires," Supplement au Journal de Physique 34, 1973, p. 45; (e) J. Iliopoulos, in Proceedings of the XVII International Conference on High Energy Physics (Rutherford Laboratory, 1974) p. III 89; (f) G. 't Hofft, invited talk at Palermo Conference, 1975 (to be published).

[5]Previous attempts to incorporate CP violation in a gauge theory include: R.N. Mohapatra, Phys. Rev. D6, 2023 (1972);

M.A. Bég and A. Zee, Phys. Rev. D8, 1460 (1973); A. Pais, Phys. Rev. D8, 625 (1973); T.D. Lee, Phys. Rev. D8, 1226 (1973); T.D. Lee, Phys. Rep. C9, 148 (1974); A. Zee, Phys. Rev. D4, 1772 (1974); R.N. Mohapatra and J.C. Pati, Phys. Rev. D11, 566 (1975); J. Frenkel and M.E. Ebel, Nucl. Phys. B83, 177 (1974). More recent papers will be cited elsewhere in this paper. For review of this subject as of 1974, see R.N. Mohapatra, in Particles and Fields - 1974, proceedings of the Williamsburg meetings of the Division of Particles and Fields of the APS, edited by Carl E. Carlson (AIP, New York, 1975), p. 127.

[6] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam in Elementary Particle Physics, edited by N. Svartholm (Almquist and Witsell, Stockholm, 1968), p. 367. See also S.L. Glashow, Nucl. Phys. 22, 579 (1961); A. Salam and J.C. Ward, Phys. Lett. 13, 1681 (1964).

[7] W.A. Bardeen, H. Fritzsch, and M. Gell-Mann, unpublished; H. Fritzsch and M. Gell-Mann, in Proceedings of the XVI International Conference on High Energy Physics (ref.4), Vol. 2, P. 134; D. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); Phys. Rev. D8, 3633 (1973); H.D. Politzer, Phys. Rev. Lett. 30, 1346 (1973); G.'t Hooft, unpublished; S. Weinberg, Phys. Rev. Lett. 31, 494 (1973).

[8] R.N. Mohapatra, J.C. Pati and L. Wolfenstein, Phys. Rev. D11, 3319 (1973).

[9]For attempts in this direction, see T.D. Lee, Ref. 5; H. Georgi and A. Pais, Phys. D10, 1246 (1974); F. Wilczek and A. Zee, to be published

[10]S. Weinberg, Phys. Rev. Lett. 37, 657 (1976).

[11]T.T. Wu and C.N. Yang, Phys. Rev. Lett. 13, 380 (1964); L. Wolfenstein, Nuovo Cimento 42, 17 (1966); T.D. Lee and C.S. Wu, Ann. Rev. Nucl. Sci. 16, 511 (1966).

[12]There are gauge theories of superweak CP violation which evade our premises: Ref. 8 and P. Sikivie, Phys. Lett. 65B, 141 (1976). In Sikivie's model, the W-boson couplings to fermions are CP conserving, and superweak CP violation occurs through very massive (or very weakly-coupled) Higgs boson exchange.

[13]I am referring here the cases in which CP violating Hamiltonian has $\Delta I=\frac{1}{2}$, or more generally satisfies certain symmetry properties. See for example S. Weinberg, Phys. Rev. 110, 782 (1958); S. Weinberg, Ref. 8; R.N. Mohapatra and J.C. Pati, in Ref. 5.

[14]N.F. Ramsey, private communication; in Proceedings of the Neutrino - 1975 Conference, (Hungarian Physical Soc., Budapest, 1975), Vol. I, p. 307.

[15]L. Wolfenstein, Nucl. Phys. B77, 375 (1974); invited talk at 1976 Neutrino Conference - Aachen.

[16]G. Goldhaber, F.M. Pierre et al., Phys. Rev. Lett. 37, 255 (1976); I. Peruzzi, M. Piccolo, et al., Phys. Rev. Lett. 37,

569 (1976); B. Knapp, P. Leung, W. Lee, et al., Phys. Rev. Lett. 37, 882 (1976).

[17]S.L. Glashow and S. Weinberg, to be published. See also, K. Kang and J.E. Kim, Phys. Lett. 64B, 93 (1976); E.A. Paschos, unpublished.

[18]We assume further that there is no zeroth order natural relation among particle masses and mixing angles. A relation of this sort arises when M is not arbitrary because of the representation contents of Higgs multiplets.

[19]K. Wilson, Phys. Rev. 179, 1499 (1969); M.K. Gaillard and B.W. Lee, Phys. Rev. Lett. 33, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. 52B, 351 (1974).

[20]A. De Rujula, H. Georgi and S.L. Glashow, Phys. Rev. D12, 3589 (1975); R.L. Kingsley, S.B. Treiman, F. Wilczek and A. Zee, Phys. Rev. D12, 2768 (1975); H. Fritzsch, M. Gell-Mann and P. Minkowski, Phys. Lett. 59B, 256 (1975); S. Pakvasa, W.A. Simmons and S.F. Tuan, Phys. Rev. Lett. 35, 703 (1975).

[21]M.K. Gaillard and B.W. Lee, Phys. Rev. D10, 897 (1974).

[22]The analysis below extends, and in some cases modifies, the analyses of previous authors to the general class of models of microweak CP violation; see, for instance, A. Pais and J.R. Primack, Phys. Rev. D8, 3063 (1973); L. Maiani, Phys. Lett. 68B, 183(1976); J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B 109, 213 (1976).

[23]K. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D6, 2923 (1972).

[24] K. Fujikawa, Phys. Rev. D7, 393 (1973). As Fujikawa notes, there are "anomalous" cases in which the contribution from unphysical Higgs scalars survives in the limit $\xi \to 0$. However, I do not think this happens in the case at hand. In any event, I have carried out a parallel analysis in the general R_ξ gauge. The description of that analysis is inordinately long, and adds no new physics. The conclusion to be reached is unaffected by the presence of unphysical Higgs scalars.

[25] M. Kobayashi and K. Maskawa, Progr. Theor. Phys. 49, 652 (1973). A concise review of this model is found in H. Harari, Les Houches lectures (1976), to be published.

[26] S. Pakvasa and H. Sugawara, Phys. Rev. D14, 305 (1976).

[27] M. Barnett, preprint, SLAC-PUB-1821 (1976).

[28] C. Albright, C. Quigg, and R.E. Shrock, in preparation.

[29] A. Benvenuti, et al., Phys. Rev. Lett. 36, 1478 (1976); ibid. 37, 189 (1976); B.C. Barish, preprint, CALT-68-544 (1976); F.A. Nezrick, preprint, FERMILAB-Conf-76/68-EXP.

[30] B.W. Lee, in Proceedings of the XVI International Conference on High Energy Physics, edited by J.D. Jackson and A. Roberts (National Accelerator Laboratory, Batavia, Illinois, 1972), Vol. IV, p. 266.

[31] S. Weinberg, Phys. Rev. Lett. 36, 294 (1976).