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Gauge Theories

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In quantum mechanics, an isolated charged particle, or a system which is electrically charged, is described by a complex wave function. The phase of such a wave function is not measurable. One may take the view that it should be possible to choose the phase of a complex wave function independently at every space-time point. This view leads naturally to the introduction of a "gauge" field, which is the electromagnetic field coupled to the electromagnetic current.

Consider a free charged particle in nonrelativistic quantum mechanics. Its behavior is governed by the Schrödinger equation ($\hbar = c = 1$)

$$- \frac{\nabla^2}{2m} \Psi(\underset{\sim}{x}, t) = i \frac{\partial}{\partial t} \Psi(\underset{\sim}{x}, t) \quad . \qquad (1)$$

In order that this equation be invariant under arbitrary change of phase

$$\Psi(\underset{\sim}{x}, t) \rightarrow \Psi'(\underset{\sim}{x}, t) = e^{-ie\Lambda(\underset{\sim}{x}, t)} \Psi(\underset{\sim}{x}, t) \qquad (2)$$

where e is a parameter (later to be identified with the unit of electric charge) and Λ is an arbitrary function of space-time), Equation (1) must be modified to read

$$- \frac{1}{2m} (\underset{\sim}{\nabla} - ie\underset{\sim}{A})^2 \Psi = (i\frac{\partial}{\partial t} + e\phi) \Psi \qquad (3)$$

where $(\phi, \underset{\sim}{A})$ are components of the four-vector electromagnetic potential, which undergo the transformation

$$\underset{\sim}{A}(\underset{\sim}{x},\underset{\sim}{t}) \rightarrow \underset{\sim}{A}'(\underset{\sim}{x},t) = \underset{\sim}{A}(\underset{\sim}{x},t) - \underset{\sim}{\nabla}\Lambda(\underset{\sim}{x},t) ,$$

$$\phi(\underset{\sim}{x},t) \rightarrow \phi'(\underset{\sim}{x},t) = \phi(\underset{\sim}{x},t) - \frac{\partial}{\partial t}\Lambda(\underset{\sim}{x},t) , \tag{4}$$

It is to be noted that the electric charge ρ and the electric current $\underset{\sim}{j}$:

$$\rho = e\Psi^{*}\Psi ,$$

$$\underset{\sim}{j} = \frac{e}{2im}\left[\Psi^{*}\underset{\sim}{\nabla}\Psi - (\underset{\sim}{\nabla}\Psi^{*})\Psi\right] - \frac{e}{m}\Psi^{*}\underset{\sim}{A}\Psi$$

and Maxwell's equations remain invariant under the gauge transformations (1) and (2).

Space-time dependent transformations of the form of Eqs. (2) and (3), which leave the equations of motion of a system form-invariant, are called gauge transformations (of the second kind). The importance of the concept of gauge invariance derives from the fact that it constrains the form of interaction of particles and fields; _a fortiori_ it gives the correct equations of motion for charged particles interacting with an electromagnetic field. The term "gauge" stems from H. Weyl's work in which he tried unsuccessfully to unify gravational and electromagnetic fields through consideration of scale (gauge) changes of space-time coördinates.

The second-quantized theory of relativistic charged particles with electromagnetic field, known as quantum electrodynamics, enjoys the quantized version of gauge invariance. Renormalizability of quantum electrodynamics, that is, the fact that infinities encountered in higher orders in perturbation expansion of the theory can be eliminated systematically by redefinitions of fundamental parameters of the theory, can be attributed ultimately to the gauge invariance that the theory possesses.

The foregoing discussion centered around the special case in which successive transformations are commutative, that is

$$\begin{aligned} \Psi \to \Psi'' &\equiv e^{-ie\Lambda_2}\left(e^{-ei\Lambda_1}\Psi\right) \\ &= e^{-ie\Lambda_1}\left(e^{-ie\Lambda_2}\Psi\right) \\ &= e^{-ie(\Lambda_1+\Lambda_2)}\Psi \quad . \end{aligned} \tag{6}$$

Gauge transformations of this type are known as <u>abelian</u>. One can consider a more general class of norm-preserving linear transformations on a multicomponent wave function. For example, an electron of negative helicity and its neutrino may be thought of as forming a two-component wave function

$$\Psi_L \equiv \begin{pmatrix} \Psi_\nu \\ \Psi_e \end{pmatrix}_L \tag{7}$$

where the subscript L refers to "left-handed," that is, negative helicity. One may consider two × two unitary transformations U acting on the two-component wave function:

$$\Psi_L \to \Psi_L' = U\Psi_L \ , \ U^{\dagger}U = 1 \quad , \tag{8}$$

wherein the unitary matrix U depends on space-time. It is possible to formulate a theory so that the energy density of the system is invariant under such transformations, as Yang and Mills showed. Two such transformations are not in general commutative, i.e., $U_1U_2 \neq U_2U_1$. Such gauge transformations are known as nonabelian. The two × two unitary matrix U may be written as

$$U = \exp i(\alpha_0 + \underset{\sim}{\tau}\cdot\underset{\sim}{\alpha})$$

where α_i, i=0,1,2,3, are real parameters, and τ_i's are two × two Pauli matrices. Two × two unitary matrices form a continuous group of transformations known as the unitary group in two dimensions, U(2). In gauge transformations of the second kind, the parameters of the group, α_i, are space-time dependent.

More generally, a gauge theory is a dynamical theory which entertains invariance under a continuous group of norm-preserving linear transformations on components of fields and particles acting independently at every space-time point. The group in question may be abelian (as in the case of electrodynamics) or nonabelian. There are as many "gauge" vector bosons, as there are parameters of the group.

The interactions amongst these gauge bosons, and of the gauge bosons with other fields, are constrained by the requirement of gauge invariance. Classically, gauge bosons are massless fields, just as the photon is massless in electrodynamics. In quantized theories, the gauge symmetry may be "spontaneously" broken, and in such a case, gauge bosons need not be massless.

Recent advances in the theory of weak interactions arose from a successful attempt to unify electromagnetism and weak interaction in a nonabelian gauge theory based on the group U(2). In this theory, the photon, charged intermediate vector bosons ($W^{\pm}$) whose existence has long been inferred, and a massive neutral vector boson (Z^0), are the physical manifestations of gauge vector mesons. The U(2) symmetry is "spontaneously" broken, by a mechanism first discussed by P. Higgs and others; this mechanism also endows gauge bosons other than the photon with masses. This theory gained credibility since the discovery of neutral current effects, in which muon-neutrinos are scattered off hadrons without turning into muons. A gauge theory of weak and electromagnetic interactions, when constructed judiciously, is renormalizable; interactions among the photon, $W^{\pm}$ and Z^0, and their interactions with other fields are constrained by the gauge invariance, and are such as to ensure renormalizability of the theory. A gauge theory of weak interactions requires the existence of a new quantum number (or "flavor" called "charm") and a quark which carries it. Particles which carry nonzero charm quantum number were definitively observed in 1976.

Strong interactions among quarks, and the manner in which quarks are confined in a hadron, may eventually find an explanation in a guage theory. Quarks carry a set of quantum numbers whimsically referred to as flavors. In addition, quarks carry a three-valued index, called color. Strong interactions among quarks are thought to be invariant under the group SU(3) acting on the three-valued index at every space-time point and are transmitted by gluons which are the gauge vector bosons associated with the color gauge symmetry. In such a theory, the infrared catastrophe due to vanishing mass of the gluons is so severe that the theory allows only colorless objects as particles that can be isolated; in configuration space, a quark and an antiquark, for instance, cannot be separated by a macroscopic distance, because the force acting between them grows progressively stronger as the separation gets larger. However, these are all conjectures at the moment. This subject, which is called quantum chromodynamics, does have a firm prediction: at short distances, the interquark forces are getting weaker, for which there is experimental evidence, albeit indirect.

There are further attempts to unify strong, weak and electromagnetic interactions all in a single gauge theory, and also to unify these interactions with gravitational interaction. It is much too early to assess the possible outcome of such theoretical endeavors today, however.

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