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SU(3) Content of the Pomeron at Very High Energies[†]

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ABSTRACT

We study the SU(3) content of the Pomeron at asymptotic energies in two types of model: s-channel absorptive models (generalisations of the eikonal model), and t-channel models (Reggeon field theory). In first case the Pomeron is a pure SU(3) singlet, and in the second case it has approximately the same admixture of octet as is observed at present energies.

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At present accelerator energies elastic scattering seems to be well described by a bare Pomeron pole with intercept above one, particularly in the case of meson-baryon scattering where the influence of absorption on total cross-sections seems to be very small. While this pole of course carries vacuum quantum numbers, it appears to have a small but significant (approximately 5%) SU(3) octet piece, which is responsible for the difference between the πp and Kp cross-sections, even when secondary trajectory effects have been subtracted out. In addition, the octet and singlet pieces have a different t-dependence, with the singlet becoming increasingly dominant in $t < 0$.[1,2]

There are two distinct types of model which attempt to reconcile the appearance of a bare pole above one with the asymptotic requirement of the Froissart bound. In the first, s-channel, type unitarity is enforced via interations in the s-channel, allowance being made for the possibility of inelastic diffractory.[3] In t-channel models, for example Reggeon field theory, the asymptotic amplitude is dominated by t-channel iterations, together with internal Reggeon interactions, at least when the bare Pomeron intercept α_o is less than or equal to its critical value α_{oc}.[4]

Despite their considerable difference in form, these models give rather similar predictions. In s-channel models, whenever the bare Pomeron intercept α_o is above one, total cross-sections grow asymptotically as $(\ln s)^2$, and also factorize. In Reggeon field theory, there is, for $\alpha_o \lesssim \alpha_{oc}$, a regime of high energies in which cross-sections grow as a power of lns, and factorize. For $\alpha_o = \alpha_{oc}$ (where α_{oc} is a model dependent number which is probably greater than one) this regime of logarithmically growing cross-sections extends to infinite energy. It may well be difficult, then, to differentiate between the two types of model on the basis of measuring the energy behaviour and factorisation properties of cross-sections.

However, their predictions of the SU(3) content of the Pomeron at asymptotic energies are quire different. We shall show that in s-channel models (where multiple re-scattering of diffractive intermediate states enforces the Froissart bound), the non-singlet pieces of the Pomeron are averaged out, leaving a pure SU(3) singlet, which implies, for example the asymptotic equality of πp and Kp cross-sections. In t-channel models (where re-scattering of non-diffractive intermediate states is dominant), we expect to see roughly the same admixture of octet as at present energies.

In what follows, we present a general argument for the suppression of SU(3) non-singlet contributions in the s-channel models and give a simple example of how this works. We then give a simple rule for determining the SU(3) structure of a general diagram in Reggeon field theory, and show how this picture simplifies at asymptotic energies. All these results are asymptotic predictions, but we also show the variation of octet admixture at foreseeable energies can in principle throw light on the relative importance of diffractive and non-diffractive re-scattering.

To define the general class of s-channel models we shall consider, we first disregard the SU(3) structure. The amplitude for elastic scattering $ab \rightarrow ab$ has the form

$$T(Y,B) = \sum_{n=1}^{\infty} T_n = - \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} g_a^{(n)} g_b^{(n)} A_1(Y,B)^n , \qquad (1)$$

where the Born term $A_1(Y,B)$ is given, as a function of rapidity Y and impact parameter B, in a simple Regge pole model as

$$A_1(Y,B) = \left(\frac{1}{4\pi\alpha' Y}\right) \exp \left[- \left(\frac{B^2}{4\alpha' Y}\right) + \delta Y \right], \qquad (2)$$

where $\delta = \alpha_o - 1$. In (1) we have neglected any t-dependence of the external couplings, as we can at high enough energies. In the case of the simple eikonal model, which takes into account only elastic re-scattering, the couplings $g_a^{(n)}, g_b^{(n)}$ have the form g_a^n, g_b^n respectively. In general the couplings may be written as integrals over the absorptive part of a 2-particle/n-Pomeron amplitude, so that the inclusion of inelastic diffraction will increase the couplings from their eikonal values, for $n \geq 2$.

The asymptotic behaviour of (1) is found, as usual, by observing that for $B^2 > R(Y)^2 = 4\alpha'\delta Y^2$, $A_1 \to 0$ as $Y \to \infty$, while for $B^2 < R(Y)^2$, $A_1 \to \infty$. The behaviour of $T(Y, B)$ as $A_1 \to \infty$ can be found by postulating, following Ref. (5), that $g_a^{(n)}, g_b^{(n)}$ are suitably analytic in n to define a unique continuation to $\mathrm{Re}\, n > -\epsilon$ where $\epsilon > 0$. (If one does not make such an assumption, it is difficult to see how (1) gives a sensible model). The sum can then be converted to a Sommerfeld-Watson integral

$$T(Y, B) = -\int_C \frac{dn}{2i} \frac{g_a^{(n)} g_b^{(n)} A_1^n}{\sin \pi n\, \Gamma(1+n)} , \tag{3}$$

where the contour C encloses the poles at $n = 1, 2, \ldots$ As $A_1 \to \infty$ we pull back the contour to pick up the leading contribution of the pole at $n = 0$, giving

$$T(Y, B) \sim g_a^{(0)} g_b^{(0)} \theta\, [R(Y)^2 - B^2] . \tag{4}$$

For reasonably behaved functions $g_a^{(n)}$, $g_a^{(0)}$ will be real, less than one, and hopefully positive. Of course, for the eikonal model, $g_a^{(0)} = 1$. Eq. (4) corresponds to a factorising grey disc picture.

We can take account of the SU(3) structure of the Pomeron in this picture in the following way. The Born term T, is equal to the matrix element between $\langle a|$ and $|b\rangle$ of an operator which we take to transform according to some representation $\underset{\sim}{\mathbb{P}}$ of SU(3). The current data suggests that

$$\underset{\sim}{\mathbb{P}} = \underset{\sim}{1} \oplus \underset{\sim}{8} . \tag{5}$$

The n-Pomeron exchange term T_n is equal to the matrix element of an operator which will transform according to the representation $(\underset{\sim}{\mathbb{P}})^n = \underset{\sim}{\mathbb{P}} \otimes \underset{\sim}{\mathbb{P}} \otimes \underset{\sim}{\mathbb{P}} \ldots$ If we decompose this into the irreducible representations $\underset{\sim}{D}^{(\mu)}$ of SU(3) we obtain T_n in the form

$$T_n = \frac{(-1)^{n-1}}{n!} A_1^n \sum_{\mu\nu} b_\nu^{(n)\mu} C_{\nu_a \nu \nu_b}^{\mu_a \mu \mu_b} M^{\mu_a \mu \mu_b} . \tag{6}$$

In the above the μ_i are representation labels, and the ν_i denote the eigenvalues of the remaining operators (I, I_3, Y) necessary to specify the states. The $b_\nu^{(n)\mu}$ are coefficients appearing in the decomposition of the n-Pomeron operator (including the external couplings) into irreducible tensors, the M's are reduced matrix elements of these tensors, and the C's are Clebsch-Gordan coefficients. Equation (6) is therefore quite general.

It is of course difficult to calculate the $b_\nu^{(n)\mu}$, even in a specific model of the couplings. However, we note that $b_\nu^{(n)\mu}$ will vanish whenever the representation $\underset{\sim}{D}^{(\mu)}$ does not appear in the decomposition of $(\underset{\sim}{\mathbb{P}})^n$. The number of times $\underset{\sim}{D}^{(\mu)}$ appears in $(\underset{\sim}{\mathbb{P}})^n$ is given by[6]

$$a_\mu^{(n)} = \int dR \; \chi^{(\mu)}(R)^* \, \chi^{(\mathbb{P})^n}(R) \quad , \tag{7}$$

where $\int dR$ represents an invariant integral over group space, normalised to unity, and χ denotes the character of the element R in the appropriate representation. Noting that $\chi^{(\mathbb{P})^n}(R) = [\chi^{\mathbb{P}}(R)]^n$, we see that $a_\mu^{(n)}$ can, in general, be continued to arbitrary n, and it will satisfy the Carlson condition since $\chi(R)$ is bounded. In particular

$$a_\mu^{(0)} = \sim \int dR \ \chi^{(\mu)}(R)^* = \delta_{\mu 0} , \tag{8}$$

that is, $a_\mu^{(0)}$ is only non-zero for the singlet representation. Therefore if we again assume that we can continue $b_\nu^{(n)\mu}$ as before (and that this is related to the above continuation of $a_\mu^{(n)}$), we obtain the asymptotic amplitude

$$T(Y, B) \sim b_0^{(0)0} C_{\nu_a 0 \nu_b}^{\mu_a 0 \mu_b} M^{\mu_a 0 \mu_b} \theta[R(Y)^2 - B^2] . \tag{9}$$

Thus the Pomeron is, in this model, asymptotically pure singlet. This somewhat heuristic argument can be illustrated by a simple example. Suppose that $T_1 \propto u_1 + u_8$ where u_1 and u_8 denote the singlet and octet pieces. Suppose that $T_n \propto (u_1 + u_8)^n$ and $u_8 << u_1$. Then the leading singlet piece in T_n is u_1^n, and the leading octet piece is $nu_1^{n-1} u_8$, neglecting other singlet and octet pieces which come from higher order terms in u_8. (Of course this assumption is not justified as $n \to \infty$.) The octet piece that we have retained explicitly vanishes at $n = 0$. The other, higher order, octet pieces will also vanish at $n = 0$ since, in this trivial example we can calculate T_0 and it is purely singlet.

This simple asymptotic result is corrected by two types of non-leading contribution. The region outside the disc, where $A_1 \to 0$, gives a constant contribution to the total cross-section, which will contain approximately the same admixture of octet as the original bare Pomeron. The region inside the disc can give a contribution $0(s^{-\delta})$ from the term at $n = -1$ (but only if $b_\nu^{(n)\mu}$ is singular there), which has an octet piece. Therefore we see that the leading octet piece is highly peripheral. This peripherality results solely from the absorptive procedure, and is detached from the peripherality of the octet piece which may reside in the 'bare' octet component. A different reason for peripherality related to π-exchange was given by Pumplin and Kane.[2]

In Reggeon field theory the situation at asymptotic energies is somewhat simpler. For an arbitrary Reggeon diagram including Reggeon interactions, we can state the following rule for its SU(3) structure: it will transform according to the representation $(\underset{\sim}{\mathbf{P}})^n$ where n is the least number of Pomerons in a t-channel intermediate state. This is so because such a diagram is equivalent, as far as quantum number structure goes, to the simple n-Pomeron exchange diagram. In stating this we have chosen a particular way to incorporate the SU(3) structure in the framework of Reggeon field theory. We assume that the SU(3) breaking manifests itself primarly in the couplings of Pomerons to the external particles. There indeed are other SU(3) breaking terms to be taken into account, but they are usually assumed to be smaller[2] and in any case do not seem to make the pomeron any more of a singlet. This point deserves more investigation in a suitable framework like the topological expansion.[7]

When $\alpha_0 = \alpha_c$ the leading contribution to the total cross-section comes from diagrams which are dressings of a single bare Pomeron--the external couplings to a single Pomeron are not renormalised. Thus we expect to see the admixture of octet as seen in the bare Pomeron at present energies. When $\alpha_0 < \alpha_c$ and when $t \neq 0$, the single Pomeron vertices are renormalised by the multi-Pomeron couplings, by an amount which tends to unity as $\alpha_0 \to \alpha_c$ and $t \to 0$. Thus there will not, in general, be a complete cancellation of the octet piece.

Recently[8] a solution has been proposed for Reggeon field theory with $\alpha_0 > \alpha_c$. It turns out that unlike the s channel model considered all multi-Pomeron Green-functions contribute equally to partial waves within a black disk and therefore again there does not seem to be a reason for the Pomeron to end up as a singlet.

We conclude, therefore, that one can distinguish between the two types of model by observing whether πp and Kp cross-sections are asymptotically equal or approximately in the same ratio as at present energies. In the case of s-channel models the increasing importance of the s-channel iterations will lead to an increasing cancellation of the octet piece, and so the ratio of the two cross-sections should tend monotonically to unity. In Reggeon field theory the situation is more complicated. At intermediate energies non-enhanced diagrams (diagrams which are one-Pomeron irreducible, for example the multi-Pomeron exchanges considered in the s-channel models) will become important. Their effect will be to decrease the ratio $(\sigma_{\pi p} - \sigma_{Kp})/\sigma_{\pi p}$, as in the pure s-channel models. At asymptotic energies, however, these diagrams are negligible, and this ratio should actually increase towards an asymptotic value approximately equal to that currently observed.

At foreseeable energies, neither of these pictures will probably apply. However, the behaviour of the ratio can in principle distinguish between the relative importance of re-scattering of diffractive and non-diffractive intermediate states. As two-Pomeron exchange diagrams become increasingly important, those which correspond to diffractive re-scattering will contain a large octet piece than the single Pomeron and non-diffractive re-scattering, one-Pomeron reducible, diagrams. (Approximately twice as much, according to our simple model above.) Since these diagrams give a negative contribution to the total cross-section, they will tend to decrease the value of the ratio. On the other hand, if the ratio is observed to remain roughly constant, this will indicate the dominance of non-diffractive re-scattering, and the enhanced diagrams of Reggeon field theory will give a more suitable description of the amplitude.

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