



Possibilities of Unconventional Resonances  
in  $\pi\Sigma$  and  $\bar{K}\Delta$  Scattering

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## ABSTRACT

It is pointed out that the use of hadronic resonances with exotic quantum numbers may be avoided by substituting the weaker symmetry for the conventional isospin symmetry. Possible experiments are suggested to test our assumptions, although they may be difficult.

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The quark model is extremely useful in classifying the hadrons including  $J/\psi$  and related particles.<sup>1</sup> However, the absence of the exotic resonances<sup>2</sup> has never been explained in a satisfactory way. One assumes the exchange degeneracy to cancel the exotic states or simply neglect them. Such procedures may cause difficulties in the baryon-antibaryon scattering.<sup>3</sup> Similar difficulties also appear in an attempt to explain the  $\Delta I = 1/2$  rule<sup>4</sup> in nonleptonic weak decays. In these examples, the products of the hadronic states or currents belonging to an ordinary  $SU(3)$  multiplets induce the higher dimensional multiplets, which do not seem to be dominant in nature. In view of this it seems worthwhile to examine these problems in their connections with the underlying group concept. In the following we will suggest a possible solution to one of these problems and will propose an experiment to test it. However, our treatment is quite elementary and much work has to be done. First we ask the question "Why is there no hadronic multiplet belonging to 12 or 18 of the ordinary  $SU(3)$ ?" The usual answer would be "Such representations of  $SU(3)$  exist but are not irreducible. If there is a hadronic multiplet belonging to 12 or 18 of  $SU(3)$ , a group different from  $SU(3)$  must be assumed at the beginning." This means that the  $SU(3)$  multiplets do not have to occur in nature if they belong to the reducible representation of the group. It will be quite natural, then, to explain the absence of the exotic states in the same way. For simplicity we restrict ourselves to the

isospin symmetry and thus the strangeness quantum numbers must be treated separately. We may suggest that the exotic states do not occur as resonances because they belong to the reducible representation of the isospin group. This may seem at first sight to be impossible and self-contradictory. That this is actually possible is understood from the following arguments. The isospin symmetry is usually formulated by a continuous group  $O(3)$  or  $SU(2)$ . In the analogous cases of the ordinary spin or the Lorentz transformation, however, the sets of the continuous parameters such as the relative angles between polarization vectors or the relative velocities between two observers can be measured by experiment to the arbitrarily high accuracy. This is in a marked contrast to the isospin symmetry, in which only the charge states of the relevant particles or the small numbers of Clebsch-Gordan coefficients are actually determined by experiment. The isospin symmetry is not a gauge symmetry.<sup>5</sup> So, each rotation angle  $\theta_i$  ( $i = 1, 2, \text{ and } 3$ ) in the isospin space must be measured by experiment to the arbitrarily high precision if the conventional  $O(3)$  or  $SU(2)$  is an underlying group. This seems to be impossible because the ordinary isospin symmetry can well be formulated by using the finite subgroups of  $O(3)$ .<sup>6</sup> To eliminate the mysterious rotation angles, we follow the finite group formulation of the isospin symmetry.<sup>7</sup> This still contains a finite number of discrete rotation angles. The physical meaning of them must be clarified eventually but will not be touched upon in this work.

Our assumptions are (a) There is no exotic stable or resonance state, (b) baryons (mesons) belong to the irreducible ray representations of the octahedral (tetrahedral) group,<sup>8</sup> and (c) the assignment of the electric charge to the members of each irreducible multiplet is identical to the conventional one. The reason for (b) is that the dimensions of the irreducible ray representations of  $O(T)$  are 1, 2, 3, and 4 (1, 2, and 3) and are consistent with the observed isospin structure of baryons (mesons). Above assumptions cannot be in contradiction with the consequences of the ordinary isospin symmetry, except for the criterion of the irreducibility of the representations, because  $O$  and  $T$  are the finite subgroups of  $O(3)$ .<sup>9</sup>

The procedure described above is, after all, a tautology. However it is rather surprising to note that until now the order of the isospin group has never been definitely determined by experiment. From the character tables for  $O$  and  $T$ , we can get the reductions of the product representations of  $O$  and  $T$  respectively. From these tables we can obtain all the  $BM$ ,  $B\bar{B}$ , and  $MM$  amplitudes ( $B$  = baryon,  $M$  = meson). The ordinary baryons are classified into  $\Gamma_0$ ,  $\Gamma_{1/2}$ ,  $\Gamma_1$ , and  $\Gamma_{3/2}$  of  $O$ . To get the  $B\bar{B}$  amplitude, we must first decompose the product of the irreducible ray representations of  $O$  into its irreducible components and then these must be further decomposed in general into the irreducible ray representations of  $T$  in conformity with the assumption (b). For the  $\pi N$  scattering, the decomposition is

identical to the conventional one,

$$\Gamma_1 \times \Gamma_{1/2} = \Gamma_{1/2} + \Gamma_{3/2}. \quad (1)$$

The characteristic feature is the appearance of  $\Gamma_x$ ,  $\Gamma_1^*$ , and  $\Gamma_{1/2}^*$  as baryon resonances. Examples are  $\pi\Sigma$ ,  $\bar{K}\Delta$ , and  $\pi\Delta$  scattering. For  $\pi\Sigma$  scattering, the decomposition is<sup>10</sup>

$$\Gamma_1 \times \Gamma_1 = \Gamma_0 + \Gamma_1 + \Gamma_1^* + \Gamma_x, \quad (2)$$

or

$$|\pi\rangle \times |\Sigma\rangle = \frac{1}{\sqrt{3}} \left( \pi^- \Sigma^+ - \pi^0 \Sigma^0 + \pi^+ \Sigma^- \right) \quad (I=0)$$

$$+ \left\{ \begin{array}{l} \frac{1}{\sqrt{2}} \left( \pi^+ \Sigma^0 - \pi^0 \Sigma^+ \right) \\ \frac{1}{\sqrt{2}} \left( \pi^+ \Sigma^- - \pi^- \Sigma^+ \right) \\ -\frac{1}{\sqrt{2}} \left( \pi^- \Sigma^0 - \pi^0 \Sigma^- \right) \end{array} \right\} \quad (I=1)$$

$$+ \left\{ \begin{array}{l} \frac{1}{\sqrt{2}} \left( \pi^- \Sigma^0 + \pi^0 \Sigma^- \right) \\ \frac{1}{\sqrt{2}} \left( \pi^+ \Sigma^+ - \pi^- \Sigma^- \right) \\ \frac{1}{\sqrt{2}} \left( \pi^+ \Sigma^0 + \pi^0 \Sigma^+ \right) \end{array} \right\} \quad (\text{missing})$$

$$+ \begin{cases} \frac{1}{\sqrt{2}i} \left( \pi^+ \Sigma^+ + \pi^- \Sigma^- \right) \\ \frac{1}{\sqrt{6}} \left( \pi^- \Sigma^+ + 2\pi^0 \Sigma^0 + \pi^+ \Sigma^- \right) \end{cases} \quad \text{(missing)}$$

The  $\Gamma_0$  and  $\Gamma_1$  are ordinary isospin states with  $I=0$  and  $I=1$  respectively. By assumption (a), doubly charged states in  $\Gamma_1^*$  and  $\Gamma_x$  will not be observed in the  $Y$  (hypercharge) = 0 channel. If we follow the conventional viewpoint, then,  $\Gamma_1^*$  and  $\Gamma_x$  will not occur in nature.<sup>11</sup> Examples are  $I=2$  state in  $\pi\pi$  scattering and  $I=2$  and  $I=3$  states in  $\Delta\bar{\Delta}$  scattering. This may be actually right. However we find no a priori reason to exclude the  $\Gamma_1^*$  and  $\Gamma_x$ . The gapped charge states will be observed if  $\Gamma_1^*$  is dominant<sup>12</sup> at some energy in the  $\pi\Sigma$  scattering. These states simulate the  $I=1$  states but are different from the latter in the point that the neutral counterpart is missing. We hope that the hyperon beam experiment will finally settle the problem.<sup>13,14</sup> If the above scheme is actually working in nature, one should ask the physical meaning of the basic rotations in isospin space.

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- <sup>4</sup>Recent attempts include: M. K. Gaillard and B. W. Lee, Phys. Rev. Letters 33, 108 (1975), E. Golowich and B. R. Holstein, Phys. Rev. Letters 35, 83 (1975).
- <sup>5</sup>The gauge symmetry or the gauge invariance means some redundancy in our theoretical framework. We think that the observed isospin symmetry is not of this type of symmetry.
- <sup>6</sup>K. M. Case, R. Karplus, and C. N. Yang, Phys. Rev. 101 874 (1956).  
We follow the notations of this reference.
- <sup>7</sup>One alternative is to introduce the quarks.
- <sup>8</sup>If the exotic resonances are observed in the  $B\bar{B}$  scattering, this assumption must be modified.

<sup>9</sup> If the isospin symmetry is really established for the reactions involving heavy ions with  $I=22$ , for example, then it may be eventually necessary to modify our approach, although the many body systems are not treated here.

<sup>10</sup> For  $\pi\rho$  scattering,  $\Gamma_x$  reduces to  $\Gamma_0' + \Gamma_0''$ , where

$$\Gamma_0' : \frac{1}{\sqrt{2}i} \left( \pi^+ \rho^+ + \pi^- \rho^- \right)$$

$$\Gamma_0'' : \frac{1}{\sqrt{6}} \left( \pi^- \rho^+ + 2\pi^0 \rho^0 + \pi^+ \rho^- \right).$$

Note that  $\Gamma_1^*$  is equivalent to  $\Gamma_1$  as a representation of T.

<sup>11</sup> This was stressed by B. W. Lee (private communication).

<sup>12</sup> The gapped charge state was originally suggested for  $\Gamma_x$  by C. N. Yang et al., in Ref. 6.

<sup>13</sup> A more realistic and systematic way to test our hypothesis is being developed by K. Yamada (in preparation).

<sup>14</sup> C. Quigg and J. L. Rosner, Phys. Rev. D (to be published),

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