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Hydrodynamic Model of Collective Resonances in Hadronic Matter

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ABSTRACT

We study the collective resonance phenomena in multiquark hadronic systems. Our analysis is essentially qualitative and based on an analogy to the well-known giant resonance phenomena in the nuclear matter.

We consider the non-relativistic hydrodynamic equations for a two-component compressible fluid describing a system of quarks and antiquarks confined to the interior of the finite volume. The confinement properties as well as the relevant phase transitions are discussed and the frequences of the hydrodynamic oscillations of the system are derived.

The problem of taking into account non-abelian colored gluon fields in the hydrodynamic equations describing motion of confined quark/antiquark fluids ("chromo-hydrodynamics") is briefly discussed and the estimation of the effect of quark-gluon interactions on the energies of collective resonances is given.

We speculate that the collective resonances or as we call them - the hadronic giant resonances may play an essential role in understanding the new resonances observed recently in e^+e^--annihilation and lepton pair production experiments in unexpectedly high mass intervals. Particularly, we show that the energies of the lowest hadronic giant resonances which could be seen in e^+e^- annihilation (E0 and E2 modes) lie at ~ 4 GeV if a size of the confinement region is about 1 GeV^{-1} (~ 0.2 fm). We should expect also an existence of colored collective states with energies of about the same order as for uncolored ones, which cannot be seen however in

the process of e^+e^--annihilation into hadrons due to the color conservation in strong interactions.

I. INTRODUCTION

The hypothesis on the existence of the collective resonance phenomena in hadronic systems which could be the analogy to the giant resonances in nuclei has been introduced in paper (1).

The main motive was the unexpected and intriguing at that time results of the experimental studies of e^+e^--annihilation into hadrons which have shown the relatively large and growing with energy yield of hadrons. While contradicting the predictions of the old-fashioned naive parton model these results and the following discovery of the ψ/J-particles have stimulated an invention of new quantum numbers and corresponding them new degrees of freedom (charm, color etc.). The authors of the paper[1] argued that for consistent explanation of the whole new events it might be useful to take into account an excitation of collective degrees of freedom beyond the elementary ones which mainly are treated by the parton model.

Considering, in particular, quark-antiquark pairs as weakly coupled quasiparticles of hadronic matter produced in e^+e^--annihilation, one may expect collective excitations to appear with the energies depending on a size of hadronic system, a radius of interaction between constituents etc.

Starting from the simplest variant of model of hydrodynamic oscillations of two incompressible fluids--one for quarks and another for antiquarks (compare with the Goldhaber-Teller model of the dipole giant resonances in nuclei[2]), the authors of paper[1] derived a qualitative estimation on energies of collective hadronic resonances and have pointed to some feasible implications.

The recent exciting discovery of the narrow resonance $\gamma(6.0)$ in e^+e^--production experiment[3] as well as the observation of the rich resonance "mini-structure" at $\sim$ 4 GeV in e^+e^--anninilation into hadrons[4] give rise us to renew the discussion of the collective resonance phenomena.

In this paper I shall study the hydrodynamic equations for the two-component compressible fluid describing a superdense system of quarks and antiquarks confined to the interior of the finite volume. Due to the assumption that masses of free unconfined quarks are very large, the consideration is mainly non-relativistic, and an analogy to the theory of the nuclear giant resonance[5] plays an essential role.

So the analogy of the well-known in nuclear physics "symmetry energy" potential is introduced which describes in our case fluctuations of energy near the point of the phase transition between the symmetric phase (all the local densities of quarks and antiquarks coincide) and the antisymmetric one (the local densities of quarks and antiquarks do not coincide).

We argue that the confined symmetric phase is stable under fluctuations of the total quark-antiquark local density $\rho = \rho_q + \rho_{\bar{q}}$, so the corresponding fluid can be considered as the incompressible one.

The frequencies of the hydrodynamic oscillations in the confined hadronic phase are derived and the estimation on the energies of the corresponding "hadronic giant resonances" (Γ-series) is done bearing in mind to give a possible explanation of the "mini-structure" in the total cross-section of e^+e^--annihilation.

Then we discuss a problem of incorporation of the gluon degrees of freedom in framework of the hydrodynamic approach. The first approximation to the so-called "chromodynamics", describing motion of the quark-antiquark fluids in the presence of colored gluon fields is derived, and an estimation is given of an effect of quark-gluon interactions on the frequencies of collective resonances.

II. GIANT RESONANCE PHENOMENA IN NUCLEAR AND HADRONIC PHYSICS

The giant resonance phenomena in nuclear photoabsorption reactions give a good example of collective excitations in systems of strong interacting particles. In spite of the properties of hadronic matter composed of confined colored quarks and gluons should be different from those of nuclear matter, we believe that physics underlying the collective resonance phenomena in nuclear and hadronic matter is quite similar. For this reason, we begin first with short introduction to the nuclear giant resonance physics.

It was known for a long time that cross sections of inelastic photo-absorption reactions (when are measured with sufficiently bad resolution) on nuclei develop the broad peaks--named as the giant resonances--which positions slightly change with nucleus mass number somewhere between 13 and 25 MeV.[5] (see Fig. 1) The rather big widths of the giant resonances as well as the strength of the absorption are considered as an indication of the collective nature of this effect, which cannot be explained by an excitation of single-particle degrees of freedom alone.

The giant electric dipole resonance was described by Goldhaber and Teller as a collective oscillation of the protons as a whole against the neutrons as a whole in the nucleus.[2] So the large absorption of γ-quanta with an electric field coupled to the dipole operator

$$\vec{D} = \sum_i e_i \vec{r}_i \rightarrow \frac{ZN}{A} (\vec{r}_p - \vec{r}_n) \tag{2.1}$$

where $(\vec{r}_p - \vec{r}_n)$ is the relative center of mass coordinate vector of the protons and the neutrons, have to appear at photon energies determined by the frequencies of collective oscillations (E1, I = 1 mode).

A necessary requirement for an excitation of collective resonances is the so-called coherence condition

$$\lambdabar >> R \tag{2.2}$$

where $\lambdabar$ is a wave vector of the electric field, and $R \sim A^{1/3}$ is a size of a nucleus.

Assuming that protons and neutrons inside nucleus can be considered as hard interpenetrating spheres and the total restoring force is proportional to the nuclear surface, Goldhaber and Teller derived the formula which determines the frequencies of collective oscillations

$$\omega \approx \left(\frac{3u_0}{Rr_0 M_p}\right)^{1/2} \approx \frac{35\ \mathrm{MeV}}{A^{1/6}} \qquad (2.3)$$

Here u_0 is an "ionization" potential of nuclear matter and r_0--the radius of the nuclear forces.

Obviously, there exist, in general, besides the dipole oscillations (E1) the others, e.g. the monopole (E0), the quadrupole (E2), etc. oscillations, as illustrated in Fig. 2.

Moreover, theories of nuclear matter show that collective oscillations can deal with all degrees of freedom such as spin and isotopic spin.[6] So, there exist modes of vibrations in which protons with spin up and neutron with spin down move against protons with spin down and neutrons with spin up (spin-isospin mode), or in which nucleons with spin up move against nucleons with spin down (spin-wave mode) etc. (see illustration in Fig. 3). If one considers excitation of basic O^+ states of nuclei this leads to Wigner's supermultiplets of giant resonances which are degenerated under assumption of spin-isospin independence of nuclear forces.[7]

An analysis of the giant resonance phenomena has shown that their most adequate description is given by a consideration of the classical motion of two-fluid system which was initially applied to the nuclear problem by A. B. Migdal (1945)[8] and by H. Steinwedel, et al. (1950).[9] The important idea of the hydrodynamical approach was that the restoring force is described by the potential energy density

$$\mathscr{H}_s = \chi(\rho_p - \rho_n)^2/\rho_0 \tag{2.4}$$

which arises from the so-called "symmetry energy" term

$$E_s = \chi(N - Z)^2/A; \qquad \chi \approx 20 \text{ MeV} \tag{2.5}$$

in the semiempirical Weizsacker's formula for nucleus masses

$$E = E_{\text{volume}} + E_{\text{surface}} + E_s + \dots \tag{2.6}$$

This more sophisticated theory gives for the frequency of nucleus dipole oscillations

$$\omega^{\text{coll.}} \sim \frac{2.08}{R}\sqrt{\frac{8\chi NZ}{m^* A^2}} \sim \frac{70 \text{ MeV}}{A^{1/2}} \tag{2.7}$$

(m^* is an effective nucleon mass in nuclear matter), which can be compared with the result of the "hard sphere" model (2.3).

The question arises: whether the hadronic matter composed of quarks (and, apparently, of gluons) can develop collective oscillations feasible experimentally, as the nuclear matter do.

The main point of the paper[1] was that the collective vibration of hadronic cluster produced via e^+e^--annihilation could explain a rather big value as well as large-scale irregularities in the energy dependence of the total annihilation cross section. We shall study here along this line the hydrodynamic picture of collective resonance phenomena in multiquark hadronic systems.

Consider the case when the numbers of quarks and antiquarks produced in high energy e^+e^--annihilation are large enough, so that the qualitative two-fluid description can be applied to the corresponding hadronic system (with the quantum numbers of photon)(Fig. 4a). We base on an idea that for large excitations hadronic vacuum can be described qualitatively as a polarizable classic medium which will resonate on the frequencies determined collective vibrations of quark-antiquark fluids. To illustrate the idea we consider here the simplest version of the model of hydrodynamic oscillations of two spinless incompressible liquids--that of quarks and antiquarks placed in a spherical volume of a radius R. That is just analogy of the Goldhaber-Teller "hard sphere" model for the dipole proton-neutron oscillations in spherical nuclei. The model is based on the assumption that the restoring force, preventing the separation of centres of masses of two liquids, is proportional to the number of separated particles (shaded regions of Fig. 4b).

For small oscillations the potential of restoring force is quadratic in ξ, so the Hamiltonian describing these oscillations has the form

$$H = \frac{1}{2} M_q \frac{n_q n_{\bar{q}}}{n_q + n_{\bar{q}}} \xi^2 + \frac{1}{2} \alpha \xi^2 \tag{2.9}$$

The rigidity parameter α is determined by the condition that at sufficiently large separations $\xi = r_0$ the restoring force potential succeeds the "ionization" energy of separated particles (see Fig. 5), i.e.

$$\alpha \frac{1}{2} r_0^{\,2} = U_0(\delta n_q + \delta n_{\bar{q}}) \tag{2.10}$$

where

$$\delta n_q = n_q \frac{\delta V}{V}, \qquad \delta n_{\bar{q}} = n_{\bar{q}} \cdot \frac{\delta V}{V}$$

$$\delta V \approx \pi R^2 r_0, \qquad \text{for } r_0 << R \tag{2.11}$$

and U_0 is an "ionization" energy per one quark.

Thus one has

$$\alpha = 3/2Rr_0 \cdot (n_q + n_{\bar{q}})U_0 \tag{2.12}$$

what gives for the frequency of the oscillations the following result:

$$\omega = \sqrt{\frac{\alpha}{\mu}} = \sqrt{\frac{3}{2Rr_0} \frac{(n_q + n_q)^2}{n_q n_q} \frac{U_0}{M}} \longrightarrow$$

$$\longrightarrow \sqrt{6/Rr_0} \begin{pmatrix} U_0/M_q \to 1 \\ n_q = n_{\bar{q}} \end{pmatrix} \tag{2.13}$$

This result has been used in paper[1] for a possible explanation of the energy structure of the total annihilation cross section $\sigma_{e^+e^- \to \text{hadrons}}$.

Assuming that the correlation length r_0 is determined by the mass of quanta mediating the interaction between quarks (gluon?), i.e.

$$1/r_0 \sim m_g \tag{2.14}$$

and the size of a hadronic system developing collective oscillations can be underestimated by the reversed total resonance width

$$R \gtrsim \Gamma_{tot}^{-1} \tag{2.15}$$

the authors of paper derived the relation[10]

$$\omega_{coll}^2 \lesssim 6 m_g \Gamma_{tot} \tag{2.16}$$

This says that for ω_{coll} and m_g of the order of few GeV/c^2 the total resonance width should be of the order of few hundreds MeV or even more. Of course, this model gives only crude picture of the hadronic collective resonance. It ignores, for instance, a discussion of confinement properties of multiquark systems in the normal hadronic phase.

III. HYDRODYNAMIC OSCILLATIONS IN CONFINED HADRONIC MATTER

Here we shall consider the hydrodynamic oscillations in hadronic matter confined to a finite volume. Due to an assumption that masses of free (unconfined) quarks are very large, we shall describe the hadronic matter by the non-relativistic hydrodynamic equations for a two-component fluid, with ρ_q and $\rho_{\bar{q}}$ being the local densities of quark and antiquark components. We start from the Hamiltonian

$$H = T + U \tag{3.1}$$

$$T = \frac{1}{2} M_q \int dr \, (\rho_q V_q{}^2 + \rho_{\bar{q}} V_{\bar{q}}{}^2); \quad U = \int dr \, \mathscr{H} \tag{3.2}$$

where the potential energy is assumed for simplicity to be local and bilinear in the densities of the quark/antiquark fluids:

$$\mathscr{H} = M_q(\rho_q + \rho_{\bar{q}}) + A(\rho_q{}^2 + \rho_{\bar{q}}{}^2) + 2B\rho_q\rho_{\bar{q}} \tag{3.3}$$

By using

$$A = \chi - g, \qquad B = -\chi - g < 0 \quad \text{(attraction)} \tag{3.4}$$

we rewrite Eq. (3.3) as

$$\mathscr{H} = (\rho_q + \rho_{\bar{q}})(M_q - U) + \chi(\rho_q - \rho_{\bar{q}})^2 \tag{3.5}$$

where $U = g(\rho_q + \rho_{\bar{q}})$--the confinement potential, and the last term in (3.5) is the analogy of the "symmetry energy" potential in the nuclear theory.

First of all we discuss the confinement properties of the hadronic systems described by the Hamiltonian (3.5). Consider first the case

$$\rho_q = \rho_{\bar{q}} \text{ ("symmetric phase")} \tag{3.6}$$

Obviously, the potential energy

$$\mathscr{H} = \rho(M_q - g\rho); \qquad \rho = \rho_q + \rho_{\bar{q}} \tag{3.7}$$

has a finite limit for superheavy quarks, as $M_q \to \infty$, for either

$$\rho = 0 \text{ (free quark-antiquark vacuum)} \tag{3.8}$$

or

$$\rho = \rho_0 = \left(\frac{g}{M_q}\right)^{-1} \text{(confined hadronic phase)} \tag{3.9}$$

Let now

$$\rho_q \neq \rho_{\bar{q}} \text{ ("non-symmetric phase")} \tag{3.10}$$

Consider, for instance, a single quark state. Due to Eq. (3.5) the energy of the state is equal

$$E\ (\text{single quark}) = M_q + \frac{\chi - g}{V} \tag{3.11}$$

and hence one has to put $g = \chi$ to avoid the dependence on the space volume V (no confinement for a single quark).

In general, the states in the symmetric and the non-symmetric phases are split in energies by the amount of

$$M_q \cdot (n_q - n_{\bar{q}})^2 / (n_q + n_{\bar{q}}) = \\ = \Delta E \equiv E(\text{symmetric}) - E(\text{non-symmetric}) \tag{3.12}$$

To the different kinds of phase transitions in hadronic matter there exist the different kinds of possible collective excitations. In our scheme one can discuss the following types of phase transitions:

A. Symmetric phase $\longleftrightarrow$ Non-symmetric phase
$(\rho_q = \rho_{\bar{q}})$ $\quad$ $(\rho_q \neq \rho_{\bar{q}})$

B. Confined phase $\longleftrightarrow$ Nonconfined phase
$(\chi\rho = M_q)$ $\quad$ $(\chi\rho \neq M_q)$

The corresponding collective excitations could be related with the hydrodynamical fluctuations of the $(\rho_q - \rho_{\bar{q}})$ or $(\rho_q + \rho_{\bar{q}})$ local densities, respectively, which should be considered in that case as <u>compressible fluids</u>. In terms of compressibility the situation can be summarized as follows:

	Compressible	Incompressible
(i)	$\rho_q \pm \rho_{\bar{q}}$	ρ_q, $\rho_{\bar{q}}$ separately
(ii)	$\rho_q - \rho_{\bar{q}}$	$\rho_q + \rho_{\bar{q}}$
(iii)	$\rho_q + \rho_{\bar{q}}$	$\rho_q - \rho_{\bar{q}}$

The first case (i) is just the "hard sphere" model developed by M. Goldhaber and E. Teller. The case (ii) corresponds to the fluctuations around the symmetric phase and its nuclear analogy was the most successful in description of the giant resonance phenomena in photonuclear reactions. The case (iii) corresponds to the fluctuations around the confined hadronic phase which is characterized by the definite value of the total local density of quarks and antiquarks $\rho_0 = M_q/g$. This type of fluctuations raises the problem of stability of the confined hadronic phase and requires a special consideration.

Now we shall only briefly discuss this problem which is, up to our present understanding, beyond the qualitative approach being described here. Consider the symmetric phase $\rho_q = \rho_{\bar{q}}$, so that

$$\mathscr{H} = g\rho(\rho_0 - \rho); \quad \rho = \rho_q + \rho_{\bar{q}} . \tag{3.13}$$

The local fluctuations of the total quark/antiquark density around two distinguished levels $\rho = 0$ and ρ_0 (see Fig. 6) are characterized by a sign and a magnitude of the derivative

$$\frac{\delta \mathscr{H}}{\delta \rho} = \begin{cases} M_q > 0; & \text{at } \rho = 0 \\ -M_q < 0; & \text{at } \rho = \rho_0 \end{cases} \tag{3.14}$$

So, it is unlikely that there could be any collective oscillations around zero level of the total local density (stability of the "frozen out" quark-antiquark matter or the vacuum). Contrary, the fluctuations around the confined phase with non-zero local density ρ_0 seem to be very probable. In fact, as we can show, these fluctuations are described under some conditions by an elliptic type partial derivation equation, what leads to either imaginary frequencies or imaginary wave lengths. Thus, there are in general exponentially increasing in space-time as well as exponentially decreasing ones. Those unwanted exponentially growing solutions which are essentially unstable, can be excluded by appropriate boundary conditions.

In what follows we shall consider only the collective oscillations around confined symmetric phase $\rho_q = \rho_{\bar{q}} = \rho_0/2$ which corresponds to fluctuations of the relative quark-antiquark local density $(\rho_q - \rho_{\bar{q}})$, i.e. the case (ii). The relevant Hamiltonian is given by:

$$\mathscr{H} = m_{eff}\rho_0 + \chi(\rho_q - \rho_{\bar{q}})^2 + \frac{1}{2\rho_0} M_q \rho_q \rho_{\bar{q}} V^2 \tag{3.15}$$

where $V = (V_q - V_{\bar{q}})$ is a local field of the quark-antiquark velocity, and $m_{eff} = M_q - g\rho_0$--an effective mass of confined quarks is assumed to be negligible (≈ 0). For small oscillations inside rigid spheric surfaces this problem was solved a long time ago.[11] The motion equations take the form

$$M_q \frac{dV}{dt} = \frac{1}{\rho_q \rho_{\bar{q}}} \rho_0 F \approx -4\chi \text{ grad } (\rho_q - \rho_{\bar{q}}) \tag{3.16}$$

and assuming that for the small hydrodynamic oscillations the velocity flow to be potential, i.e.

$$V = - \text{grad } \phi \tag{3.17}$$

one gets

$$\dot{\phi} = \frac{\partial \phi}{\partial t} \approx \frac{4\chi}{M_q} (\rho_q - \rho_{\bar{q}}) \tag{3.18}$$

Using Eqs. (3.17) and (3.18) it is easy to rewrite the Hamiltonian in the form

$$\mathscr{H} = \frac{1}{8} M_q \rho_0 \left\{ (\text{grad } \phi)^2 + \frac{M_q}{2\chi \rho_0} \dot{\phi}^2 \right\} \tag{3.19}$$

which immediately leads to the harmonic oscillations with frequencies

$$\omega = k \left(\frac{2\chi \rho}{M_q} \right)^{1/2} \tag{3.20}$$

The values of the wave vector k are determined from the boundary condition (at $r = a$)

$$\begin{aligned} V_n &= 0 \quad \text{or} \quad (r \cdot \text{grad } \phi) = 0 \quad , \quad \ell \geq 1 \\ V_t &= 0 \quad \text{or} \quad \phi = 0 \quad , \quad \ell = 0 \end{aligned} \tag{3.21}$$

where ℓ is an angular momentum. The solutions of the problem are of the form

$$\phi \,\alpha\, Y_\ell^m(\cos\theta,\ \phi) j_\ell(kr) \tag{3.22}$$

so the eigenvalues of the wave number $(ka)_\ell$ determined by the roots of the Bessel functions (for $\ell = 0$) or its first derivatives (for $\ell \geq 1$). We list for convenience a few first solutions for the wave number $(ka)_\ell$ [12] (see Fig. 7):

		$n_r = 1$	$n_r = 2$
$j_\ell(ka) = 0$	$\ell = 1$	2.08	5.95
	$\ell = 2$	3.31	7.30
	....	....	....
$j_{\ell=0}(ka) = 0$	$\ell = 0$	3.14	6.25

IV. SPECULATIONS ON COLLECTIVE QUARK RESONANCES IN e^+e^--ANNIHILATION AND LEPTON PAIR PRODUCTION PROCESSES

In spite of the main goal of this paper is the qualitative description of the collective resonances in hadronic matter, we would like to speculate here on the possible role of this phenomena in explanation of the observed "mini-structure" of the total cross section of e^+e^--anninilation into hadrons[4] as well as of an excess of prompt leptons in hadron collisions.[3]

As it is known from SLAC experimental data the total cross section $\sigma_{e^+e^-\to \text{hadrons}}$ develops two prominent peaks at about 4.1 and 4.4 GeV with a possible mini-structure around these regions.[4] We shall assume here that these two resonance regions centered at ~4.1 and ~4.4 GeV correspond respectively to the electric monopole (E0) and the quadrupole (E2) collective resonances with quantum numbers 1^{--}. For constituents with spin $\frac{1}{2}$ the corresponding couplings (the electric dipole operator) are respectively

$$\text{E0:} \qquad \sim \vec{\sigma} \qquad \left({}^3S_1\right)$$

$$\text{E2:} \qquad \sim \vec{r}(r \cdot \sigma) - \vec{\sigma}\cdot r^2/3 \qquad \left({}^3D_1\right)$$

We should notice, however, that for quarks with spin $\frac{1}{2}$ the dipole oscillations $\left({}^1P_1\right)$ are of pure magnetic type (M1) with quantum numbers 1^{++} and hence cannot be observed in e^+e^--annihilation or lepton pair production experiments.

On the contrary, if there is any necessity of boson-like constituents in description of hadronic vacuum fluctuations, the electric dipole collective resonances coupled to a photon via

$$\text{E1:} \qquad \sim \vec{r} \qquad \left({}^1P_1\right)$$

could be seen in these experiments. Returning to the discussion of the observed peaks in the total cross section of e^+e^--anninilations, we note that the difference in peak energies, which by an assumption is related to the difference in the angular momenta, could give an estimation on the mass of heavy quarks. Really, from the relation

$$E_{\ell=2} = E_{\ell=0} \approx \left. \frac{\ell(\ell+1)}{2M_q a^2} \right|_{\ell=0}^{\ell=2} \approx 300 \text{ MeV} \tag{4.1}$$

one finds for $1/a \sim 1$ GeV that $M_q \sim 10$ GeV. The ratio of the energies

$$E_{\ell=2}/E_{\ell=0} \approx 4.4/4.1 \approx 1.07 \tag{4.2}$$

is very close to the ratio of the frequencies of quadropole and monopole collective oscillations listed in the previous section (see p. 18)

$$(ka)_{\ell=2}/(ka)_{\ell=0} \rightarrow 3.31/3.14 \approx 1.05 \tag{4.3}$$

In absolute scale the required energies of the observed peaks correspond to the following values of a reverse radius of hadronic clusters:

$$\frac{1}{a} \approx 0.94 \text{ GeV } (\ell = 2)$$
$$\frac{1}{a} \approx 0.92 \text{ GeV } (\ell = 0) \tag{4.4}$$

By the way, the energy of dipole oscillations can be estimated from the ratio of frequencies

$$\frac{\omega_{\ell = 0} + \omega_{\ell = 2}}{\omega_{\ell = 1}} \approx 3.1 \tag{4.5}$$

that gives $E_{\ell = 1} \approx 2.74$ GeV or in terms of the reverse value of the radius $\frac{1}{a} \approx 0.94$ GeV.

Besides the angular momentum splitting in energy one should expect also an appearence of some resonance sub-structure in accordance with isotopic spin and hypercharge assignment of collective resonances. This "mini-structure" is seen, apparently, in the recent SLAC experiments at energies around and above 4 GeV. Obviously, the "hidden" charm gives the highest jump in energy, say,

$$\Delta E^2 \approx m_\psi^{\ 2} - m_\omega^{\ 2} \approx 8.97 \text{ GeV}^2 \tag{4.6}$$

leading to the new peaks ("charm-mirror") with the energies[13]:

$$4.1 \text{ GeV} \rightarrow 5.08 \text{ GeV}$$
$$4.4 \text{ GeV} \rightarrow 5.32 \text{ GeV} \tag{4.7}$$

which we will denote by $E_\psi(E0)$ and $E_\psi(E2)$.

The next question is what can one say about widths of collective resonances? An experience of theory of nuclear giant resonance phenomena teaches us that this problem is the most hard one for the qualitative hydrodynamical description. Evidently, the total widths of collective resonances must exceed an average energy of excitation of separate single quark degrees of freedom in a region with a finite size a:

$$\Delta E_q \sim \frac{1}{2M_q} \frac{1}{a^2} \tag{4.8}$$

Thus, one has

$$\Gamma_{tot} > \Delta E_q \sim \alpha \omega^2 / M_q \tag{4.9}$$

where

$$\begin{aligned} \alpha &= 2.28 \times 10^{-2} \; (\ell = 2) \\ &= 2.54 \times 10^{-2} \; (\ell = 0) \\ &= 5.78 \times 10^{-2} \; (\ell = 1) \end{aligned} \tag{4.10}$$

So, for example, for the values

$$\frac{1}{a} \sim 1 \text{ GeV} \qquad \text{and} \qquad M_q \sim 10 \text{ GeV}$$

one gets for the lower bound of total resonance widths the value 50 MeV.[14] More general situations are shown on Fig. 8.

It is worth noting that there could exist, in general, collective oscillations in hadronic matter with non-zero color and frequencies comparable to that of uncolor ones. We emphasize however that colored collective resonances should be stable with respect to strong decays because of the color conservation and can be observed (if any exists) only through their leptonic decays (Obviously, under condition that the electromagnetic or weak currents have colored counterpart[15]). Thus the previous estimation on resonance widths does not work for colored collective excitations in hadronic matter.

V. "CHROMO-HYDRODYNAMICS"

In the QCD-Bag theory,[16] the most popular at the present time approach to quark confinement, there are colored vector gluons confined to the same interior as quark and antiquark constituents. Thus, the problem arises how gluon degrees of freedom can be incorporated into our scheme?

Here we shall not introduce a condensed phase for gluons, but shall assume a dassical description of gluon field interaction with quark-antiquark matter. Apparently, this approach which we shall call, hereafter as the "chromo-hydrodynamics", could be based on the eq.:

$$\partial_\nu T_{\mu\nu} = 0 \quad \text{(inside a finite volume V)}, \qquad (5.1)$$

for the energy-momentum tensor

$$T_{\mu\nu} = T_{\mu\nu}^{\text{quark fluids}} + T_{\mu\nu}^{\text{gluon fields}}, \qquad (5.2)$$

with appropriate boundary conditions which ensure confinement. Unfortunately there is no theory of a non-abelian field interacting with macroscopic medium.[17] For this reason, we consider here as a first step the chromo-hydrodynamic eqs. in the lowest (non-vanishing) order in the quark-gluon coupling constant g. Obviously, in this approximation the gluon fields become effectively abelian ones like the electromagnetic field. The underlying physics is very simple. Assume that there are a number of fluctuation regions, where $(\rho_q - \rho_q) \neq 0$, the fluctuations generate the gluon fields, say E^a and B^a, satisfying the Maxwell eqs., e.g.

$$\begin{aligned} \text{div } E^a &= 4\pi g^a(\rho_q - \rho_{\bar{q}}) \\ \text{div } B^a &= 0, \quad \text{etc.} \end{aligned} \qquad (5.3)$$

where a = 1, 2, ..., 8--the color index. In its turn these fields influence the motion of quarks and antiquarks and hence on developing of the fluid fluctuations. The relevant notion describing this influence is the so-called "ponderomotive" or Lorentz force[18]:

$$\vec{F} = \sum_a g^a(E^a + \frac{1}{c} V x B^a) \qquad (5.4)$$

In terms of the action principle we have

$$\delta A = \delta \int dt(T + U) + \int dr\, \vec{F}\, (\rho_q \delta\vec{\xi}_q - \rho_{\bar{q}} \delta\vec{\xi}_{\bar{q}}) = 0 \qquad (5.5)$$

where $\delta\xi_q$ and $\delta\xi_{\bar{q}}$ are the fields of displacements of quark and antiquark positions, and

$$T = \frac{1}{2} M_q \int dr \left\{ \frac{\rho_q \rho_{\bar{q}}}{\rho_0} v^2 + \rho_0 V^2 \right\}$$

$$U = \chi \int dr (\rho_q - \rho_{\bar{q}})^2 \qquad (5.6)$$

Here we use the notations

$$\delta\vec{\xi}_q = \delta\vec{R} + \delta\vec{\xi} \cdot \rho_{\bar{q}}/\rho_0$$

$$\delta\vec{\xi}_{\bar{q}} = \delta\vec{R} - \delta\vec{\xi} \cdot \rho_q/\rho_0 \qquad (5.7)$$

where $\delta\xi = (\delta\xi_a - \delta\xi_{\bar{q}})$, $\rho_0 = (\rho_q + \rho_{\bar{q}}) = \text{const}$, so that $v = \frac{\delta\xi}{\delta t}$; $V = \frac{\delta R}{\delta t}$. From the continuity equations it follows that

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + (v\ \mathrm{grad})\rho = -\rho\ \mathrm{div}\ V$$

for q and q, and hence we can put in the action principle

$$\delta\rho_q = -\rho_q \cdot \mathrm{div}\ \delta\xi_q$$

$$\delta\rho_{\bar{q}} = -\rho_{\bar{q}} \cdot \mathrm{div}\ \delta\xi_{\bar{q}} \qquad (5.9)$$

so that

$$\delta U = \chi \int dr(\delta R \cdot \text{grad})(\rho_q - \rho_{\bar{q}})^2 + \frac{4\chi}{\rho_0} \int dr\ \rho_q \rho_{\bar{q}} (\delta \xi \cdot \text{grad})(\rho_q - \rho_{\bar{q}}) \tag{5.10}$$

Now one obtains the following motion equation for the velocity of relative oscillations of the quark and antiquark fluids:

$$\frac{dV}{dt} = -\frac{4\chi}{M_q} \text{grad}\,(\rho_q - \rho_{\bar{q}}) + \frac{2}{M_q} \cdot F \tag{5.11}$$

The boundary condition for a spheric shape of the volume V occupied by $q/\bar{q}$ fluids is

$$(r \cdot v)_{r=a} = 0 \tag{5.12}$$

so, no effects of surface vibrations are considered.

By applying the divergence operation to both sides of the eq. (5.11) and using the relation

$$\text{div } V = \text{div } (v_q - v_{\bar{q}}) = -\frac{1}{\rho_q} \frac{d\rho_q}{dt} + \frac{1}{\rho_{\bar{q}}} \frac{d\rho_{\bar{q}}}{dt} = -\frac{1}{2} \frac{\rho_0}{\rho_q \rho_{\bar{q}}} \frac{d}{dt} (\rho_q - \rho_{\bar{q}}) \tag{5.13}$$

one gets the equation

$$\left[\frac{d}{dt}\;\frac{1}{c^2}\frac{d}{dt}\;-\Delta+\frac{2\pi}{M_q}\,g_a{}^2\right](\rho_q-\rho_{\bar{q}}) = 0, \tag{5.14a}$$

or

$$\left[\frac{1}{c^2}\frac{d^2}{dt^2}+\frac{16\chi}{M_q\rho_0}\left(\frac{1}{c^2}\frac{d}{dt}\rho_q\right)^2-\Delta+\frac{2\pi}{M_q}\;g_a{}^2\right](\rho_q-\rho_{\bar{q}}) = 0 \tag{5.14b}$$

where $c^2 = \frac{8\chi}{M_q\rho_0}\;\rho_q\rho_{\bar{q}}$.

We shall assume for small oscillations that

$$\frac{d\rho}{dt}\approx\frac{\partial\rho}{\partial t}\qquad\text{or}\quad(v\text{ grad})\rho << \frac{\partial\rho}{\partial t} \tag{5.15}$$

which means

$$\tau V_q << \ell \tag{5.16}$$

where τV_q is an average path of particles during one period of oscillations $\tau\sim 1/\omega$, and ℓ --a length on which $\rho_{q/\bar{q}}$ varies more or less considerably.

The resulting equation for linearized hydrodynamical oscillations of the quark and antiquark fluids interacting with gluon fields takes the form of the wave equation, e.g.

$$\left(\Delta-\frac{1}{c^2}\;\frac{\partial^2}{\partial t^2}\;-c^2\mu^2\right)\delta\rho = 0;\ \delta\rho = (\rho_q-\rho_{\bar{q}}) \tag{5.17}$$

Here $c = \left(\frac{8\chi}{M_q\rho_0}\,\rho_q{}^{(0)}\rho_{\bar{q}}{}^{(0)}\right)^{1/2}$ is an effective velocity and $\mu = \left(\frac{\pi g_a{}^2}{\chi}\right)^{1/2}$ -- an effective mass of quanta of the collective excitations. For the frequency of the collective oscillations we get

$$\omega = \left(\frac{8\chi}{M_q}\ \frac{\rho_q \rho_{\bar{q}}}{\rho_0}\right)^{1/2} \cdot \left(k^2 + \frac{2\pi g_a{}^2}{M_q}\right)^{1/2} \tag{5.18}$$

where the values of k^2 are determined by the boundary condition

$$0 = (r \cdot \mathrm{grad})\delta\rho \Bigg|_{\substack{r=0 \\ r=a}} \longrightarrow \begin{array}{ll} j_e'\,(ka) = 0 & e \geq 1 \\ j_0\,(ka) = 0 & e = 0 \end{array} \tag{5.19}$$

By using $\chi\,\rho_0/M_q \to 1$ and $\rho_q = \rho_{\bar{q}} = \rho_0/2$ as it should be for confined hadronic phase we get finally

$$\omega = \sqrt{2k^2 + \omega_p{}^2} \tag{5.20}$$

where

$$\omega_p{}^2 = \frac{4\pi g_a{}^2}{M_q}\,\rho_0 \tag{5.21}$$

It is interesting to note that the equation determining ω_ρ is the well-known formula for the frequency of collective oscillations in plasma[19] (here the "plasma" made of the massive quarks and antiquarks interacting through the gluon fields!). This amusing analogy brings an idea of a possible existence of the new state of hadronic matter, namely the gas of free (unconfined) quarks and antiquarks interacting with the colored gluon fields emboded in the confined hadronic phase. Obviously, no such a "plasma" type state can be realized in the vacuum.

Now we estimate the value of the hadronic "plasma" frequency $\omega\rho$. First of all, we should take into account the color group structure of the quark-gluon coupling constant. We guess that this can be done by substitution

$$g_a^{\ 2} \longrightarrow \frac{1}{3}\,\mathrm{Spur}\,(g_* \lambda_a)^2 = g_*^{\ 2} C_2^{\ \mathrm{color}}$$

where $C_2^{\ \mathrm{color}}$ is the quadratic Casimir operator for the SU_3'-color group, so that

$$C_2^{\ \mathrm{color}}\ (\text{single quark}) = 16/3$$

Here g_*--the rationalized coupling constant--is to be taken in the Gauss units, so that

$$g_*^{\ 2} = g^2/4\pi$$

where g is the usual quark-gluon coupling constant determined by the Lagrangian $g\bar{\psi}\lambda^a\gamma_\mu\psi A_\mu^{\ a}$. One has now

$$\omega p^2 = \frac{g^2\rho_0}{M_q} C_2^{\ \mathrm{color}} = \left(\frac{g^2}{4\pi}\right)\frac{16}{M_q a^3}$$

This can be compared with the energy of self-interaction of a quark through the gluon field:

$$\Delta M(\text{self-inter.}) = \left(\frac{g^2}{4\pi}\right)\frac{C_2^{\ \mathrm{color}}}{a} = \left(\frac{g^2}{4\pi}\right)\frac{16}{3a}$$

which for $1/a \approx 0.94$ GeV and $g2/4\pi \sim 1$ gives $\Delta M \sim 5$ GeV. By using the Eq. (4.9) one gets the relation

$$\omega_p^2 = 6\,\Gamma\Delta M$$

which gives under the previous conditions the value $\omega_p^2 \sim \Gamma \cdot 30$ GeV or $\omega_p \sim 1.2$ GeV for $\Gamma \sim 50$ MeV.

Note Added: During the completion of this work, a paper by J. W. Moffat (University of Toronto preprint) was called to my attention in which a "quark-nucleus" shell model of hadrons is proposed on the basis of a generalization of the empirical nucleus mass formula with the symmetry-energy term. This paper does not consider the collective structure of multiquark hadronic states.

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FIGURE CAPTIONS

Fig. 1: The photoneutron cross section of 0 up to 30 MeV, [taken from Bramblett, Caldwell, Harvey, Fultz (64)].

Fig. 2: A schematic representation of giant resonances as hydrodynamic oscillations.

Fig. 3: Scheme of collective multipole vibrations of nuclear matter when spin (s) and isospin (i) are taken into account (modes of the generalized Goldhaber-Teller model): (a) E0: (I) i mode, (II) si, (III) s, (IV) protons and neutrons in phase; (b) E1: (I) i mode, (II) si, (III) s; (c) E2: (I) i mode, (II) si, (III) s, (IV) in phase.

Fig. 4: (a) Schematic representation of hadron matter production process in one-photon e^+e^--annihilation. (b) Illustration to the "hard-sphere" model: quark-antiquark splitting in uniform electric field of the virtual photon in the center of the mass system.

Fig. 5: Graphic demonstration of the energy balance of two potentials: one of the restoring force $U = \frac{1}{2}\alpha r^2$ and another of the quark ionization of hadronic matter $U_{\text{ioniz.}} = U_0(\delta n_q + \delta n_{\bar{q}}) \approx U_0 \rho \delta V$.

Fig. 6: Pictures illustrating the different phases of hadronic matter and different types of fluctuations: (1) around quark/antiquark and gluon vacuum, (2) around confined hadronic phase. The

sign of the derivative $\delta\mathscr{H}/\delta\rho$ characterizes whether the fluctuation is being stable or not.

Fig. 7: The analytic expressions and the graphics of the first few Bessel functions for n = e = 0, 1 and 2.[11]

Fig. 8: The illustration of the relations between the energies of the collective resonances for modes E0 (L = 0), E1 (L = 1) and E2 (L = 2), the unconfined quark mass M_q and the lower bound on the total resonance widths Γ_{min}. The arrows indicate the positions of the collective resonances for the particular value of a size of hadronic system $a \approx 0.2$ fm.

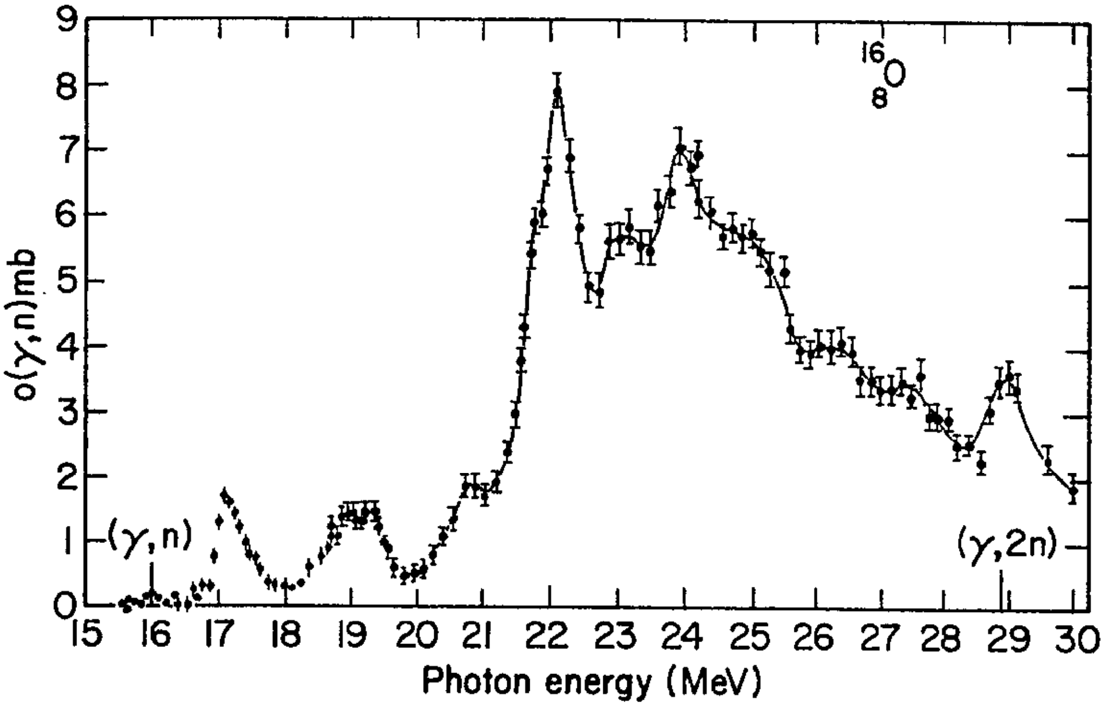

$^{16}_{8}O$
$\sigma(\gamma,n)$mb
Photon energy (MeV)
(γ,n)
$(\gamma,2n)$
0
1
2
3
4
5
6
7
8
9
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30

Fig. 1

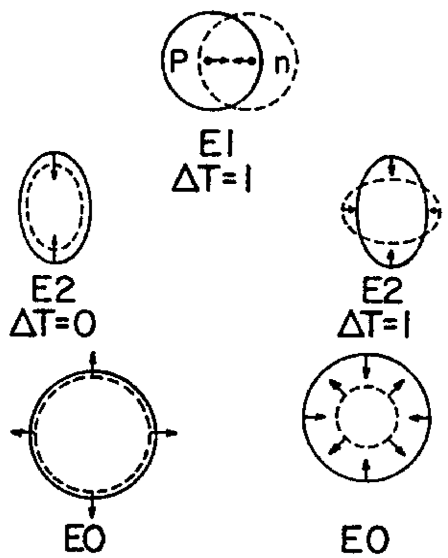

P
n
E1
$\Delta T=1$
E2
$\Delta T=0$
E2
$\Delta T=1$
E0
$\Delta T=0$
E0
$\Delta T=1$

Fig. 2

The Giant Resonance

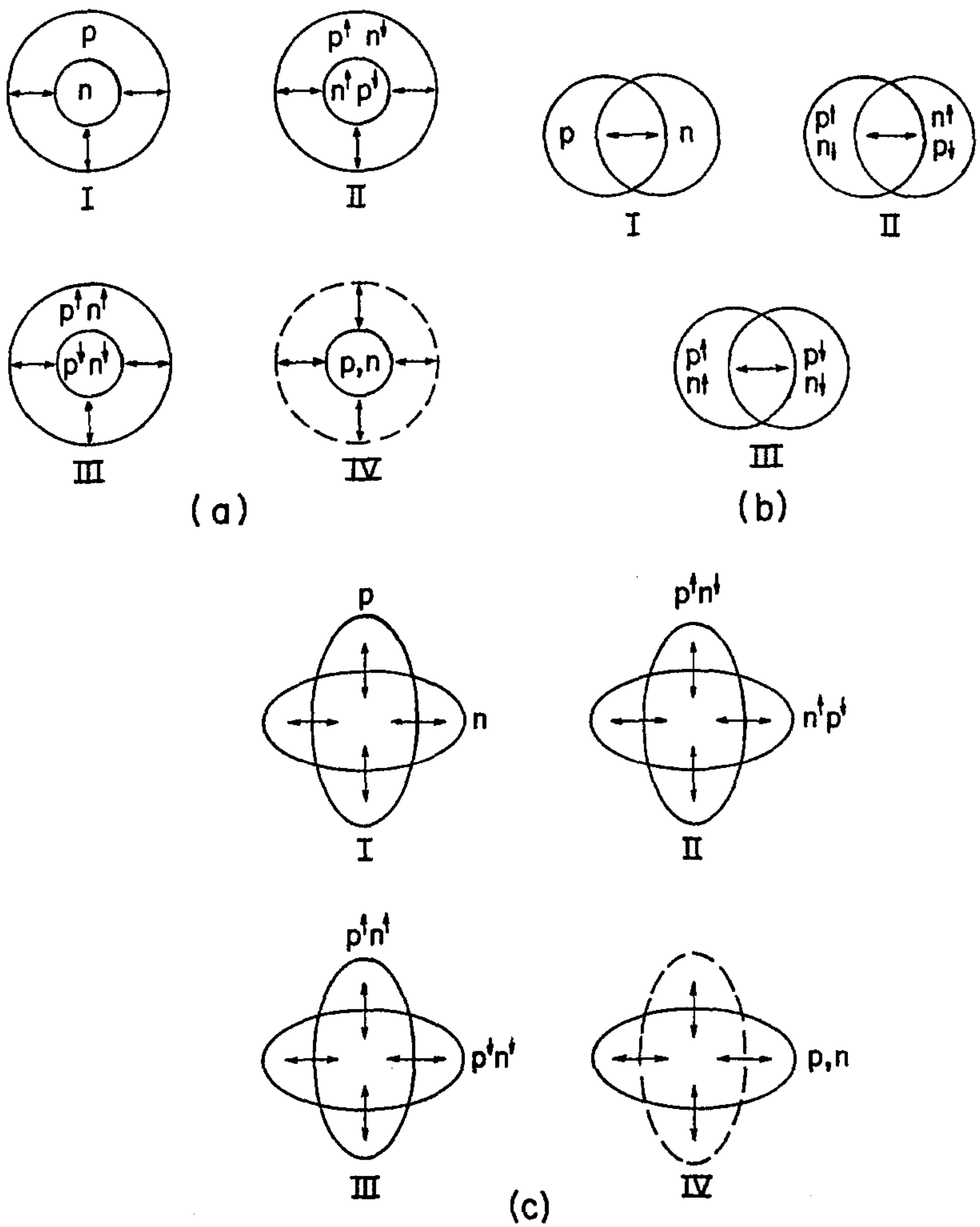
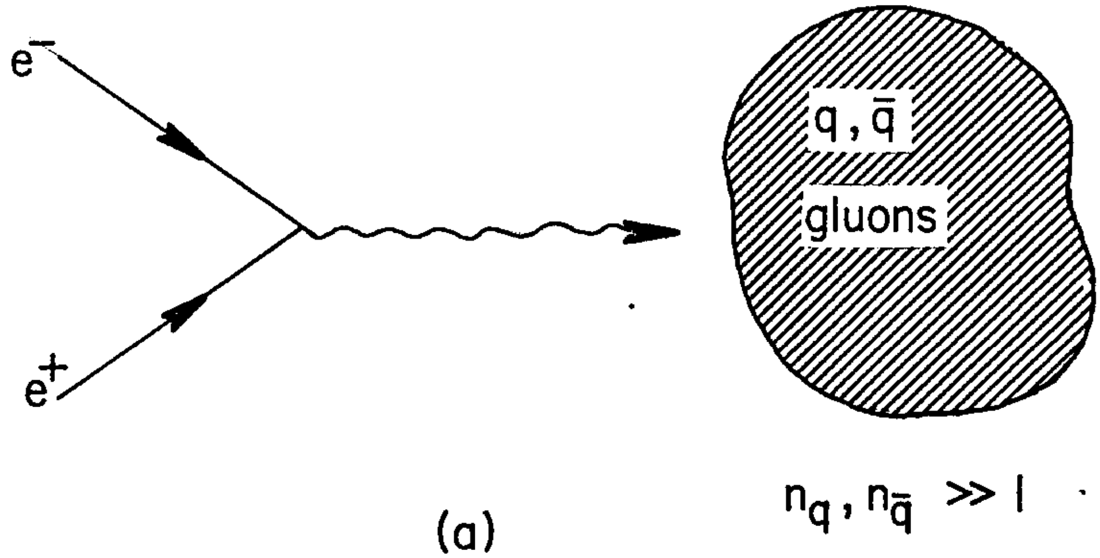
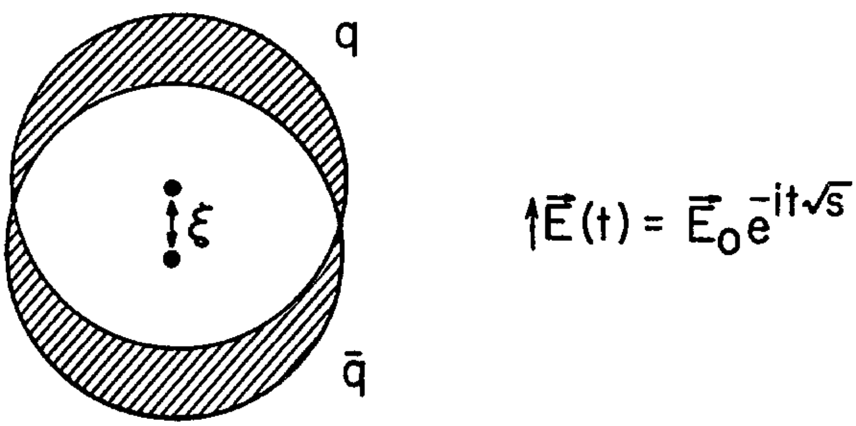


Fig. 3

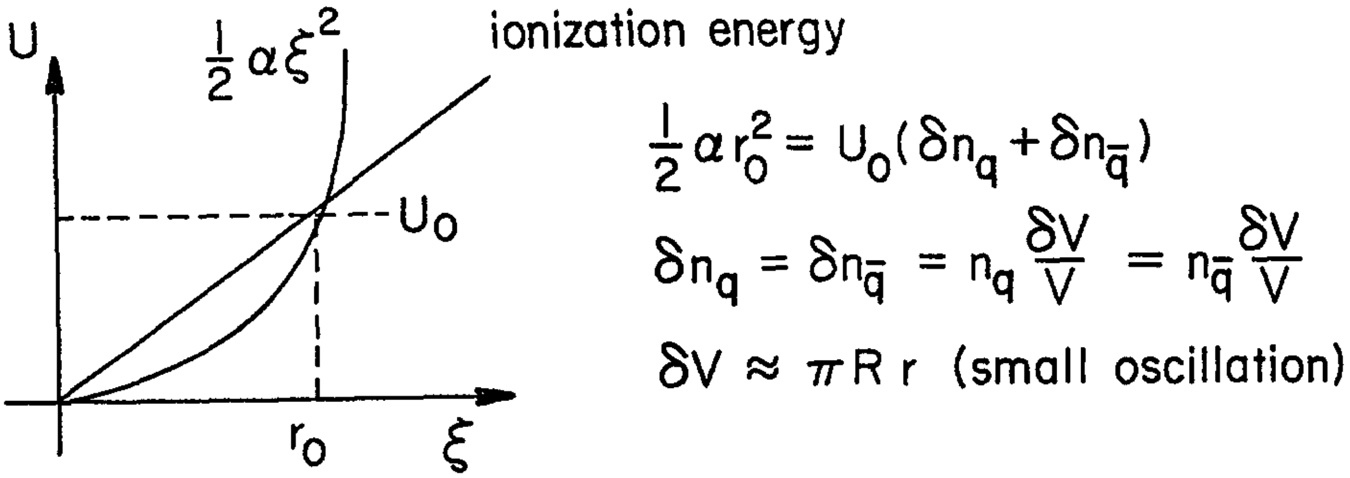


(a)



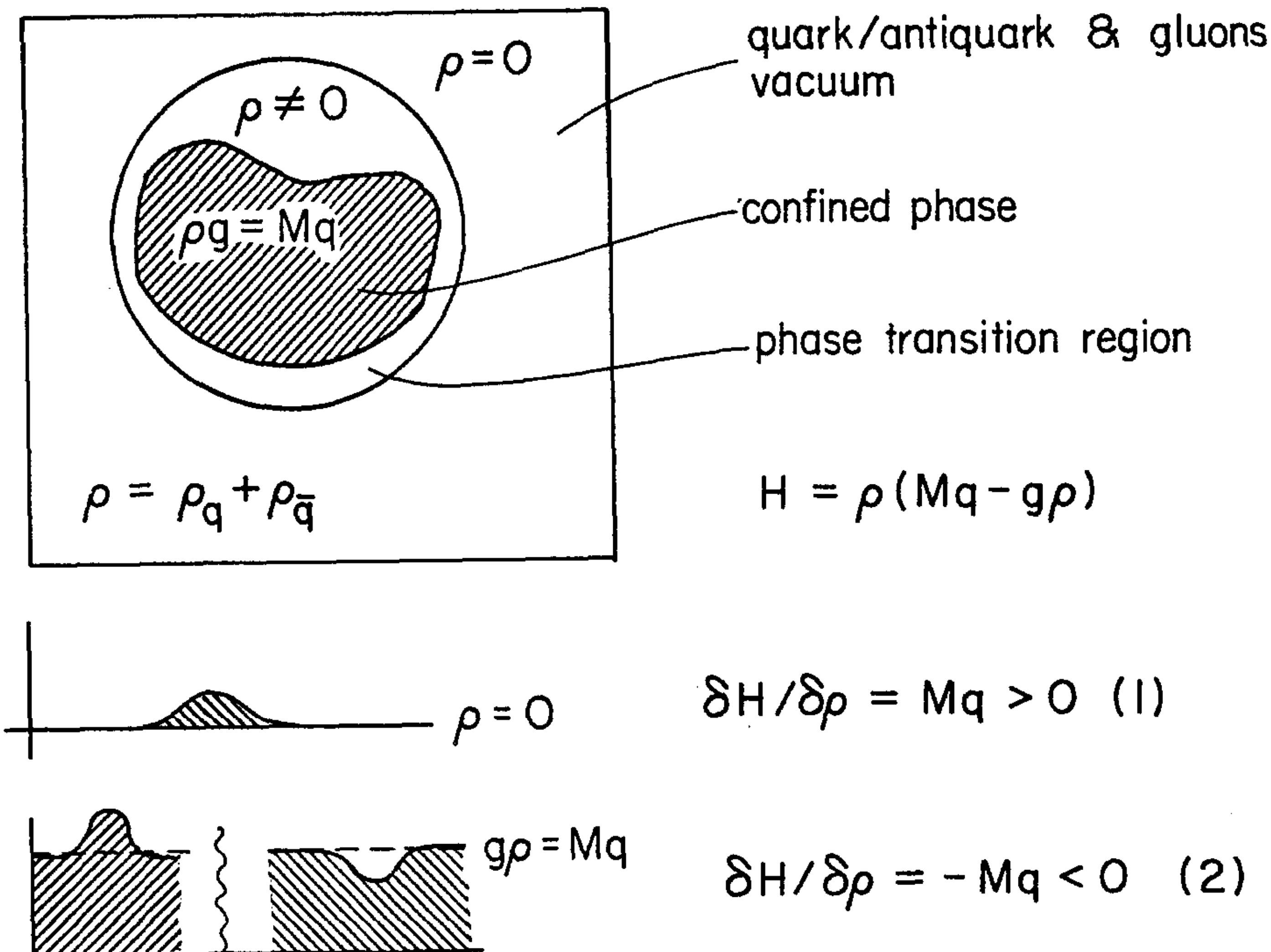
(b)

Fig. 4



U
$\frac{1}{2}\alpha\xi^2$
ionization energy
U_0
r_0
ξ
$\frac{1}{2}\alpha r_0^2 = U_0(\delta n_q + \delta n_{\bar{q}})$
$\delta n_q = \delta n_{\bar{q}} = n_q \frac{\delta V}{V} = n_{\bar{q}} \frac{\delta V}{V}$
$\delta V \approx \pi R r$ (small oscillation)

Fig. 5



$\rho = 0$
$\rho \neq 0$
$\rho g = Mq$
quark/antiquark & gluons vacuum
confined phase
phase transition region
$\rho = \rho_q + \rho_{\bar{q}}$
$H = \rho(Mq - g\rho)$
$\rho = 0$
$\delta H/\delta\rho = Mq > 0$ (1)
$g\rho = Mq$
$\delta H/\delta\rho = -Mq < 0$ (2)

Fig. 6

The functions $J_n(z), Y_n(z)$ for $n = 0, 1, 2$

$$J_0(z) = \frac{\sin z}{z}$$

$$J_1(z) = \frac{\sin z}{z^2} - \frac{\cos z}{z}$$

$$J_2(z) = \frac{3}{z^2} - \frac{1}{z} \sin z - \frac{3}{z^2} \cos z$$

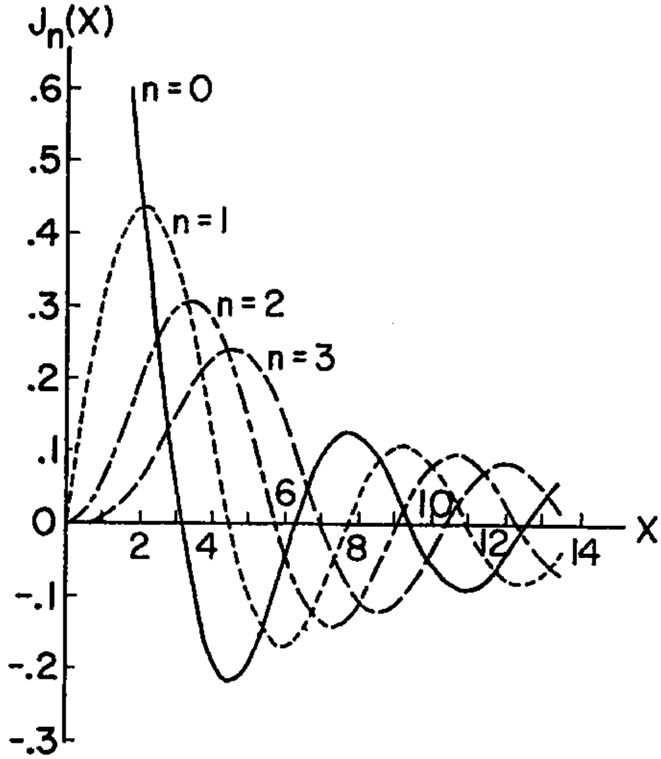
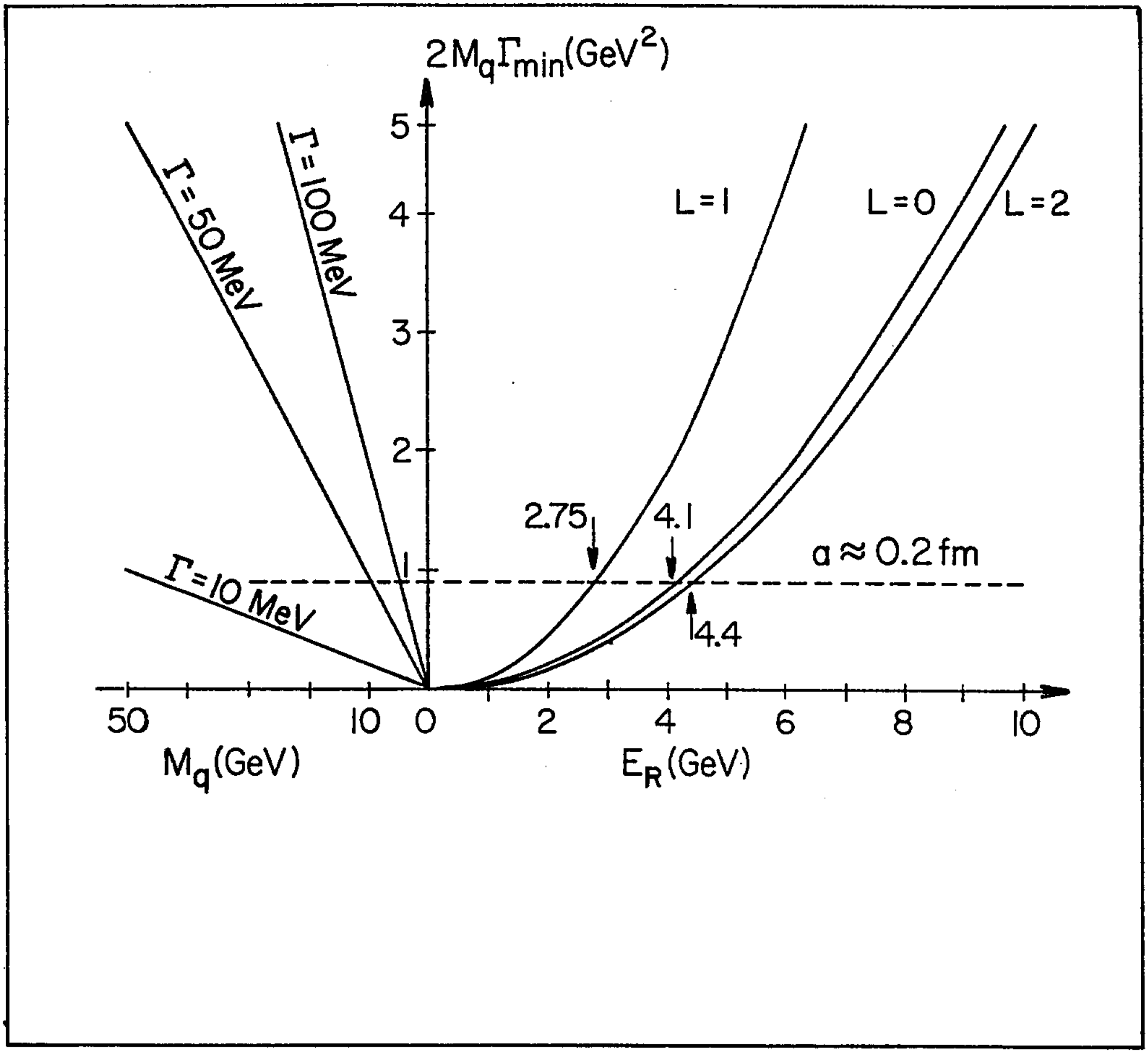


Fig. 7



2 Mq Γmin (GeV²)
5
4
3
2
1
Γ = 50 MeV
Γ = 100 MeV
Γ = 10 MeV
L = 1
L = 0
L = 2
2.75
4.1
4.4
a ≈ 0.2 fm
50
10
0
2
4
6
8
10
Mq (GeV)
ER (GeV)

Fig. 8