Fermi National Accelerator Laboratory

FERMILAB-Pub-76/24-THY
February

Local Gauge Invariance and the Bound State Nature of Hadrons

WILLIAM A. BARDEEN and ROBERT B. PEARSON
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

ABSTRACT

We analyze those features of non abelian color gauge theories which lead to confinement. A consistent picture of hadrons as bound states of quarks and gluons emerges when the vacuum is gauge invariant. The introduction of a transverse lattice approximation leads to a description of the theory in terms of basic hadronic degrees of freedom and a tractable method for calculation of properties of hadrons.

Operated by Universities Research Association Inc. under contract with the Energy Research and Development Administration

Quarks have formed the basis of much of our understanding of strong interaction phenomenology. Whether quarks may also form the basis of a dynamical theory of hadrons has become one of the most important questions in elementary particle physics.

The most attractive theory for the dynamics of quarks is a gauge field theory with a non abelian color gauge group. This theory has been used to study the short distance behaviour of current operators where the predicted asymptotic freedom[1] provides an understanding of the approximate scaling observed in deep inelastic electron scattering and e^+e^- annihilation. In this paper we will show that this theory also provides the basis for a complete dynamical theory of hadrons.

The color gauge theory is compactly described by the action

$$A = \int dt \int d\underset{\sim}{x} \left\{ \bar{q}\,(i \not{D} - m)\, q - \frac{1}{4} \vec{G}_{\mu\nu}^{\,2} \right\}, \tag{1}$$

where the quark fields, q, carry both color and flavor indices, D_μ is the gauge covariant derivative, and $\vec{G}_{\mu\nu}$ is the Yang-Mills field strength tensor. The quark mass matrix, m, is singlet in color but depends on the quark flavor.

This theory has been studied order by order in perturbation theory, and has been shown to give a consistent theory of quarks and gluons, but not hadrons.[2] Wilson and others[3] have suggested that quarks and gluons are the physical particles in one possible phase of the theory, and that hadrons are the physical particles in another phase which cannot be reached from perturbation theory.

Much confusion exists in the literature concerning the nature and existence of such a phase transition. This confusion can be traced to the fact that the action is invariant under local gauge transformations and that the gauge field $\vec{A}_\mu(x)$ describes two physical degrees of freedom, not four. We may study the theory either by choosing a gauge which eliminates the redundant degrees of freedom, or by considering only gauge invariant quantities. To study the continuum theory we choose the axial gauge $\vec{A}_z = 0$, and eliminate $\vec{A}_0$ for each of the color gauge fields thereby eliminating the two redundant degrees of freedom. The Hamiltonian for the system, neglecting quarks, becomes

$$\vec{H} = \int dz dx_\perp \left\{ \frac{1}{2} \vec{P}_\alpha^2 + \frac{1}{4} \vec{G}_{\alpha\beta}^2 + \frac{1}{2} (\partial_z A_\alpha)^2 \right\}$$

$$- \int dz dz' dx_\perp \frac{1}{4} g^2 \vec{J}(z, x_\perp) \left| z - z' \right| \vec{J}(z', x_\perp) , \tag{2}$$

where $\alpha, \beta = \hat{x}, \hat{y}$. $\vec{P}_\alpha$ is the cannonical momentum for the transverse gluons, and $\vec{J}(z, x_\perp)$ is the local color charge density. The theory remains invariant under gauge transformations which are local in x and y but are global with respect to z and t. The conserved charge which generates these transformations is

$$\vec{Q}(x_\perp) = \int dz\ \vec{J}(t, z, x_\perp)\ , \tag{3}$$

where the charge density is given by

$$\vec{J}(t, z, x_\perp) = \frac{1}{g}\ \partial_\alpha \vec{P}_\alpha + \vec{A}_\alpha \times \vec{P}_\alpha\ . \tag{4}$$

Here the term linear in gluon fields is a reflection that the theory is described by a nonlinear realization of the transverse gauge symmetry in terms of the gluon fields. The ordinary (perturbative) state of this theory with physical gluons results from a spontaneous breakdown of transverse gauge invariance in direct analogy to the Higgs mechanism.

We now consider the possibility that another phase exists for which the symmetry is not spontaneously broken. Since there is a symmetry associated with each point (x, y), it is useful to introduce a transverse lattice by keeping t and z as continuous variables but treating the x and y coordinates as a square lattice with lattice spacing a. The transverse lattice serves two purposes as it provides a gauge invariant ultraviolet cutoff for the theory, and, at the same time, allows us a method of studying the gauge symmetry at each discrete point in transverse space.

The lattice variables are defined in a manner analogous to those used by Kogut and Susskind[4] in their formulation of lattice gauge theories. The gauge fields are defined as

$$\vec{A}_{\underset{\sim}{n},\mu}(t,z) = \vec{A}_{\mu}(t,z,\underset{\sim}{n}a)\,a \qquad \mu=\hat{t},\hat{z}$$

$$\vec{A}_{\underset{\sim}{n},\alpha}(t,z) = \vec{A}_{\alpha}(t,z,\underset{\sim}{n}a) \qquad \alpha=\hat{x},\hat{y} \quad , \tag{5}$$

where $\vec{A}_{\underset{\sim}{n},\mu}$ is identified with the site with lattice vector $\underset{\sim}{n}$, and and $\vec{A}_{\underset{\sim}{n},\alpha}$ with a link between $\underset{\sim}{n}$ and $\underset{\sim}{n}+\hat{\alpha}$. The quark fields are identified with lattice sites.

There are technical difficulties associated with the description of fermions in lattice theories. The approximation of the linear derivative in the Dirac equation by finite differences leads to an increase in the number of fermions in the naive continuum limit.[5] We partially overcome this problem by splitting the four components of the fermion spinor placing the spin up quarks on even and spin down quarks on odd lattice sites. The remaining spurious degree of freedom could be removed by adding a second derivative term which vanishes in the naive continuum limit, or by further splitting the fermion components. This has not been done here. The spin projected fermion fields are given by

$$\chi_{\underset{\sim}{n}}(t,z) = \sqrt{2}\,a\,\frac{1+\sigma_z}{2}\,\psi\,(t,z,\underset{\sim}{n}a)\,, \quad \underset{\sim}{n} \text{ even}$$

$$\chi_{\underset{\sim}{n}}(t,z) = \sqrt{2}\,a\,i\beta\,\frac{1-\sigma_z}{2}\,\psi\,(t,z,\underset{\sim}{n}a)\,, \quad \underset{\sim}{n} \text{ odd} \tag{6}$$

The gauge theory action of Eq. (1) when written in terms of lattice fields becomes

$$A = \int dtdz \sum_{\underset{\sim}{n}\mu\nu} \left\{ -\frac{1}{4} \vec{G}^{\,2}_{\underset{\sim}{n}\mu\nu} \right\}$$

$$+ \int dtdz \sum_{\underset{\sim}{n}\alpha} \mathrm{tr} \left[D_\mu M_{\underset{\sim}{n}\alpha} D_\mu M^+_{\underset{\sim}{n}\alpha} \right]$$

$$+ \int dtdz \sum_{\underset{\sim}{n}\alpha\beta} \frac{H}{a^2} \mathrm{tr} \left[M_{\underset{\sim}{n}\alpha} M_{\underset{\sim}{n}+\hat{\alpha},\beta} M^+_{\underset{\sim}{n}+\hat{\beta},\alpha} M^+_{\underset{\sim}{n}\beta} \right]$$

$$+ \int dtdz \sum_{\underset{\sim}{n}} \overline{\chi}_{\underset{\sim}{n}} (i\gamma^\mu D_\mu - m) \chi_{\underset{\sim}{n}}$$

$$- \int dtdz \sum_{\underset{\sim}{n}} \frac{G}{2a} \left[\overline{\chi}_{\underset{\sim}{n}} S_{\underset{\sim}{n}\alpha} i\gamma_5 M_{\underset{\sim}{n}\alpha} \chi_{\underset{\sim}{n}+\hat{\alpha}} + \overline{\chi}_{\underset{\sim}{n}+\hat{\alpha}} S^*_{\underset{\sim}{n}\alpha} i\gamma_5 M^+_{\underset{\sim}{n}\alpha} \chi_{\underset{\sim}{n}} \right] , \qquad (7)$$

where $\vec{G}_{\mu\nu}$ is the longitudinal field strength tensor, $\chi_{\underset{\sim}{n}}$ is written as a two component spinor, and γ_μ and γ_5 are the two dimensional γ matrices. The field, $M_{\underset{\sim}{n}\alpha}$, is related to the transverse gauge fields by

$$M_{\underset{\sim}{n}} = \frac{1}{g} \exp\left(iag\vec{T}\cdot\vec{A}_{\underset{\sim}{n}\alpha}\right) \quad . \tag{8}$$

The fermion spin factors are given by $S_{\underset{\sim}{n}\hat{x}} = i \quad S_{\underset{\sim}{n}\cdot\hat{y}} = (-1)^n$.
The covariant derivatives are defined by

$$D_\mu M_{\underset{\sim}{n}\alpha} = \partial_\mu M_{\underset{\sim}{n}\alpha} + i\frac{g}{a}\vec{T}\cdot\vec{A}_{\underset{\sim}{n}\mu} M_{\underset{\sim}{n}\alpha} - i\frac{g}{a} M_{\underset{\sim}{n}\alpha}\vec{T}\cdot\vec{A}_{\underset{\sim}{n}+\hat{\alpha},\mu}$$

$$D_\mu\chi_{\underset{\sim}{n}} = \partial_\mu\chi_{\underset{\sim}{n}} + i\frac{g}{a}\vec{T}\cdot\vec{A}_{\underset{\sim}{n}}\chi_{\underset{\sim}{n}} \quad . \tag{9}$$

This completes the definition of the lattice gauge theory. The naive continuum limit is recovered by taking the limit $a\to 0$, making the identifications: $g\to g$, $H\to g^2$, $G\to g$, and using the field identifications, Eqs. (5, 6, 8).

We would now like to discuss some of the properties of the lattice action Eq. (7). The action is invariant under the complete set of local color gauge transformations. It also preserves the global flavor symmetries of vector and axial vector charges with only the quark mass term breaking these symmetries. The transformation properties are $V\chi_{\underset{\sim}{n}}V^{-1} = e^{iF}\chi_{\underset{\sim}{n}}$ for vector charges and $A\chi_{\underset{\sim}{n}}A^{-1} = e^{i(-1)^n F\gamma_5}\chi_{\underset{\sim}{n}}$

for axial charges where F is the matrix representation of the transformation which only acts on the flavor indices of the quarks. The lattice theory clearly breaks many of the space time symmetries; however, Lorentz transformations and continuous translations in the longitudinal direction are clearly preserved. We believe these features to be advantages of the transverse lattice.

To study some of the implications of the transverse lattice theory, we again wish to focus our attention on the physical degrees of freedom by eliminating redundant degrees of freedom. For this purpose the light cone gauge $\vec{A}_{\underset{\sim}{n}-} = 0$, where $\vec{A}_{\underset{\sim}{n}\pm} = (\vec{A}_{\underset{\sim}{n}t} \pm \vec{A}_{\underset{\sim}{n}z})/\sqrt{2}$, is a convenient choice. We may then eliminate the field $\vec{A}_{\underset{\sim}{n}+}$ using the equations of motion. In terms of the physical degrees of freedom, the lattice action of Eq. (7) becomes

$$\begin{aligned}
A = &\int dx_+ dx_- \sum_{\underset{\sim}{n}\alpha} \mathrm{tr}\,(\partial_\mu M_{\underset{\sim}{n}\alpha} \partial_\mu M^+_{\underset{\sim}{n}\alpha}) \\
&+ \int dx_+ dx_- \sum_{\underset{\sim}{n}} \bar{\chi}_{\underset{\sim}{n}} (i \not\partial - m) \chi_{\underset{\sim}{n}} \\
&- \int dx_+ dx_- \sum_{\underset{\sim}{n}\alpha} \frac{G}{2a} \left\{ \bar{\chi}_{\underset{\sim}{n}} S_{\underset{\sim}{n}\alpha}\, i\gamma_5 M_{\underset{\sim}{n}\alpha} \chi_{\underset{\sim}{n}+\hat{\alpha}} + \bar{\chi}_{\underset{\sim}{n}+\hat{\alpha}} S^*_{\underset{\sim}{n}\alpha}\, i\gamma_5 M^+_{\underset{\sim}{n}\alpha} \chi_{\underset{\sim}{n}} \right\} \\
&+ \int dx_+ dx_- \sum_{\underset{\sim}{n}\alpha\beta} \frac{H}{a^2}\, \mathrm{tr}\,(M_{\underset{\sim}{n}\alpha} M_{\underset{\sim}{n}+\hat{\alpha},\beta} M^+_{\underset{\sim}{n}+\hat{\beta},\alpha} M^+_{\underset{\sim}{n}\beta}) \\
&+ \int dx_- dx_+ dx'_+ \sum_{\underset{\sim}{n}} \frac{g^2}{4a^2} \vec{J}_{\underset{\sim}{n}-}(x_-, x_+) \left| x_+ - x'_+ \right| \vec{J}_{\underset{\sim}{n}-}(x_-, x'_+) \;, \qquad (10)
\end{aligned}$$

where $x_\pm \equiv (t \pm z)/\sqrt{2}$ and the charge density $\vec{J}_{\underset{\sim}{n}-}(x_-, x_+)$ is given by

$$\vec{J}_{\underset{\sim}{n}-} = \sum_\alpha \mathrm{tr}\left[\vec{T}(M_{\underset{\sim}{n}\alpha}\, i\overleftrightarrow{\partial}_-\, M^+_{\underset{\sim}{n}\alpha} + M^+_{\underset{\sim}{n}-\hat{\alpha},\alpha}\, i\overleftrightarrow{\partial}_-\, M_{\underset{\sim}{n}-\hat{\alpha},\alpha})\right]$$

$$- \bar{\chi}_{\underset{\sim}{n}}\gamma_- \vec{T}\chi_{\underset{\sim}{n}} \quad . \tag{11}$$

As in the continuous case the action is invariant under a set of gauge transformations whose generators are the conserved charges

$$\vec{Q}_{\underset{\sim}{n}} = \int dx_+ \, \vec{J}_{\underset{\sim}{n}-}(x_-, x_+) \ . \tag{12}$$

In the continuous case a linear term in the charge density signaled the spontaneous breakdown of transverse gauge invariance. Here there are two possibilities. If the operators $M_{\underset{\sim}{n}\alpha}$ have a non zero vacuum expectation value as one would expect naively to recover the continuum limit, the transverse symmetry is spontaneously broken, and the theory would describe quarks and gluons as physical excitations. If $M_{\underset{\sim}{n}\alpha}$ has zero vacuum expectation value, the vacuum remains invariant under transverse gauge rotations. It is clearly the second possibility which must obtain if the theory is to describe hadrons. The mechanism which generates the second phase is analogous to that of the nonlinear σ model in two dimensions studied recently by several authors.[6] It is shown that the nonlinear O(N) σ model exists in two dimensions only as a linear realization of the symmetry, with a full degenerate multiplet of massive scalars generated as bound states of the nonlinear degrees of freedeom. In the closely related nonlinear O(N) σ model with gauge fields in two dimensions a similar result is found,[7] except for the existence of a nonlinear Higgs phase with a first order transition to the linear phase which again describes a full multiplet of massive excitations generated dynamically from the nonlinear degrees of freedom. The longitudinal dynamics of the transverse gluons are precisely of the latter form.

With the above motivation we will assume that hadronic physics is described by the symmetric phase and a linear realization of the transverse gauge symmetry. Therefore we modify the action by allowing all of the degrees of freedom of the complex matrices $M_{\underset{\sim}{n}\alpha}$ to be dynamical. We must also add a local potential in the fields $M_{\underset{\sim}{n}\alpha}$ to the action. The role of this linearization is to correctly describe the important degrees of freedom when the lattice spacing is large. A further advantage is that we may study both phases of the system by adjusting the parameters of the local potential. These parameters and the parameters of the action are not really free parameters but must be determined by a renormalization group from the continuum limit. This subject will not be discussed here.

The potential which must preserve the local gauge symmetries has the general form

$$V = \int dx_- \int dx_+ \sum_{\underset{\sim}{n}\alpha} \left\{ \mu^2 \, \mathrm{tr}\,(M_{\underset{\sim}{n}\alpha} M^+_{\underset{\sim}{n}\alpha}) \right.$$

$$+ \lambda_1 \, \mathrm{tr}\,(M_{\underset{\sim}{n}\alpha} M^+_{\underset{\sim}{n}\alpha} M_{\underset{\sim}{n}\alpha} M^+_{\underset{\sim}{n}\alpha})$$

$$+ \lambda_2 \left[\mathrm{tr}\,(M_{\underset{\sim}{n}\alpha} M^+_{\underset{\sim}{n}\alpha}) \right]^2$$

$$\left. + \lambda_3 \, (\det M_{\underset{\sim}{n}\alpha} + \det M^+_{\underset{\sim}{n}\alpha}) \right\} . \tag{13}$$

The linearized transverse lattice theory is a two dimensional continuum field theory in the longitudinal direction with quark fields associated with each site and meson fields associated with each link of the transverse lattice. If the vacuum is invariant under transverse gauge transformations, the linear "Coulomb" potential generated by integrating out the longitudinal gauge fields will confine the local color charges. States which are not locally color singlet are completely decoupled from the spectrum of physical states. We emphasize that the states must be singlet with respect to color rotations at each transverse site.

The confinement of quarks is a direct result of this Coulomb potential. For quarks at a given site the binding in the longitudinal direction comes directly from the potential. Bound states of quarks separated in the transverse direction must include enough link mesons so that the state is color singlet at each site between the quarks. The energy will depend on the minimum number of link mesons needed to form a color singlet state, and will grow with the distance between the quarks. The actual energy of the state will depend on the mass of the links, and the energy associated with the binding of neighboring links in the chain.

While the theory clearly confines quarks, we must see if it really makes hadrons. The Hamiltonian for the theory may be constructed by standard methods. Since we have chosen a light cone gauge we use light cone quantization for the quarks and link meson fields. The Hamiltonian for the system becomes

$$\begin{aligned} H = & \int dx_+ \sum_{\underset{\sim}{n}\alpha} \mu^{*2}\, \mathrm{tr}\,(M_{\underset{\sim}{n}\alpha} M^+_{\underset{\sim}{n}\alpha}) \\ & + \int dx_+\, \chi_1^+ \left\{ -im + \frac{G}{2a}\mathscr{M} \right\} \frac{1}{2i\partial_-} \left\{ im + \frac{G}{2a}\mathscr{M} \right\} \chi_1 \\ & - \int dx_+ \frac{g^2}{\pi a^2} C_N \chi_1^+ \frac{1}{2i\partial_-} \chi_1 \\ & - \int dx_+ \sum_{\underset{\sim}{n}\alpha\beta} \frac{H}{a^2}\, \mathrm{tr} \left(M_{\underset{\sim}{n}\alpha} M_{\underset{\sim}{n}+\hat{\alpha},\beta} M^+_{\underset{\sim}{n}+\hat{\beta},\alpha} M^+_{\underset{\sim}{n},\beta} \right) \\ & - \int dx_+ \int dx'_+ \sum_{\underset{\sim}{n}} \frac{g^2}{4a^2} : \vec{J}_{\underset{\sim}{n}-}(x_+) \left| x_+ - x'_+ \right| \vec{J}_{\underset{\sim}{n}-}(x'_+) : \end{aligned} \tag{14}$$

Not all of the terms in the potential have been indicated. χ_1 is a single component dirac field given by the projection $\chi_1 = 2^{-5/4}(1-\gamma_5)\chi$, $\vec{T}^2 = C_N I$, and $\mathscr{M}$ is a matrix in color and lattice indices given by

$$\mathscr{M}_{\underset{\sim}{n}\underset{\sim}{n}'} = \sum_\alpha \left\{ \delta_{\underset{\sim}{n}',\underset{\sim}{n}+\hat{\alpha}}\, S_{\underset{\sim}{n}\alpha} M_{\underset{\sim}{n}\alpha} + \delta_{\underset{\sim}{n}'+\hat{\alpha},\underset{\sim}{n}}\, S^*_{\underset{\sim}{n}'\alpha} M^+_{\underset{\sim}{n}'\alpha} \right\} . \tag{15}$$

The correction to the quark mass term comes from normal ordering the Coulomb interaction. The link meson mass μ^{*2} is the renormalized mass. The fields have the plane wave expansions and commutation relations

$$M(x_+) = \int_0^\infty \frac{dk}{2k} \left\{ A_k f_k(x_+) + B_k^+ f_k^*(x_+) \right\}$$

$$[A_k, A_{k'}^+] = [B_k, B_{k'}^+] = 2k\, \delta(k-k')$$

$$\chi_1(x_+) = \int_0^\infty \frac{dk}{\sqrt{2k}} \left\{ a_k f_k(x_+) + b_k^+ f_k^*(x_+) \right\}$$

$$\{a_k, a_{k'}^+\} = \{b_k, b_{k'}^+\} = 2k\, \delta(k-k')$$

$$f(x_+) = \frac{1}{\sqrt{2\pi}}\, e^{-ikx_+} \quad , \tag{16}$$

where we have suppressed the site and color labels.

In order to systematically study the Hamiltonian of Eq. (14) we must separate the part of the Coulomb interaction which acts as the potential between particles. This potential acts independently at each transverse site and is confining, and thus inherently non-perturbative. This part can be diagonalized if one keeps in H_0 only the longitudinal kinetic terms and the Coulomb interactions which do not produce pairs. The remaining interactions may be treated perturbatively. The states of H_0 are a spectrum of transversely static bound states associated with each configuration of quarks and link mesons. The perturbation theory which results is one of "bare" hadrons and their interactions. At any level of sophistication in describing the localized states one may compute the couplings between these states and those of neighboring configurations reducing the problem to one of finding the normal modes on the transverse lattice. This generates a continuous spectrum of excitations with definite transverse momentum and masses of the form $2P_+P_- = M^2 + P_\perp^2/c^2$ for $P_\perp$ small compared to the inverse lattice spacing. We expect this procedure to be highly tractable since we expect the physical hadrons to be composed of bare hadrons of approximately the same mass for suitable choice of lattice spacing. The full implementation of the program discussed above will be treated in a subsequent publication.

The simplest bare hadron is a quark-antiquark bound state at a single site. There is a discrete spectrum of such states, and they are precisely the states of the two dimensional 't Hooft model.[8] The wave functions may be written as $|p\rangle = \int_0^1 dx\, \phi(x)[2x(1-x)]^{-1/2} a^+_{xp} b^+_{(1-x)p} |0\rangle$ where $\langle p'_- | p_- \rangle = 2p_- \, \delta(p_- - p'_-)$, and $\int_0^1 dx |\phi(x)|^2 = 1$. $\phi(x)$ satisfies the wave equation

$$2p^+p^-\phi(x) = m^{*2}\left(\frac{1}{x} + \frac{1}{1-x}\right)\phi(x)$$

$$- \frac{g^2}{\pi a^2} C_N \fint_0^1 dy\, \phi(y) \frac{1}{|x-y|^2} , \qquad (18)$$

where the renormalized quark mass is given by $m^{*2} = m^2 - g^2 C_N/\pi a^2$. The principal value integral is to be taken.

'tHooft has shown that these states have approximately linear spacing in the mass squared, $2p_+p_-$. The ground state meson is pseudoscalar, and its mass goes to zero as the bare quark mass goes to zero as one would expect for a Goldstone realization of chiral symmetry. As we have noted the transverse lattice theory preserves chiral symmetry. Hence in the phase where the vacuum is invariant under the local gauge symmetry the vacuum is not invariant under the global chiral symmetry and vice versa.

Another bare hadronic bound state consists of a link meson and its antiparticle. The wave function for this state is given by

$$|p> = \int_0^1 dx\,\Phi(x)\;[2x(1-x)]^{-1/2} A^+_{xp} B^+_{(1-x)p}|0>$$

$$<p'|p> = 2p\delta(p-p'), \int_0^1 dx\;|\Phi(x)|^2 = 1 \;, \tag{19}$$

and $\Phi\,(x)$ satisfies the wave equation

$$2p^+p^-\;\Phi(x) = \mu^{*2}\left(\frac{1}{x}+\frac{1}{1-x}\right)\Phi(x)$$

$$-\frac{2g^2}{\pi a^2}\, C_N \int_0^1 \frac{dy}{|x-y|^2}\;\Phi(y)\;\frac{(x+y)(2-x-y)}{4[y(1-y)x(1-x)]^{1/2}}\;. \tag{20}$$

We interpret these states as daughters of the bare pomeron trajectory. We note that the WKB solutions for the meson and pomeron bound states yield a spacing for the pomeron just twice that of the meson, i.e. half the slope.

An amusing feature of the local bound states emerges when we use a $1/N$ expansion to restrict the Coulomb interactions to planar topology. A bound state of n link mesons at a given site is directly analogous to the longitudinal Virasoro string[9] in the limit that the link meson mass vanishes. Similarly a bound state of n link mesons and a quark-antiquark pair is the analog of the n break longitudinal string with quarks at the ends.[10] We note that when we include the interaction terms, the quarks can emit and absorb link mesons even when we restrict ourselves to one site for the link. Hence strings with different numbers of breaks are coupled together.

Other configurations for the bare hadrons may be studied in a similar fashion. The bound state equations follow directly from the application of H_0 to the appropriate states. We defer discussion of these states to a future paper where we will discuss the formation of physical hadrons.

It is important to note that the bare hadrons cannot be directly identified with the physical hadrons as they do not propagate in the transverse direction. The physical hadrons must necessarily involve those linear combinations of bare hadrons which have normal propagation in the transverse lattice.

In this paper, we have focused on the important physical concepts which result from a careful study of local color gauge theories. We have clarified the nature of the phase transition which leads to a gauge invariant ground state. The confinement of quarks and gluons occurs in the symmetric phase and is not a result of an infinite coupling strength, bag, or soliton solution to the field theory. The theory is studied through the introduction of a transverse lattice and linear realizations for the gluon fields. The nonperturbative effects which lead to the formation of the hadronic bound states are easily isolated. We believe that the transverse lattice theory represents a tractable method for a systematic study of those features of hadronic physics which do not involve large transverse momentum.

ACKNOWLEDMENT

We would like to thank the members of the Fermilab theory group for constructive criticism and encouragement. We would especially like to thank R. Savit, E. Rabinovici, B. W. Lee, and M. Einhorn for many useful discussions.

REFERENCES

[1] G. 't Hooft, Marseilles Conference on Gauge Theories, 1972 (unpublished); H. D. Politzer, Phys. Rev. Lett., **30**, 1345 (1973); D. J. Gross, F. Wilczek, Phys. Rev. Lett., **30**, 1343 (1973).

[2] T. Appelquist, J. Carazzone, H. Kluberg-Stern, M. Roth, Fermilab-Pub-76/16-THY, to be published.

[3] K. Wilson, Phys. Rev., **D10**, 2445 (1974); R. Balian, J. Drouffe, C. Itzykson, Phys. Rev., **D10**, 3376 (1974).

[4] J. Kogut, L. Susskind, Phys. Rev., **D11**, 399 (1975).

[5] K. Wilson, Erice lectures (1975), unpublished.

[6] The symmetry properties of the σ model have been discussed by A. Polyakov, Phys. Lett., **59B**, 79 (1975); A. Migdal, Landau Institute of Theoretical Physics preprint, 1975; E. Brezin, J. Zinn-Justin, to be published. The precise nature of the phase transition in the large N limit is given by W. Bardeen, B. Lee, R. Shrock, Fermilab-preprint 76/33-THY.

[7] These results were obtained by one of the authors (W.A.B.) and M. Bander, to be published.

[8] G. 't Hooft, Nucl. Phys., **B75**, 461 (1974).

[9] M. A. Virasoro, Phys. Rev., **177**, 2309 (1969); C. Rebbi, Phys. Lett., **12C**, 1 (1974).

[10] A. Patrasciou, Nucl. Phys. B81, 525 (1974), W. A. Bardeen, I. Bars, A. J. Hansen, R. D. Peccei, Phys. Rev. D13, 2364 (1976).