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Particle Physics

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(Notes taken by Professor Fayazuddin, University of Islamabad)

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PREFACE

In the following four lectures, I will give a bird's eye view of particle physics for physicists who do not specialize in this subject. The discussions I shall give are necessarily incomplete as to details and rigor. The purpose is to provide a background necessary to appreciate recent developments in particle physics. If I can convey to you the sense of excitement surrounding this subject if nothing else through these lectures, I will have succeeded in my aim.

I will cover the following four topics in these lectures:

1. Classification of hadrons,
2. Spectroscopy of hadrons,
3. Unification of weak and electromagnetic interactions,
4. Phenomenology of new particles.

I will try a quick tour through the labyrinth of particle physics to the very recent discoveries of charmed particles.

I have enjoyed my stay in Nathiagali. I wish to express my gratitude to Professor Riazuddin, Dr. Munir Ahmad Khan, Chairman of the Pakistan Atomic Energy Commission, and especially Professor Abdus Salam, for their impeccable hospitality, and for a glimpse of the majestic Nanga Parbat.

September, Batavia, Illinois B.W.L.

FIRST LECTURE - CLASSIFICATION OF HADRONS

1.1 Isospin and Strangeness.

We list the well-known hadrons.

i). Baryons: we know eight baryons of spin ½. They are fermions, and carry one unit of baryon number (B).

$S = 0$	:	p, n
$S = -1$	:	$\Sigma^+, \Sigma^0, \Sigma^-$;
		Λ ,
$S = -2$	:	Ξ^0, Ξ^- .

ii). Mesons; again, there are eight known mesons of spin 0. They are bosons.

$S = 0$	:	π^+, π^0, π^-;
		η ,
$S = +1$	;	K^+, K^0,
$S = -1$	;	$\bar{K}^0, K^-$.

We used the symbol S to denote strangeness to be discussed presently. We see that hadrons occur in mass multiplets; for example, p and n are nearly degenerate in mass. They appear in singlets (Λ, η), doublets and triplets. Each multiplet carries isospin I, $I = \frac{1}{2}$ for a doublet, and $I = 1$ for a triplet. For example, the proton and neutron are two states of $I_3 = \frac{1}{2}$ and $-\frac{1}{2}$, respectively, of the nucleon. Isospin symmetry is a symmetry of nuclear interactions.

Strangeness is a quantum number, conserved in strong and electromagnetic interactions, but not in weak interactions. It is related to the electric charge Q by

$$Q = I_3 + \tfrac{1}{2}(B+S) \quad .$$

Since Q, I_3 and B are conserved in strong and elctromagnetic interactions, so is S . The important point is that members of the same isomultiplet have the same strangeness. The combination $y = B+S$ is known as hypercharge.

Strange particles are always produced in pairs (associated production) in strong (and electromagnetic) interactions initiated by nonstrange particles. For example

$$\begin{aligned} \pi^- + p \rightarrow \; & \underset{(S=+1)}{K^0} + \underset{(-1)}{\Lambda} \\ \rightarrow \; & \underset{(S=-2)}{\Xi^-} + \underset{(+1)}{K^0} + \underset{(+1)}{K^+} \quad . \end{aligned}$$

1.2 SU(3) Classification.

The eight baryons and mesons we discussed can be unified in single multiplets of a group larger than the isospin group.

Let $\phi_i (i=1,2,3)$ be a three dimensional complex vector. Consider a unitary, unimodular (special) transformation in ϕ.

$$\phi \rightarrow \phi' = U\phi \, ,$$
$$UU^\dagger = U^\dagger U = 1 \quad ,$$
$$\det U = 1 \quad .$$

In longhand, we can write

$$\phi_i \rightarrow \phi_i' = U_i{}^j \phi_j$$

(cont.)

$$\sum_j (U_i{}^j)(U_k{}^j)^* = \delta_{ik}$$

$$\varepsilon^{ijk} U_i{}^{\ell} U_j{}^m U_k{}^n = \varepsilon^{\ell mn} \quad .$$

Unitary unimodular transformations in three dimensions form a continuous group called SU(3). Since U is unitary, unimodular, it can be written as

$$U = \exp i \sum_{a=1}^{8} \alpha_a \lambda_a$$

where the α are real parameters and the λ are eight linearly independent 3×3 Heimitian traceless matrices:

$$\det U = \exp i \sum \alpha_a \mathrm{Tr}\lambda_a \quad ,$$

$$\mathrm{Tr}\ \lambda_a = 0 \quad .$$

The group SU(3) is an eight-parameter group. The three matrices λ_1, λ_2, and λ_3 :

$$\lambda_i = \left(\begin{array}{c|c} \tau_i & \\ \hline & 1 \end{array}\right) \quad , \qquad i = 1, 2, 3$$

have the same commutation relations as the Pauli matrices. The matrix λ_8, defined by

$$\lambda_8 = \sqrt{\frac{1}{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} ,$$

commutes with λ_1, λ_2 and λ_3 .

Abstractly, the group SU(3) is generated by eight generators, just as the rotation group is generated by three angular momentum operators L_x, L_y and L_z . The eight generators satisfy the same commutation relations as the eight λ's. In fact, the λ's are the three dimensional realizations of the eight generators. We can regard $\lambda_i/2$, $i=1,2,3$, as the realizations of the three isospin operators, and $\frac{1}{\sqrt{3}}\lambda_8$ as that of the hypercharge operator $y = S+B$. It it not difficult to see that only two operators can be diagonalized simultaneously, the ones corresponding to λ_3 and λ_8 for example. These are the third component of the isospin operators and the hypercharge operator .

We have to learn something about SU(3) and its representations. The basic representations are

$$\underset{\sim}{3} \equiv \{\phi_i\} \quad ,$$

$$\underset{\sim}{\bar{3}} \equiv \{\phi_i^{\,*}\} \equiv \{\phi^i\} \quad ;$$

$$\phi_i = U_i^{\,j}\phi_j \quad ,$$

$$\phi^i = U^i{}_j\phi^j \quad , \text{ where } \quad U^i{}_j = (U_i^{\,j})^* \quad .$$

We can build up higher representations from the basic ones. Consider for example the tensor ϕ_{ij} which transforms like $\phi_i\phi_j$. It gives us a nine-dimensional representation, but it is not irreducible. The reduction of a product representation to irreducible ones is based on the observation that symmetretization and antisymmetrization of indices are preserved under linear transformations. Thus

$$\underset{\sim}{3} \otimes \underset{\sim}{3} = \underset{\text{antisymmetric}}{\bar{\underset{\sim}{3}}} \oplus \underset{\text{symmetric}}{\underset{\sim}{6}}$$

It is not difficult to see that the antisymmetric part transforms like the basic complex conjugate representation. I leave it to you to prove this. Now consider $\phi_i{}^j \sim \phi_i \phi^j$. The trace $\phi_i \phi^i$ remains invariant under SU(3), so we have

$$\underset{\sim}{3} \otimes \bar{\underset{\sim}{3}} = \underset{\sim}{1} \oplus \underset{\sim}{8}$$

The octet representation is irreducible.

Consider now $\phi_{ijk} \sim \phi_i \phi_j \phi_k$.

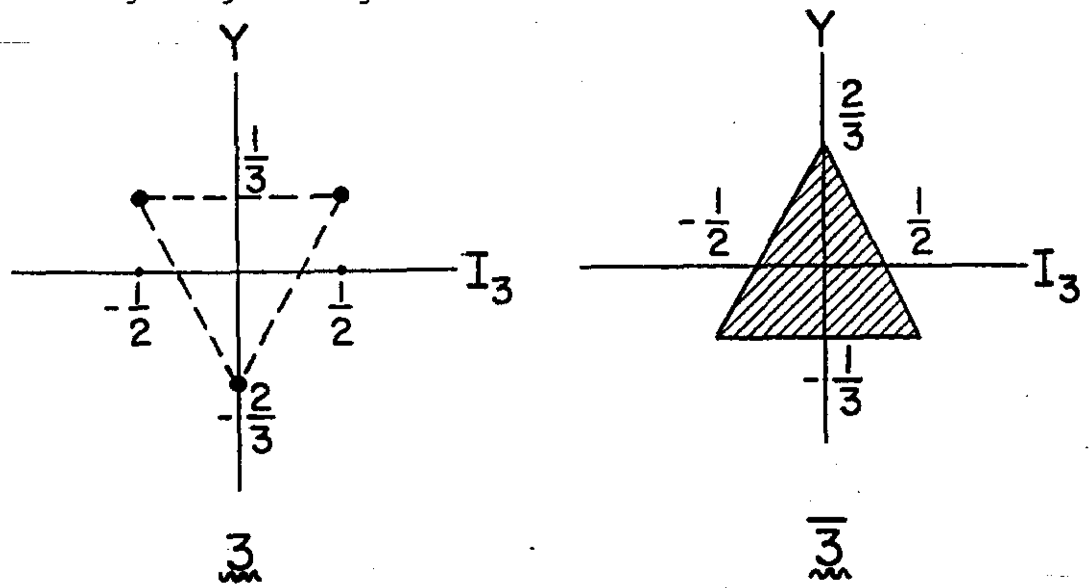
$$\underset{\sim}{3} \otimes \underset{\sim}{3} \otimes \underset{\sim}{3} = \left[\bar{\underset{\sim}{3}} \oplus \underset{\sim}{6}\right] \otimes \underset{\sim}{3}$$

$$= \left[\bar{\underset{\sim}{3}} \otimes \underset{\sim}{3}\right] \oplus \left[\underset{\sim}{6} \otimes \underset{\sim}{3}\right] .$$

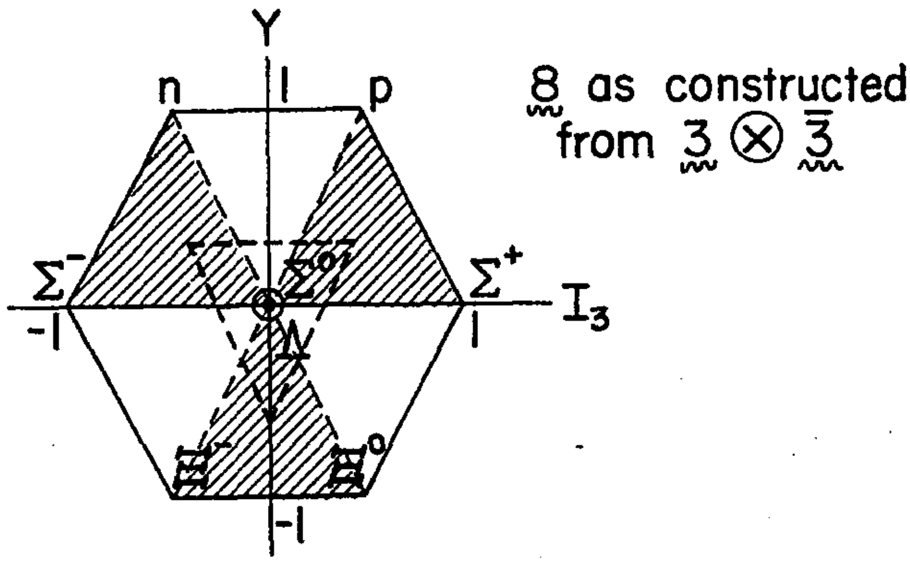
We have worked out $\bar{\underset{\sim}{3}} \otimes \underset{\sim}{3}$. $\underset{\sim}{6} \otimes \underset{\sim}{3}$ decomposes into $\underset{\sim}{10}$ and $\underset{\sim}{8}$, where $\underset{\sim}{10}$ is the part of ϕ_{ijk} completely symmetric in the three indices;

$$\begin{array}{lcl}
\phi_{111}, \phi_{222}, \phi_{333} & ; & 3 \\
\frac{1}{\sqrt{3}}(\phi_{112} + \phi_{121} + \phi_{211}), & & \\
\quad \text{etc.} & ; & 6 \\
\frac{1}{\sqrt{6}}(\phi_{123} + \phi_{132} + \phi_{213} + \phi_{231} & & \\
\quad + \phi_{312} + \phi_{321}) & ; & \underline{1} \\
 & & 10
\end{array}$$

A pictorial way of showing the content of a representation is to draw the so-called weight diagram, in which the hypercharge Y and I_3 of states belonging to the representation can be read off. Thus, $\underset{\sim}{3}$ and $\bar{\underset{\sim}{3}}$ are represented by the following weight diagrams:

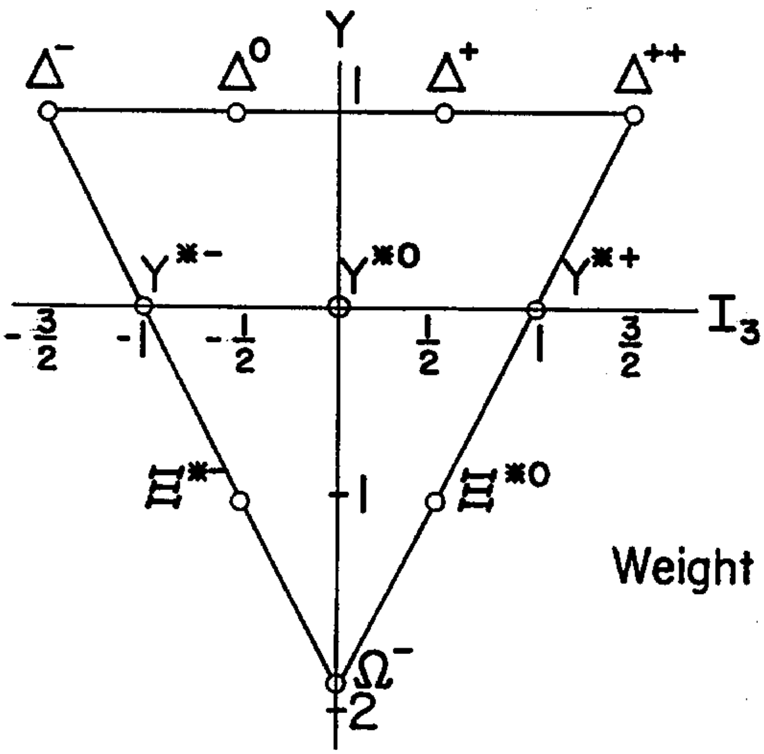


The weight diagram for an octet is shown below:



We have labeled the states by the octet baryons. For the spin 0 octet mesons, we need only to replace p, n, Σ, Λ, Ξ^0 and Ξ^- by K^+, K^0, π, η, $\bar{K}^0$ and K^-. Historically the classification of hadrons in terms of octets and deciments (see below) of SU(3) was proposed by Gell-Mann and Neeman. This scheme is called the Eightfold Way. At that time, it must be recalled, the eighth spin 0 meson, η, was not discovered yet.

The decimet representation, $\underset{\sim}{10}$, is shown in the weight diagram shown below. The excited spin 3/2 baryons are assigned to this multiplet.



Weight diagram for $\underset{\sim}{10}$

The existence of Ω^- was predicted by the Eightfold Way, which was subsequently discovered.

We recall a basic theorem in quantum mechanics: If the Hamiltonian commutes with generators of a group, then eigenstates of the Hamiltonian can be classified into irreducible representations of the gorup; states belonging to the same irreducible representation (multiplet) are degenerate. Thus if SU(3) is a symmetry of strong interactions, then members of an SU(3) multiplet must be degenerate.

As the following table shows, members of a multiplet are not exactly degenerate in mass, but are only approximately so. In other words, SU(3) is an approximate symmetry of strong interactions.

		I	Y	Mass(GeV)
Baryons				
Spin 1/2	N	1/2	1	0.94
	Σ	1	0	1.19
(octet)	Λ	0	0	1.12
	Ξ	1/2	-1	1.32
Spin 3/2	Δ	3/2	1	1.23
	Y^*	1	0	1.38
(decimet)	Ξ^*	1/2	-1	1.53
	Ω	0	-2	1.67
Mesons				
Spin 0	π	1	0	0.14
(octet)	η	0	0	0.55
	K	1/2	± 1	0.50

(Table Continued)

Table Continued:

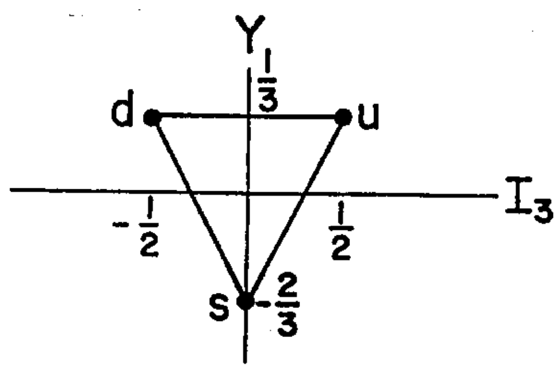
Mesons		I	Y	Mass (GeV)
Spin 1	ρ	1	0	0.77
(octet	ω	0	0	0.78
singlet)	K^*	1/2	±1	0.89
	ϕ	0	0	1.02

1.8 Quark Model - SU(6) and Color

The classification scheme for hadrons just discussed can be explained if there are three fundamental constituents of hadrons-quarks, corresponding to the basic triplet representation of SU(3):

$$\phi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

whose weight diagram is shown below:



u is called the up quark (isospin up, i.e., $I_3 = +\frac{1}{2}$), d the down quark. These carry S=0, and B=1/3. s is called the strange quark and carries S=-1 and B=1/3. The electric charges of these quarks are given by $Q = I_3 + Y/2 = I_3 + (S + B)/2$; these are 2/3, -1/3, -1/3 , respectively.

Now $\underset{\sim}{3} \otimes \underset{\sim}{\bar{3}}$ contains an octet and a singlet. Spin 0 mesons can be constructed out of a quark and an antiquark which transform like 3 and $\underset{\sim}{\bar{3}}$, respectively. Thus

$$\begin{aligned} \pi^+ &\sim u\bar{d} \; ; && I = 1,\ I_3 = 1 \; , \\ K^+ &\sim u\bar{s} \; ; && I = \tfrac{1}{2},\ I_3 = \tfrac{1}{2} \; , \\ \pi^0 &\sim \frac{1}{\sqrt{2}} (u\bar{u} - d\bar{d}) ; && I = 1,\ I_3 = 0 \; , \\ \eta &\sim \frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} - 2s\bar{s}) \; ; && I = 0,\ I_3 = 0 \; , \end{aligned}$$

etc.

Baryons are compounds of three quarks. An octet corresponds to three quarks in a mixed symmetry. For example,

$$\begin{aligned} p &\sim u(1)\left[u(2)d(3) - d(2)u(3)\right] \\ &\quad + u(2)\left[u(1)d(3) - d(1)u(3)\right] \; , \\ \Sigma^+ &\sim u(1)\left[u(2)s(3) - s(2)u(3)\right] \\ &\quad + u(2)\left[u(1)s(3) - s(1)u(3)\right] \; . \end{aligned}$$

We can build a decimet by completely symmetrizing the three quark states. Thus

$$\begin{aligned} \Delta^{++} &\sim u(1)u(2)u(3) \; , \\ \Omega^- &\sim s(1)s(2)s(3), \quad \text{etc.} \end{aligned}$$

Further consequences will follow if one assumes that forces among quarks are approximately spin- and SU(3)-independent, that is to say, the dynamics of quarks in a hadron is approximately invariant under SU(6), acting on a six-component object:

$$\phi_a = \begin{pmatrix} u\uparrow \\ u\downarrow \\ d\uparrow \\ d\downarrow \\ s\uparrow \\ s\downarrow \end{pmatrix} \qquad a = 1,2,\cdots,6 \quad .$$

where $\uparrow$ and $\downarrow$ denote spin up and down.

Let us assume that baryons are states of three quarks with $L = 0$, completely symmetric in SU(6) indices. The number of states of this symmetry type is given by $(6+2)\cdot(6+1)\cdot(6)/3! = 56$. These 56 states consist of an SU(3) decimet of spin 3/2, and an SU(3) octet of spin 1/2;

$$10 \times (2\times \frac{3}{2} + 1) + 8 \times (2 \times \frac{1}{2} + 1) = 56 \ ,$$

just as the observed baryon spectrum.

Mesons can be constructed from a quark and an antiquark. The low lying mesons of spin 0 and 1 are grouped together in an $\underset{\sim}{35}$ representation of SU(6). The SU(3) and spin contents of this $\underset{\sim}{35}$ are

$$\begin{aligned} S = 0 \quad &: \quad \underset{\sim}{8} \ , \\ S = 1 \quad &: \quad \underset{\sim}{1} + \underset{\sim}{8} \ , \end{aligned}$$

again just like the observed meson spectrum.

There are many remarkable successes of SU(6). Let me just mention one more. In this picture, the magnetic moment of the baryon is completely determined up to an overall normalization. This model gives the ratio of the proton and neutron magnetic moments to be -3/2, to be compared with the experimental values 2.79/(-1.91).

Despite these successes, the quark model formulated as above is logically inconsistent. We have assumed quarks to have spin 1/2, but we have also assumed that baryons are made of three quarks in a completely symmetric configuration with respect to position,spin and flavor (that is, the SU(3) attributes u, d and s), in violation of the Pauli exclusion principle. To overcome this difficulty, we endow quarks with another attribute which is usally called color. Thus, each quark - u, d or s - comes in three colors. <u>We require that all hadrons to be color singlets</u>. For baryons there is only one way to make a three quark system a color singlet. Let i be the color index. Then the color singlet state of three quarks is

$$\varepsilon^{ijk} q_i(1) q_j(2) q_k(3)$$

which is completely antisymmetric in color indices. Such a system, then, must be totally symmetric with respect to other attributes, as the quark model assumes.

What are the electric charges of quarks in the color scheme? There are two options which have been considered. One possibility is that the quark charge is independent of color; in this case quarks are fractionally charged, and the color symmetry can be exact. Since fractionally charged objects have never been seen, despite intensive searches, we must assume that the dynamics of quarks is such that color nonsinglet objects do not exist in isolation, or else it takes a very large amount of energy to isolate them.

The second option which has been discussed is the so-called Han-Nambu scheme in which quarks come in as

$$\begin{pmatrix} u_B^{\ +} & u_Y^{\ +} & u_R^{\ 0} \\ d_B^{\ 0} & d_Y^{\ 0} & d_R^{\ -} \\ s_B^{\ 0} & s_Y^{\ 0} & s_R^{\ -} \end{pmatrix}$$

where the subscripts B, Y, R stand for three colors - blue, yellow and red, and the superscripts denote electric charges. Note that the average charges of the u-,d-, and s-quarks are 2/3, -1/3, -1/3. In this scheme quarks are integrally charged, and the color symmetry is approximate. Quarks can be isolated, or might have already been isolated at big accelerators, but they cannot be identified by fractional charges. The second view has been championed by Pati and Salam in recent years. It is consistent with all known facts.

I will nevertheless adhere to the first view and develop a theory of hadrons and their interactions based on it in the next lectures. We will assume that quarks are fractionally charged and the color symmetry is exact. We will then argue why quarks cannot be isolated, and why physical hadrons are necessarily color-singlets.

SUGGESTED READINGS

Particle physics in general:

S. Gasiorowicz, Elementary Particle Physics, (John Wiley & Sons, Inc., New York, 1966).

D.H. Perkins, Introduction to High Energy Physics, (Addison-Wesley Pub. Co., 1972).

SU(3):

M. Gell-Mann and Y. Neeman, The Eightfold Way, (W.A. Benjamin Inc., New York, 1965).

P. Carruthers, Introduction to Unitary Symmetry, (Interscience Publishers, New York, 1966).

SU(6):

B.W. Lee, SU(6) in Particle Physics in 1965 Brandeis University Summer Institute in Theoretical Physics, (Gordon & Breach, New York, 1963), Vol. II.

J.J.J. Kokkedee, The Quark Model, (W.A. Benjamin Inc., New York, 1969).

SECOND LECTURE - SPECTROSCOPY OF HADRONS

2.1 Gauge Principle.

One of the fundamental differences between classical and quantum mechanics is that in quantum mechanics one deals with complex numbers. A quantum mechanical system is described by a complex wave function, $\psi(x)$. We may demand that physical laws are invariant under space-time dependent phase transformations (gauge transformations of the second kind) on an electrically charged system,

$$\psi(x) \to \psi'(x) = e^{ie\Lambda(x)}\psi(x)$$

where $\Lambda(x)$ is an arbitrary real function of space-time.

Consider the free-particle Schrödinger equation:

$$-\frac{1}{2m}\nabla^2\psi = \frac{1}{i}\frac{\partial}{\partial t}\psi \quad .$$

This is not invariant under gauge transformations. To implement gauge invariance, it is necessary to postulate the existence of a four-component field $(\underset{\sim}{A},\phi)$ and make substitutions

$$\begin{aligned} \underset{\sim}{\nabla} &\to \underset{\sim}{\nabla} - ie\underset{\sim}{A} \\ \frac{\partial}{\partial t} &\to \frac{\partial}{\partial t} - ie\phi \quad . \end{aligned}$$

If we assume that, simultaneously with the phase transformation of ψ, $\underset{\sim}{A}$ and ϕ undergo the transformations

$$\begin{aligned} \underset{\sim}{A} &\to \underset{\sim}{A} + \underset{\sim}{\nabla}\Lambda \\ \phi &\to \phi + \frac{\partial}{\partial t}\Lambda \quad . \end{aligned}$$

then the Schrödinger equation

$$-\frac{1}{2m}(\underset{\sim}{\nabla} - ie\underset{\sim}{A})^2\psi = -\frac{1}{i}(\frac{\partial}{\partial t} - ie\phi)\psi$$

is left invariant. Of course $(\underset{\sim}{A},\phi)$ is the electromagnetic potential. The fields $\underset{\sim}{E}$ and $\underset{\sim}{H}$

$$\underset{\sim}{E} = \frac{\partial}{\partial t}\underset{\sim}{A} - \underset{\sim}{\nabla}\phi;\ H = \underset{\sim}{\nabla} \times \underset{\sim}{A}$$

are invariant under the above transformations.

This is the gauge principle. It gives correctly the form of interaction of a non-relativistic charged particle with electromagnetic field. The energy density is given by

$$H = \psi^* \frac{1}{2m} p^2\psi - e(\rho\phi - \underset{\sim}{j}\cdot\underset{\sim}{A}) + \tfrac{1}{2}(\underset{\sim}{E}^2 + \underset{\sim}{H}^2)$$

which is gauge invariant, where the electric charge density and current $\underset{\sim}{j}$ are given by

$$\rho = \psi^*\psi \ ;\ \underset{\sim}{j} = \frac{1}{2im}(\psi^*\underset{\sim}{\nabla}\psi - (\underset{\sim}{\nabla}\psi^*)\psi) - e\underset{\sim}{A}\psi^*\psi\ .$$

They satisfy the continuity equation

$$\partial\rho/\partial t + \underset{\sim}{\nabla}\cdot\underset{\sim}{j} = 0$$

which implies the conservation of the electric charge $Q = \int d^3x\rho(x)$.

We want to extend this idea to strong interactions of quarks. Each quark, say q=u comes in three colors:

$$q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

where the subscript denotes different colors. Our premise that strong interactions are color-symmetric implies that they must be invariant under unitary unimodular transformations acting on q :

$$q \to q' = Uq \ .$$

We now assert the gauge principle of color symmetry; dynamics of quarks is invariant under U which depends on space-time:

$$UU^\dagger = U^\dagger U = 1 \ ,$$

$$\det U = 1 \ ,$$

$$U = U(x) = \exp ig \sum_{a=1}^{8} \alpha_a(x)\lambda_a/2 \quad ,$$

where $\alpha_a(x)$ are space-time dependent parameters.

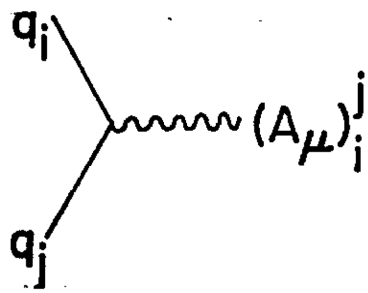
This gauge principle can be implemented with an octet of vector fields $A_\mu{}^a(x)$, $\mu = 0,1,2,3$ and $a=1,\cdots,8$, as many fields as there are parameters of the group SU(3). The substitution rule is

$$\partial_\mu q(x) \to \left[\partial_\mu - iq(\frac{\lambda_a}{2})A_\mu{}^a(x)\right]q(x) \ .$$

very much like the case of electromagnetism. There is a complication here, due to the fact that SU(3) transformations are not commutative (i.e., are non-Abelian): $U_1U_2 \neq U_2U_1$. The vector fields $A_\mu{}^a$ are called gluons. Sometimes, it is more convenient to think of gluons as carrying two kinds of color indices:

$$(A_\mu)_i{}^j = A_\mu{}^a \left[\frac{\lambda}{2}\right]_i{}^j \ , \ (A_\mu)_i{}^j = 0 \ .$$

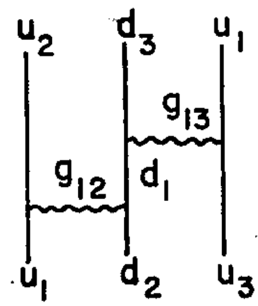
The gluon $(A_\mu)_i{}^j$ is coupled to the color-changing current; another way of saying this is that a quark j can turn into a quark i by emission or absorption of the gluon $(A_\mu)_i{}^j$ as shown in the Feynman diagram below:



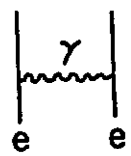
As the indices on $(A_\mu)_i{}^j$ imply, gluons carry colors. Gluons are coupled to color currents. These currents are conserved, just as the electromagnetic current is. The fact that gluons themselves carry colors implies that they may be emitted or absorbed by another gluon. That is to say that non-Abelian gauge theories necessarily imply self-interactions of gluons. This circumstance is to be contrasted with electromagnetism: the photon is coupled to the charge current, but the photon itself carries no electric charge. Thus, photons do not interact with one another (except in higher orders in e).

2.2 Color Confinement.

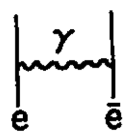
In the gauge theory of color symmetry, quarks are bound by forces generated by exchange of gluons. Thus the proton may be viewed as a bound state of three quarks, in which the basic binding forces are generated by diagrams such as shown below.



This is a typical example of exchange of gluons. Here analogy with electromagnetic case is in order.

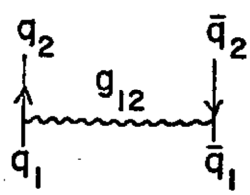


$\frac{ee}{r}$: repulsive force for like charges



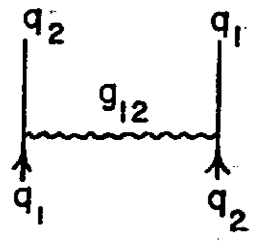
$-\frac{ee}{4}$: attractive force for opposite charges.

Consider the case of color gluon exchanges. The force between a quark and an antiquark is attractive.



attractive

The force between quark and antiquark is attractive

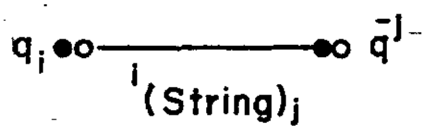


For two quarks here the force is attractive if q_1, q_2 are antisymmetric in color and is repulsive if q_1 and q_2 are in a symmetric state in color. If three quarks are in a totally antisymmetric state in color, the forces acting between any two quarks are all attractive and this is the reason how three quarks are bound.

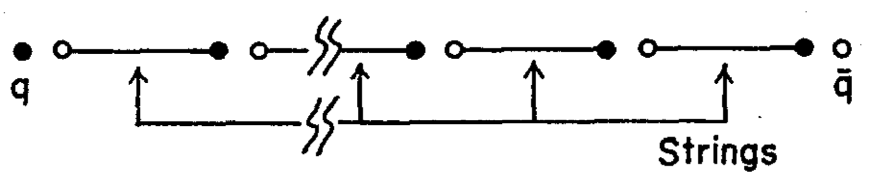
However, very little is known about the nature of the exact solution to color gauge theory (this is often called quantum chromodynamics - QCD for short). In perturbation theory, infrared divergences in QCD are so severe, that it is not expected that perturbation theory is a reliable guide to long-distance behavior of the theory. In fact, it is very plausible that the effective coupling strength of interactions increases without bound at large distances. On the other hand, it _is_ known that, at short distances, the effective strength decreases, so that perturbation theory is a reasonable guide to understanding interquark forces at short distances. The statements made here are a crude summary of the results of the so-called renormalization group analysis applied to QCD.

Progress has been made in our understanding of the long-distance behavior of QCD in a lattice version of the theory. The four dimensional space-time continuum is replaced by a lattice space, with lattice separation a . Quarks are allowed to exist only on lattice sites. Gluon fields are replaced by strings, which connect adjacent sites.

A string carries color indices at its ends. Gauge invariance implies that each site must be color-singlet. Thus, an allowed configuration of a quark and an antiquark on adjacent sites is the one in which the quark and antiquark are linked by a string so that



the color index of the quark (antiquark) and the color index of the string at that end are contracted to form a color singlet. When a quark and an antiquark are far apart, many strings have to be excited to connect the two sites.



Calculation based on this theory shows that the energy stored in this configuration is

$$E = T_0\left(\frac{L}{a}\right)$$

where L is the quark-antiquark separation, for $L \gg a$. To isolate a quark, for example, the antiquark in the above illustration has to be moved away to infinity; it clearly takes an infinite amount of energy to do this. This is the basis of color confinement. The confining potential is of the form

$$V(r) \sim ar$$

for $r > 1/M$, where M is a typical hadronic mass scale, say 300 MeV. The confining potential is spin- and flavor-independent.

Eventually, one has to probe the nature of lattice QCD as $a \to 0$. This has not been done. One would like to show that results obtained for $L/a \gg 1$ are insensitve to the limiting process $a \to 0$; that as $a \to 0$, then $r \to 0$, we recover the classical limit of continuum QCD; and that in the limit $a \to 0$, the theory recovers Lorentz invariance. We shall assume that these propositions will be shown correct.

As mentioned before, the forces between two quarks, and a quark and an antiquark become weaker at short distances. This has to do with the so-called asymptotic freedom in non-Abelian gauge theory: for processes involving large virtual momenta (or short distances), the effective coupling strength becomes weak. Under these circumstances, the short distance behavior of the interquark potential is predominantly Coulombic, reflecting exchange of a massless gluon. There are additional spin-dependent interactions arising from nonrelativistic reduction of relativistic two-body interaction. The resultant nonrelativistic Hamiltonian is

$$
\begin{aligned}
H = & \sum_i (m_i + p_i^2/2m_i) + V_c(r_1, r_2, \cdots) \\
& + \sum_{i>j} k_{ij}\alpha_s \Bigg\{ \frac{1}{r} - \frac{1}{m_i m_j}\left(\frac{\vec{p}_i\cdot\vec{p}_j}{r} + \frac{\vec{r}(\vec{r}\cdot\vec{p}_i)\cdot\vec{p}_j}{r^3}\right) \\
& - \frac{\pi}{2}\,\delta^3(\vec{r})\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{16}{3m_i m_j}\,\vec{s}_i\cdot\vec{s}_j\right) \\
& - \frac{1}{2r^3}\left(\frac{1}{m_i^2}\,(\vec{r}\times\vec{p}_i)\cdot\vec{s}_j - \frac{1}{m_j^2}\,(\vec{r}\times\vec{p}_j)\cdot\vec{s}_i\right) \\
& + \frac{2}{m_i m_j}\left[(\vec{r}\times\vec{p}_i)\cdot\vec{s}_j - (\vec{r}\times\vec{p}_j)\cdot\vec{s}_i + 3\left(\frac{(\vec{s}_i\cdot\vec{r})\cdot(\vec{s}_j\cdot\vec{r})}{r^2} - \right.\right. \\
& \qquad\qquad \left.\left.\frac{1}{3}\,\vec{s}_i\cdot\vec{s}_j\right)\right]\Bigg\}
\end{aligned}
$$

where summation is over constituent quarks in confinement, $V_c(r_1, \cdots)$ is the central potential responsible for confinement, $\alpha_s = g^2/4\pi$ is the strong-interaction analog of the fine structure constant $\alpha = e^2/4\pi$, m_i is the <u>effective</u> mass of the i-th quark, and $\vec{r} = \vec{r}_i - \vec{r}_j$. The non-Abelian nature of the exchanged quanta leaves only a pale reminder:

$$
\begin{aligned}
k_{ij} &= -4/3 \text{ for } q\bar{q} \\
&= -2/3 \text{ for } [qq]_{\text{antisymmetric}}.
\end{aligned}
$$

In the center of mass system of two particles, we have $\vec{p}_i + \vec{p}_j = 0$. The derivation of the above result may be gleaned from J. Schwinger, <u>Particles, Sources and Fields</u>, Vol. II, (Addison-Wesley Publishing Co., 1973) p. 349. The use of this Hamiltonian to hadron spectroscopy, which we will take up next, was pioneered in A. DeRujula, H. Georgi and S.L. Glashow, Phys. Rev. D<u>12</u>, 147 (1976).

2.3 Spectroscopy of Hadrons.

We shall use the above Hamiltonian for s-wave baryons and mesons. We split up the Hamiltonian into two pieces:

$$H = H_0 + H_I \quad ,$$

$$H_0 = \sum_i (m_i + p_i^{\,2}/2m_i) + V_c(r_1, \cdots) + d_s \sum_{i>j} k_{ij} \frac{1}{r} \, ,$$

$$H_I = -\sum_{i>j} \alpha_s k_{ij} \left\{ \frac{1}{m_i m_j} \left(\frac{\vec{p}_i \, \vec{p}_j}{r} + \frac{\vec{r}(\vec{r} \, \vec{p}_i) \, \vec{p}_j}{r^2} \right) + \frac{\pi}{2} \delta^3(\vec{r}) \left(\frac{1}{m_i^{\,2}} + \frac{1}{m_j^{\,2}} + \frac{16}{3 m_i m_j} \vec{s}_i \, \vec{s}_j \right) \right\}$$

where we have dropped the part of the Hamiltonian which, when acted upon an s-wave, vanishes. We imagine solving the "unperturbed" Hamiltonian H_0, and doing perturbation theory in H_I. To the extent that we ignore the difference between $m_u = m_d$ (isospin symmetry) and m_s, the 36 states of the s-wave mesons (and the 56 states of the s-wave baryons) are degenerate in the first step.

The splitting of the S=0 and S=1 mesons are entirely due to the spin-spin interaction term. Since we have

$$\vec{s}_q \, \vec{s}_{\bar{q}} = -3/4, \text{ for } S = s_q + s_{\bar{q}} = 0,$$
$$= 1/4, \text{ for } S = 1 \, ,$$

we deduce

$$\rho - \pi = \frac{4}{3} \alpha_s \frac{\pi}{2} |\Psi(0)|^2 \frac{16}{3} \frac{1}{m_u^{\,2}}$$
$$K^* - K = \frac{4}{3} \alpha_s \frac{\pi}{2} |\Psi(0)|^2 \frac{16}{3} \frac{1}{m_u m_s}$$

where we have used particle labels for the corresponding particle masses, and $\Psi(r)$ is the unperturbed wave function of the s-wave mesons. We obtain

$$\frac{m_u}{m_s} \quad \frac{K^*-K}{\rho-\pi} = 0.63 \quad .$$

We can deduce the ratio m_u/m_s also from baryon masses. For Σ^+, we have

$$\begin{aligned} (\vec{s}_u + \vec{s}_{u'})^2 &= 2 \quad , \\ (\vec{s}_s + \vec{s}_u + \vec{s}_{u'})^2 &= \frac{3}{4} \quad , \text{ for } \Sigma^+ \end{aligned}$$

because the two up-quarks, u and u', are in a symmetric state. Similarly, we have

$$\begin{aligned} (\vec{s}_u + \vec{s}_d)^2 &= 0 \quad , \\ (\vec{s}_u + \vec{s}_d + \vec{s}_s)^2 &= \frac{3}{4} \quad , \text{ for } \Lambda: \\ (\vec{s}_u + \vec{s}_{u'})^2 &= 2 \quad , \\ (\vec{s}_u + \vec{s}_{u'} + \vec{s}_s)^2 &= \frac{15}{4} \quad , \text{ for } Y_1^{**} \quad . \end{aligned}$$

We have therefore

$$Y_1^* - \Sigma \propto \frac{3}{2} \frac{1}{m_s m_u} \quad ,$$

$$2Y_1^* + \Sigma - 3\Lambda \propto 3 \frac{1}{m_u^2} \quad ,$$

or

$$\frac{2(y_1^*-\Sigma)}{2y_1^*+\Sigma-\Lambda} = \frac{m_u}{m_s} = 0.62 \quad ,$$

which is close enough to 0.63, deduced from the meson masses. We do understand the splittings of the J=0 and J-1 mesons, and of the J=1/2 and J=3/2 baryons in this picture.

SU(3) symmetry breaking effects are largely due to the quark mass dependence of the unperturbed Hamiltonian. If we expand it in lowest order of the mass difference $m_u - m_s$, we obtain

$$K^* - \rho = K - \pi = (\phi - \omega)/2$$

$$\rho = \omega$$

and

$$2N + 2\Xi = 3\Lambda + \Sigma \quad ,$$

$$\Delta - Y_1^* = Y_1^* - \Xi^* = \Xi^* - \Omega$$

which are reasonably well-satisfied.

One can explain all mass differences within L=0 hadrons in this way, except for the J=0 mesons η and η'(958MeV). The reason for this is that our Hamiltonian does not take into account quark-antiquark annihilation: the J=0, isosinglet mesons can virtually make transition to a two gluon state, and therefore mix. On the other hand, the J=1, isosinglet mesons (ω and ϕ) are coupled to a three gluon state. This means that the ω-ϕ mixing is much less important than the η-η' mixing, first because the three-gluon annihilation occurs with two more powers of α_s, and second because the average mass of ω, ϕ is higher than that of η, η', so that the effective value of α_s is smaller for the J=1 case. In any case, ω and ϕ have the quark constitutions of $(u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ to a good approximation, respectively.

What is the mass scale of m_u and m_s? As we have mentioned in the previous lecture, the magnetic moments of the nucleons are $\mu(p) = 2.79$, $\mu(n) = -1.92$ in nuclear magnetons. In the model under consideration, in which the proton is an s-wave bound state of three quarks, one has

$$\mu(p) = \frac{1}{2m_u} \quad ,$$

which should be equated to $2.79\ (2m_p)^{-1}$. From this it follows that

$$m_u \simeq .34 \text{ GeV} ,$$

$$m_s \simeq .54 \text{ GeV} .$$

The utility of De Rujula-Georgi-Glashow Hamiltonian lies not only in explaining the spectroscopy of hadrons, including p-wave hadrons which we have not discussed, but more importantly, in its predictive power with regards to charmed hadrons which we will discuss in the last lecture.

SUGGESTED READINGS

Gauge Theories:

E.S. Abers and B.W. Lee, Gauge Theories, Phys. Repts. 9C, 1, (1973).

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K. Wilson, Quarks and Strings on a Lattice, in Gauge Theories and Modern Field Theory, (The MIT Press, Cambridge, Mass., 1976).

H.D. Politzer, Asymptotic Freedom: An Approach to Strong Interactions, Phys. Repts., 4C, 129 (1974).

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A. De Rujula, H. Georgi and S.L. Glashow, Phys. Rev. D12, 147, 1976).

J.J.J. Kokkedee, loc. cit.

THIRD LECTURE - UNIFICATION OF ELECTROMAGNETIC AND WEAK INTERACTIONS

3.1 Weak Interactions.

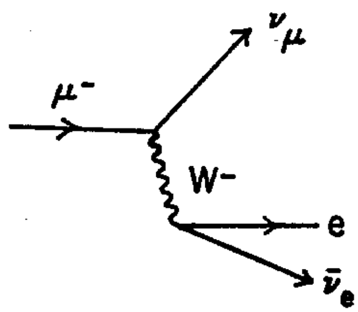
We first consider two well-known examples of weak decays:

(1) μ decay: $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$

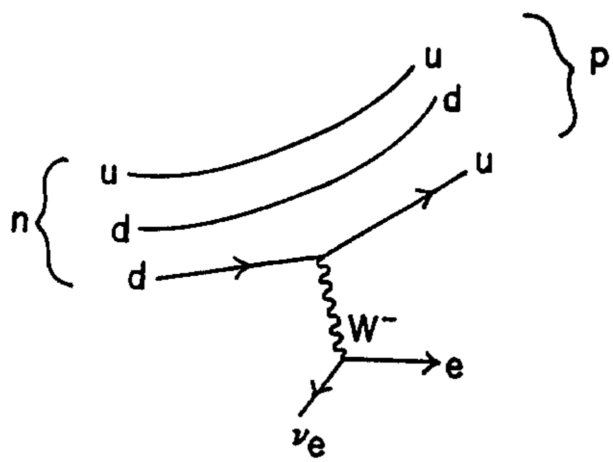
(2) β decays: $n \to p + e^- + \bar{\nu}_e$

$\Lambda \to p + e^- + \bar{\nu}_e$

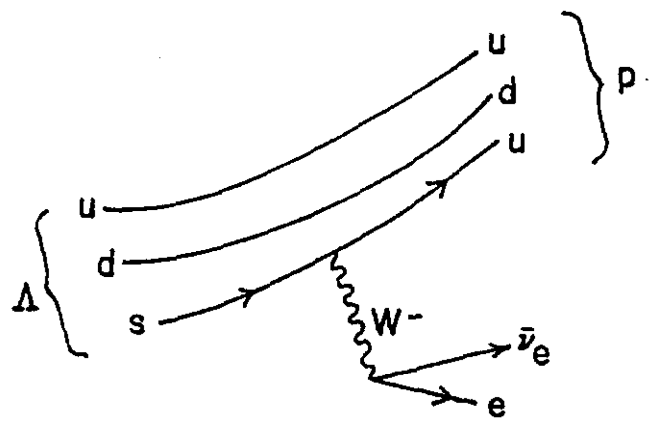
We imagine that weak interaction behaves like electromagnetic interaction and is mediated by vector bosons:



We extend this picture to β-decay (2). We describe the weak interaction at fundamental level in terms of quarks as shown below



The Neutron β-Decay



The Λ β - Decay

If we normalize the amplitude of μ-decay to 1, then the amplitude of β-decay ($n \to p + e^- + \nu_e$) is proportional to cos θ_c and that of $\Lambda \to p + e^- + \nu_e$ is proportional to sin θ_c:

$$\cos^2 \theta_c + \sin^2 \theta_c = 1 .$$

This is the Cabibbo universality and θ_c is called the Cabibbo angle. Experimentally

$$\sin \theta_c \simeq 0.2 .$$

There are other known strangeness changing β-decays, such as $K^\pm \to \pi^0 + e^\pm + \binom{\nu}{\bar{\nu}}$, $\Sigma^- \to n + e^- + \bar{\nu}$, etc. In all these strangeness charges by one unit, and we have the rule $\Delta S = \Delta Q$. This feature can be explained if we say that the fundamental processes at the quark level responsible for these decays are

$$s \to u + e^- + \bar{\nu} \; ; \; \bar{s} \to \bar{u} + e^+ + \nu .$$

3.2 Gauge Theory of Weak and Electromagnetic Interactions.

A unified understanding of weak interactions is possible if we assume

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \quad \begin{pmatrix} u \\ d_c \end{pmatrix}_L$$

where L denotes left chiral components, and

$$d_c = d \cos \theta_c + s \sin \theta_c ,$$

form doublets; these are doublets in weak isospin space which should be distinguished from the isospin space of strong interactions. The weak currents are associated with the weak isospin raising and lowering operators $t^{\pm}$, and the electric charge is given by $Q = t_3 + y/2$, where t_3 and y are weak isospin and hypercharge (This defines weak hypercharge). With respect to weak interactions d and s are not eigentstates but d_c is a member of a doublet, and

$$s_c = -d \sin \theta_c + s \cos \theta_c$$

is a singlet.

Consider unitary transformations

$$U(\alpha) = e^{i\left[\alpha_0 + \alpha \cdot \tau/2\right]}$$

acting on doublets in weak isospin space. Gauge theory based on the U(2) group of unitary transformations in 2 × 2 space (weak isospin) was first proposed by Weinberg and Salem in the context of a spontaneously broken gauge theory.

Now under a gauge transformation U(2), the weak doublet transforms as

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \rightarrow U(x) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad .$$

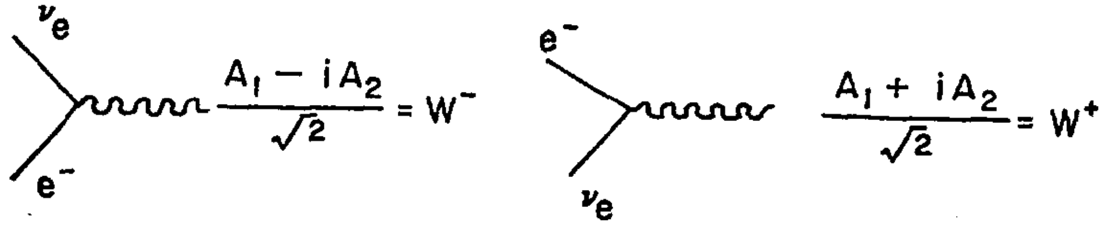
We require the basic Lagrangian describing weak and electromagnetic interactions to be invariant under generalized phase transformations generated by four parameters $\alpha_0, \alpha_1, \alpha_2, \alpha_3$. For SU(2), $\alpha_0 = 0$. $\alpha_1, \alpha_2, \alpha_3$ generate SU(2). Thus

$$U(2) = SU(2) \otimes U(1) \quad .$$

The one parameter U(1) corresponds to hypercharge gauge transformations. There have to be four gauge vector bosons corresponding to $\alpha_0, \alpha_1, \alpha_2, \alpha_3$. The form of coupling of these gauge bosons to a doublet is

$$\bar{\psi}_L \left[i\gamma_\mu (\partial^\mu - ig\, \vec{\tau}/2\cdot\vec{A}^\mu - ig'\tfrac{1}{2}B^\mu)\right] \psi_L \quad .$$

The coupling for charged vector bosons is graphically shown below



Note that there are two coupling constants g and g'

corresponding to SU(2) and U(1) and that there are two neutral vector bosons A_3 and B, in addition to $W^{\pm}$.

In a gauge theory, gauge fields must be massless, because the mass term for a gauge field $m^2 A_\mu{}^2$ in electromagnetism, for example, is not invariant under the gauge transformation $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$. However, the W bosons, if they exist, must be massive. Also, we see only one massless vector boson - the photon - and not two neutral massless vector bosons.

This impasse is overcome by invoking the Higgs-Kibble mechanism, that is, spontaneous breakdown of gauge symmetry. To convey the idea involved, we consider a very simple abelian gauge theory, given by

$$H = |(\partial_t - ieA_0)\phi|^2 + |(\underset{\sim}{\nabla} - ie\underset{\sim}{A})\phi|^2$$
$$+ \alpha|\phi|^2 + \frac{\beta}{2}|\phi|^4 + \frac{1}{2}(\underset{\sim}{E}^2 + \underset{\sim}{H}^2)$$

where ϕ is a complex field. Since we want the energy to be bounded from below, we require β to be positive. If $\alpha<0$, the minimum of H occurs at

$$|\phi| = \sqrt{|\alpha|/\beta}$$

This is the classical approximation to the vacuum expectation value of ϕ:

$$<0|\phi|0> = <0|\phi^*|0> = \sqrt{|\alpha|/\beta} \quad .$$

The Hamiltonian is invariant under gauge transformations of the form $\phi \rightarrow e^{ie\Lambda}\phi$, $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$. But a nonvanishing expectation value of ϕ means that the gauge symmetry is broken, i.e., the vacuum (ground state) is not invariant under gauge transformations. (Let $U(\Lambda)$ be the generator of gauge transformation in the q-number theory: $U^{-1}(\Lambda)\phi U(\Lambda) = e^{ie\Lambda}\phi$. <u>If</u> the vacuum state is gauge invariant, $U(\Lambda)|0\rangle = |0\rangle$, then $\langle 0|U^{-1}\phi U|0\rangle = \langle 0|\phi|0\rangle = e^{ie\Lambda}\langle 0|\phi|0\rangle$ for any Λ - a contradiction if $\langle 0|\phi|0\rangle$ is nonzero). Under this circumstance, the gauge boson acquires a mass. It can be easily seen by substituting $\phi = \phi^* = \sqrt{|\alpha|/\beta}$ in the Hamiltonian; there results

$$H = \tfrac{1}{2}(\underset{\sim}{E}^2+\underset{\sim}{H}^2) + \tfrac{1}{2}\mu^2 A_\mu{}^2 + \text{constant} \quad ,$$

where $\mu^2 = 2e^2|\alpha|/\beta$.

The above example is for an abelian group. The U(2) case is more complicated. Suffice it to say that in the SU(2) case, it is possible to arrange scalar fields in such a way that only the subgroup U(1) of the form $e^{i\alpha_e(t_3+y/2)}$ is preserved as a gauge symmetry. That is, only the gauge transformation associated with electric charge is an invariance of the vacuum. The charged fields $W^\pm = (A_1 \mp iA_2)/\sqrt{2}$ become massive, and couple to charged currents:

$$W_\mu{}^+\left[(\nu_e\gamma_\mu e)_L + \cdots\right] + \text{h.c.}$$

A linear combination of A_3 and B, the combination associated with the surviving U(1) gauge symmetry of electromagnetism, becomes A_μ, the massless photon field; the orthogonal combination of A_3 and B becomes a massive neutral vector boson $Z_\mu{}^0$, which couples to a neutral current.

The interactions of vector fields with currents is given by the expression

$$\frac{gg'}{\sqrt{g^2+g'^2}} A_\mu j^\mu_{em} + g\left[j_\mu{}^+ W^{+\mu} + j_\mu{}^- W^{-\mu}\right] + \sqrt{g^2+g'^2}\, Z_\mu{}^0 \left[j^{3\mu} - \sin^2\theta_W j^\mu_{em}\right]$$

where j^μ_{em} is the electric current, and

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} .$$

The charged currents $j^\pm_\mu = (j^1_\mu \pm i j^2_\mu)$ and j^3_μ are the three currents associated with the weak isospin group:

$$j^i_\mu = (\bar{\nu}_e, \bar{e})_L \gamma_\mu \frac{\tau_i}{2} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L + \cdots$$

so that

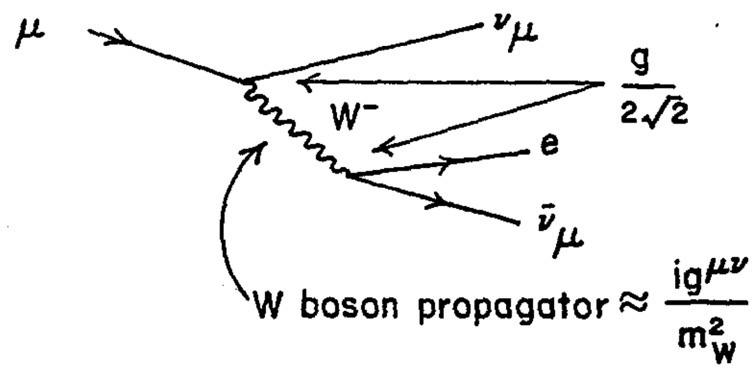
$$j_\mu{}^+ = \frac{1}{2\sqrt{2}} \bar{\nu}_e \gamma_\mu (1-\gamma_5) e + \bar{\nu}_\mu \gamma_\mu (1-\gamma_5) \mu + \cdots ,$$

$$j_\mu{}^- = (j_\mu{}^+)^\dagger , \text{ etc.}$$

Since the coupling constant e is defined as the coefficient of the term $A_\mu j^\mu{}_{em}$, we have

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} = g \sin\theta_W .$$

Consider now the matrix element for μ-decay:



Neglecting spinors, we obtain

$$T(\mu\to e\bar{\nu}_e\nu_\mu) = \left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{m_W^{\;2}} \ .$$

This is the universal Fermi coupling constant:

$$G_F/\sqrt{2} = g^2/(8m_W^{\;2}) \ ,$$

$$G_F = 10^{-5}(m_p)^{-2} \quad .$$

Since $g = e/\sin\theta_W$, we have

$$m_W = \frac{e}{2}\left(\frac{1}{\sqrt{2}G_F}\right)^{\frac{1}{2}} \frac{1}{\sin\theta_W} \geq 38 \text{ GeV} \quad .$$

This model predicts a rather large W boson mass, much larger than today's accelerators can produce. With the simplest Higgs-Kibble mechanism, one obtains a further condition that $m_Z = m_W/\cos\theta_W$.

Before the advent of the Weinberg-Salam gauge theory, there was no compelling reason to introduce Z^0 coupled to a neutral current. In the old-fashioned theory of weak interactions, the so-called neutral current effects:

$$\nu + N \rightarrow \nu + N + \cdots$$

can proceed only in second order in G_F. The Weinberg-Salam model predicts such processes to occur with the strength of first order weak interactions. Indeed, the neutral currents effects have been observed at various laboratories since 1973. The inclusive neutral current effects $\nu N \rightarrow \nu$ + anything were first observed at CERN and Fermilab. Recently, the elastic scattering of neutrinos and antineutrinos have been observed at Brookhaven National Laboratory.

Finally, we note that neutral current interaction can cause parity violating effects in atomic physics. This is so because the couplings of Z^0 to electrons and nucleons are parity-violating, so that Z^0 exchange between electrons and a nucleus can cause parity admixture in atomic levels. Such effects are being sought for in experiments done at the University of Washington at Seattle, Oxford and Paris.

3.3 Charm.

There is a problem when we try to extend the theory to the hadronic sector. In the following discussion, we ignore the color degrees of freedom since color has nothing to do with weak interactions. For hadrons, the weak doublet is

$$\begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix} .$$

The third component of the weak isospin current j_μ^3 is of the form

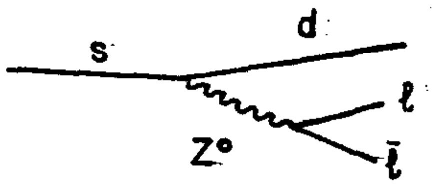
$$(\bar{u},\ \bar{d}_c)\begin{pmatrix} \frac{1}{2} & \\ & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} u \\ d_c \end{pmatrix}$$

$$= \tfrac{1}{2}(\bar{u}u - \bar{d}_c d_c)$$

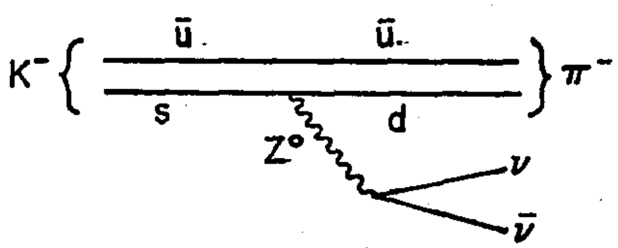
$$= \tfrac{1}{2}\Big[\bar{u}u - \cos^2\theta_c(\bar{d}d) - \sin^2\theta_c(\bar{s}s)$$

$$- \cos\theta_c\ \sin\theta_c(\bar{d}s + \bar{s}d)\Big] \ .$$

The neutral current to which Z^0 couples is $(j_\mu{}^3 - \sin^2\theta_W j_\mu{}^{em})$. Therefore, the last term, which has $S=\pm1$, predicts the strangeness-changing quark process, $s\to d+\ell+\bar{\ell}$ where ℓ stands for any lepton:



This implies, for example, the existence of the process $K^-\to\pi^-+\nu+\bar{\nu}$ in first order of weak interactions.



Processes of this sort have been looked for and have been found to occur ,if at all, at rates much lower than first order weak interactions: this means that these processes do not occur in first order and can occur only in higher orders. Something has to be done to eliminate the term $\bar{s}d + \bar{d}s$. Charm has to be introduced here. First people to discuss this in the context of gauge theory were Glashow, Iliopoulos and Maiani (GIM). Postulate in addition to $\begin{pmatrix} u \\ d_C \end{pmatrix}_L$, another doublet $\begin{pmatrix} c \\ s_C \end{pmatrix}_L$, where c is the fourth quark called charmed quark which has charge 2/3, and

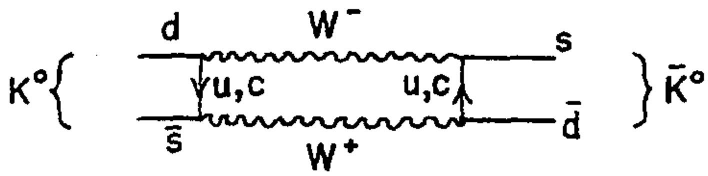
$$\begin{pmatrix} d_C \\ s_C \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix} .$$

In j_μ^3, there is now a contribution from the second quark doublet, so it is of the form:

$$\begin{aligned} &\tfrac{1}{2}(\bar{u}u - \bar{d}_C d_C) + \tfrac{1}{2}(\bar{c}c - \bar{s}_C s_C) \\ &= \tfrac{1}{2}(\bar{u}u + \bar{c}c) - \tfrac{1}{2}(\bar{d}_C d_C + \bar{s}_C s_C) \\ &= \tfrac{1}{2}(\bar{u}u + \bar{c}c) - \tfrac{1}{2}(\bar{d}d + \bar{s}s) \quad . \end{aligned}$$

Thus we see that if another doublet as above is postulated, we can eliminate strangeness-changing neutral current.

What is the mass of the charmed quark? If it is low, and of order of 0.5 GeV, then charmed hadrons, which contain one or more charmed quarks as constituents must have long been seen. The absence of such observations argues strongly that the charmed quark should be much more massive than ordinary quarks. But how heavy is it? It cannot be very massive, or we will have trouble with certain higher order weak interactions. To see this, consider K^0-$\bar{K}^0$ transition. This is a $\Delta S=2$ process and proceeds by second order weak interactions.



The rate of this transition is experimentally known, and is very small. It is not difficult to see that in the Glashow-Iliopoulos-Maiani scheme, this process would vanish identically were the up and charmed quarks degenerate. Therefore this amplitude is proportional to $m_c - m_u$, and is a sensitive measure of the size of m_c. Comparison with the experimental value suggest that

$$m_W >> m_c >> m_u$$

and $m_c \approx 1.5$ GeV.

When a sharp resonance at 3.1 GeV was discovered two years ago at Brookhaven and SPEAR, which we call now J or ψ, it was immediately conjectured that this was the J=1 bound state of $c\bar{c}$. Evidence since then supports very strongly this assignment. The charmed quark can combine with ordinary quarks and antiquarks to produce a new family of charmed hadrons.

Among them are J=0 and 1 charmed mesons of the form $D^+ \equiv (c\bar{d})_{J=0}$, $D^0 \equiv (c\bar{u})_{J=0}$, $F^+ \equiv (c\bar{s})_{J=0}$; $D^{*+} \equiv (c\bar{d})_{J=1}$, $D^{*0} \equiv (c\bar{u})_{J=1}$, $F^{*+} \equiv (c\bar{s})_{J=1}$, and their antiparticles. Very recently, candidates for $D^\pm$, D^0, and possibly their J=1 counterparts have been observed at SPEAR. I will talk more about them in the next lecture.

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FOURTH LECTURE - NEW PARTICLES

4.1 Charmed Particles.

We have discussed some of the low lying charmed meson states. In addition to those, there ought to be a J=0 $c\bar{c}$ state. Whether it has been seen is, in my mind, still problematic.

For s-wave charmed baryons, we expect

$$
\begin{array}{lll}
J = 1/2\ , & \Lambda_c\left[C_0{}^{++}\right] & c\left[ud\right]_{I=0}\ , \\
 & \Sigma_c{}^{**}\left[C_1{}^{++}\right] & c\left[uu\right]_{I=1} \\
 & \Sigma_0{}^{*}\left[C_1{}^{+}\right] & c\left[ud\right]_{I=1} \\
 & \Sigma_0{}^{0}\left[C_1{}^{0}\right] & c\left[dd\right]_{I=1}\ , \\
J = 3/2 & Y_c{}^{*++}\left[C_1{}^{*++}\right] & \\
 & Y_c{}^{*+}\left[C_1{}^{*+}\right] & \\
 & Y_c{}^{*0}\left[C_1{}^{*0}\right]\ . &
\end{array}
$$

In addition, we expect s-wave baryons which contain a strange quark and a charmed quark, two strange quarks and a charmed quark, etc.

4.2 Production and Decays.

The charm quantum number is conserved in strong and electromagnetic interactions. Therefore in $e\bar{e}$ collisions, hadron-hadron collisions, and photoproduction, charmed particles are produced in pairs. For example

$$p + p \rightarrow D^- + C_0^+ + \cdots \quad ,$$

or

$$e + \bar{e} \rightarrow D^+ + D^{*-} \quad ,$$

or

$$\gamma + p \rightarrow D^0 + \bar{D}^0 + \cdots \quad .$$

In neutrino interactions, charmed particles may be produced singly,

$$\nu + p \rightarrow \mu^- + C_1^{++}$$

for example, reflecting the quark process

$$\nu + d \rightarrow \mu^- + c \quad .$$

Since the charm quantum number is conserved by strong and electromagnetic interactions, at least the least massive charmed baryon and meson must be stable against strong and electromagnetic decays, and must therefore decay weakly. Weak decays of charmed hadrons are triggered by a charmed quark decaying weakly. There are semileptonic decays of the charmed quark:

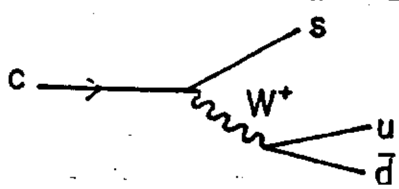
$$c \rightarrow s + \begin{pmatrix} e^+ \\ \mu^+ \end{pmatrix} + \nu \text{ (Cabibbo-favored)} \; ,$$

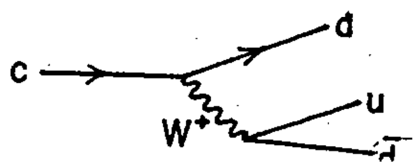
$$c \rightarrow d + \begin{pmatrix} e^+ \\ \mu^+ \end{pmatrix} + \nu \text{ (Cabibbo-disfavored)} \quad ,$$

and nonleptonic decays of the charmed quark:

$$c \rightarrow s + u + \bar{d} \qquad \text{(Cabibbo-favored)} \quad ,$$

$$c \rightarrow d + u + \bar{d} \qquad \text{(Cabibbo-disfavored)} \quad .$$





Cabibbo-favored processes are proportional to $\cos^2\theta_c$ and obey the selection rule $\Delta C=\Delta S=\pm 1$. Since $\sin\theta_c \simeq 0.2$, Cabibbo-favored processes should predominate. Thus $D^0 \sim (c\bar{u})$ is expected to decay predominantly into

$$D^0 \to (\bar{K} + m\pi\text{'s})^0 \; ,$$
$$(K + m\pi\text{'s})^- + \begin{pmatrix} e^+ \\ \mu^+ \end{pmatrix} + \nu \; .$$

Similarly we expect that

$$D^+ \to (\bar{K} + m\pi\text{'s})^+ \; ,$$
$$(\bar{K} + m\pi\text{'s})^0 + \begin{pmatrix} e^+ \\ \mu^+ \end{pmatrix} + \nu \; ,$$

and

$$F^+ \to (m\pi\text{'s})^+ \; , \; (m\pi\text{'s} + K\bar{K})^+$$
$$(m\pi\text{'s})^0 + \begin{pmatrix} e^+ \\ \mu^+ \end{pmatrix} + \nu \; , \text{ etc.}$$

Decays of D^+ would show up as a sharp peak in the $K^-\pi^+\pi^+$ mass spectrum. The final state has Q=+1 and S=-1; such a set of quantum numbers is not possible for hadrons which belong to an octet. This is a convenient signature for $D^{\pm}$: decay products of this state carries an "exotic" quantum number. The charmed baryon C_0^+ can decay hadronically:

$$C_0^+ \to \Lambda + (m\pi\text{'s})^+ \; ,$$
$$\to (\bar{K}N + m\pi\text{'s})^+ ,$$

or semileptonically.

What are the lifetimes of charmed particles? Since charmed particles are expected to decay into many different channels, because many of them are energetically open, attempts to estimate partial decay widths and sum them have been futile. Instead, we will make a very crude estimate of the inclusive decay width of a charmed particle by the following consideration. Consider a charmed quark confined within a small region in space. When the charmed quark decays, three lighter quarks are created which carry on the average a large amount of kinetic energy. This configuration, in which many energetic quarks are confined in a small region, is unstable, and it must break up into small pockets of regions, each containing a stable configuration of quarks, with 100% probability. Under this assumption, the generic decay rate of a charmed particle is just the rate for the charmed quark. It is given by

$$\Gamma_c \simeq 5 \times \left(\frac{G_F{}^2}{192\pi^3} \; m_c{}^5 \right) .$$

Numerically, this is about 10^{-13} sec. We expect charmed particles which are stable against strong and electromagnetic decays to live this long.

4.3 Masses of Charmed Particles.

For s-wave charmed hadrons, we can apply the considerations developed in the second lecture. Phenomenologically, we write the mass of an s-wave meson as

$$M = M_0 + m_1 + m_2 + a\left(\frac{1}{m_1} + \frac{1}{m_2}\right) + b\,\frac{1}{m_1 m_2}$$

(cont.)

$$+ c\left(\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{16}{3}\frac{1}{m_1 m_2}\vec{s}_1\ \vec{s}_2\right)$$

where 1 and 2 refer to the constituent quark and antiquark, respectively. We can determine unknown parameters M_0,a,b,c by fitting the known masses of π, ρ, K, K^* and ϕ. As noted earlier $m_u = 0.34$ GeV and $m_s = 0.54$ GeV. The charmed quark mass m_c can now be determined by assuming ψ(3.1 GeV) is the $c\bar{c}$ s-wave bound state of J=1. In this way one obtains $m_c \simeq 1.6$ GeV.

One is now in a position to predict charmed particle masses. One obtains

$$m_D \simeq 1.8 \text{ GeV}$$

$$m_F \simeq 2.1 \text{ GeV.}$$

Since $m_{D*} - m_D = (16/3)(c/m_u m_c)$ and $m_{K*} - m_K = (16/3)(c/m_u m_s)$,

$$m_{D*} - m_D = \left(\frac{m_s}{m_c}\right)(m_{K*} - m_K)$$

$$= 0.12 \text{ GeV.}$$

Similar considerations give, for charmed baryons,

$$m_{C^0} \simeq 2.2 \text{ GeV} \quad ,$$

$$m_{C_1^*} - m_{C_1} \simeq \left(\frac{m_s}{m_c}\right)(m_{Y_1^*} - m_\Sigma)$$

$$\simeq 0.07 \text{ GeV,}$$

and

$$\frac{1}{3}(2m_{C_1^*} + m_{C_1}) - m_{C_0}$$

(cont.)

$$= \frac{1}{3}(2m_{Y*} + m_{\Sigma}) - m_{\Lambda}$$

$$\simeq 0.2 \text{ GeV}.$$

4.4 Discoveries.

Discoveries of the J,ψ particle, and of a family of states connected to this by radiative transitions, are now a legend, and have been well documented, for instance, in Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies (SLAC, Stanford University, Stanford, California, 1975).

More recently, two states have been found at SPEAR which have all the characteristics expected of $D^{\pm}$, and D^0 as far as can be presently ascertained.

(1). $D^0, \bar{D}^0$ candidates. Narrow peaks have been observed in the invariant mass plots of $K^{\pm}\pi^{\mp}$ and $K^{\pm}\pi^{\mp}\pi^{+}\pi^{-}$ produced in $e\bar{e}$ annihilation at SPEAR. The peak is at 1.865 ± .015 GeV; the width of the state is consistent with being zero (less than 40 MeV). The recoil mass spectrum (the mass spectrum of the object recoiling against the 1.87 GeV object) has a peak around $\sim$2 GeV (perhaps with some structures), indicating that D^0 or $\bar{D}^0$ is produced in association with systems of comparable or larger mass. It is indicative of production mechanism

$$\begin{aligned} e + \bar{e} + \gamma &\to D^0 + \bar{D}^0 \\ &\to D^0 + \bar{D}^{*0} , \\ &\to D^{*+} + D^- \\ &\quad \hookrightarrow D^0 + \pi^+ , \end{aligned}$$

etc. The last process is possible if $m(D^{*+}) > m(D^0) + m(\pi^+)$.

(2). D$^\pm$ candidates. Again at SPEAR, narrow charged states were observed with a mass of 1.876 ± .015 GeV in the exotic channels $K^\mp\pi^\pm\pi^\pm$. These states are produced primarily in association with a system of mass 2.01 ± 0.02 GeV. It is likely that the following production mechanism is responsible:

$$e + \bar{e} \to \gamma \to D^\pm + (D^*)^\mp \quad .$$

(3). C_0^+, C_1^{++}, C_1^{*++} candidates. About a year ago, the following reaction was observed at Brookhaven National Laboratory:

$$\nu + p \to \mu^- + \Lambda + \pi^+ + \pi^+ + \pi^+ + \pi^-$$

It can be interpreted as the production and subsequent decay of a charmed baryon:

$$\begin{aligned} \nu + p \to \mu^- + &C_1^{++} \\ &\hookrightarrow C_0^+ + \pi^+, \text{(strongly)} \\ &\qquad \hookrightarrow \Lambda + \pi^+ + \pi^+ + \pi^-, \text{ (weakly)}. \end{aligned}$$

Interpreted this way, the masses of C_1^{++} and C_0^+ are consistent with being about 2.4 and 2.25 GeV respectively. Interpreted instead in the absence of charm, this event would mark the first instance of a semileptonic weak process with $\Delta S=-\Delta Q$ (see Section 3.1).

Very recently, a peak has been observed at 2.25 GeV in the effective mass distribution of $\bar{\Lambda}\pi^-\pi^-\pi^+$ produced in the reaction

$$\gamma + Be \to \bar{\Lambda} + \pi^- + \pi^- + \pi^+ + \cdots$$

at Fermilab by the Columbia-Fermilab-Hawaii-Illinois collaboration. The mass coincides with one of the $\Lambda\pi^+\pi^+\pi^-$ combinations of the Brookhaven event. There is in addition an indication of a state near 2.5 GeV which decays into $\pi^\pm + (\bar{\Lambda}\pi^-\pi^-\pi^+)$.

The mass estimates of De Rujula, Georgi and Glashow have been very remarkable. A priori, agreement with experiments to within, say. 0.1 GeV is not expected, because of the necessarily perturbative nature of the theory. Therefore the predicted values $m_D \simeq 1.8$ GeV, $m_{C_0} \simeq 2.1$ GeV must be considered in agreement with the experimental values $m_D \simeq 1.87$ GeV and $m_{C_0} \simeq 2.25$ GeV.

For excited states, we have the prediction that

$$m_{D*} = m_D + \left(\frac{m_s}{m_C}\right)(m_{K*} - m_K) = 1.86 + 0.13 \simeq 2.0 \text{ GeV}$$

which seems to agree with the observed recoil mass in $e\bar{e}$ annihilation. For baryons, we deduce from the relations derived in the last section and the input $m_{C_0} \simeq$ 2.25 GeV:

$$m_{C_1} \simeq 2.4 \text{ GeV},$$

$$m_{C_1^*} \simeq 2.48 \text{ GeV} .$$

Again, these values seem to agree, roughly, with observation.

Are there proofs that the decays of $D^{\pm}(1.87)$, D^0, $\bar{D}^0(1.87)$ and $C_0(2.25)$ involve weak interactions? Only circumstantial ones, so far. First of all, their widths are very narrow. Suppose the $K^{\pm}\pi^{\pm}$ and $K^{\mp}\pi^{\pm}\pi^{\pm}$ peaks correspond to the decays of members of an isomultiplet with spin J=0. Then the parity must be violated since $(K\pi)_{J=0}$ has positive parity where $(K\pi\pi)_{J=0}$ has negative parity. There is some hint (private communication from the CFHI group) that $\bar{\Lambda}$ may be longitudinally polarized, which would imply parity violation in the process $\bar{C}_0 \to \bar{\Lambda} + \pi^- + \pi^- + \pi^+$.

There has also been considerable circumstantial evidence for semileptonic decays of these objects. One is the neutrino- (and antineutrino-) induced dimuon events, which can be interpreted as

$$\nu + \text{nucleon} \to \mu^- + \underset{\displaystyle \llcorner\!\!\to \mu^+ + \nu + \cdots ,}{(\text{charmed object})} + \cdots$$

which was discovered by the Harvard-Pennsylvania-Wisconsin-Fermilab collaboration and the $K_{e\mu}$ events in bubble chamber experiments, reported by the Gargamelle collaboration at CERN and the LBL-CERN-Hawaii-Wisconsin collaboration at Fermilab:

$$\nu + \text{nucleon} \rightarrow \mu^- + (\text{charmed object}) + \cdots$$
$$\hookrightarrow \bar{K}^0 + e^+ + \nu + \cdots ,$$

In conclusion, particle physics faces new challenges of understanding charmed particles, of unifying strong, weak and electromagnetic interactions (and gravitational, too) in a single conceptual framework. There is further indication that four quarks are not the end of the story of flavors. I believe that the truth is inexhaustible, and we will have many more excitements in our lifetime.

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