national accelerator laboratory

FERMILAB-Pub-74/99-THY
UM HE 74-31
October 1974

Are There Rescattering Corrections to Inclusive Reactions?

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ABSTRACT

We construct and solve a model for multiparticle production which obeys unitarity exactly. We find, in agreement with some earlier results, that there are no rescattering corrections to inclusive production in this model. We examine the assumptions which lead to this result, and we discuss the consequence of this result to the phenomenology of the parton model. We find that, in a wide class of factorizable models, the inclusive cross section for heavy virtual photon emission is independent of the strong interactions.

*Work supported in part by the Atomic Energy Commission.

It has been argued that there are no absorption corrections to some inclusive reactions. Abramovskii, Kancheli, and Gribov[1] made this claim for inclusive production in the pionization region. Cardy and Winbow[2], in response to the claim for absorption by Henyey and Savit,[3] argued for the absence of absorption in inclusive heavy lepton pair production. The result of Cardy and Winbow has been substantiated from another approach by DeTar, Ellis and Landshoff.[4] All these arguments involved the consideration of discontinuities of infinite sums of Feynman diagrams.

In this paper, we examine this situation from another point of view. We construct an exactly soluble model incorporating unitarity and calculate the inclusive cross sections. The model we consider, being unitary, includes absorption of exclusive processes. After discussing this model, we discuss its generality and the implications of the results for parton models.

Let us begin by motivating the assumptions of our model by considering Feynman's discussion of the parton model.[5] Consider $pp \to \mu^- \mu^+ X$ but, for now, neglect effects due to the strong interactions. The heavy virtual photon is produced by the electromagnetic annihilation of a parton from one of the protons with its antiparton from the other photon. The various hadrons contributing to the missing mass X consist of fragments of the proton plus particles with finite momenta in the center of mass. Next consider the effect of strong interactions. In the limit of infinite energy, it is assumed in Feynman's model that we may neglect the interactions among hard partons. Only the wee partons (with finite longitudinal momenta) of each proton undergo scattering, which leads to the redistribution of pionization products in the collision. So long as we kinematically require that the virtual photon be formed by the annihilation of hard partons, it would seem that the inclusive cross section for heavy photon emission will be independent of the strong interactions.

The argument of Feynman, repeated above, leads us to construct a model in which the dynamical mechanism responsible for the photon production and that responsible for pionization are independent. First we incorporate these properties into a "coherent state" model of the type extensively studied.[6, 7] Then, we generalize to arbitrary, but independent, mechanisms.

The model is as follows: Two "protons" collide at a given impact parameter $\vec{b}$, creating a classical source of "pions" and "photons". The sources are classical in the sense that the "protons" are assumed to be so massive that their recoil can be ignored. The sources are $\rho_\pi(\vec{p}_\pi, \vec{b})$ and $\rho_\gamma(\vec{p}_\gamma, \vec{b})$. We ignore spin and internal quantum numbers, and make no assumptions on the functional form of ρ_π and ρ_γ.

The classical source problem can be solved exactly.[8] The S-matrix element for $p+p \to p+p+n_1\pi's + n_2\gamma's$ is

$$S_{n_1 n_2} = e^{-\frac{1}{2}\left(n_\pi(b)+n_\gamma(b)\right)} \prod_{j=1}^{n_1} \rho_\pi(\vec{p}_{\pi_j}, \vec{b}) \prod_{k=1}^{n_2} \rho_\gamma(\vec{p}_{\gamma_k}, b) \tag{1}$$

where

$$\bar{n}_\pi(\vec{b}) = \int |\rho_\pi(\vec{p}_\pi, \vec{b})|^2 \frac{d^3 p_\pi}{E_\pi}$$

$$\bar{n}_\gamma(\vec{b}) = \int |\rho_\gamma(\vec{p}_\gamma, \vec{b})|^2 \frac{d^3 p_\gamma}{E_\gamma} \quad . \qquad (2)$$

$S_{n_1 n_2}$ depends on the total energy, on $\vec{b}$, and on the momenta of **all** "pions" and "photons". Unitarity for pp → pp can easily be checked. One takes the absolute square of $S_{n_1 n_2}$, integrates (d^3P/E) over each π and γ momentum, divides by $n_1! n_2!$, since phase space is multiply counted this number of times, and sums over n_1 and n_2, obtaining the value 1 for the sum. In fact, the model obeys unitarity not only for pp → pp, but for every process.

Every exclusive amplitude is absorbed in this model. Every S matrix element includes a factor

$$S_{oo} = e^{-\frac{1}{2}\bar{n}_\gamma(b) - \frac{1}{2}\bar{n}_\gamma(b)} \qquad (3)$$

so that

$$S_{n_1 n_2} = S_{oo}\, S^{\text{unabsorbed}}_{n_1 n_2} \qquad (4)$$

where $S^{\text{unabsorbed}}_{n_1 n_2}$ is just the lowest order contribution, i.e., the "Born approximation". One might anticipate that, since every exclusive process is absorbed, the inclusive processes also should be.

We now calculate the single γ inclusive production. Since the momentum of the observed γ is not integrated, phase space is counted only $n_1!(n_2-1)!$ times. We have

$$E_\gamma \frac{d\sigma_\gamma}{d^3p_\gamma} \propto \sum_{n_1=0}^{\infty} \sum_{n_2=1}^{\infty} \int \prod_{j=1}^{n_1} \frac{d^3p_{\pi j}}{E_{\pi j}} \prod_{k=1}^{n_2-1} \frac{d^3p_{\gamma k}}{E_{\gamma k}}$$

$$|S_{n_1 n_2}|^2 / n_1!(n_2-1)!$$

$$= \sum_{n_1=0}^{\infty} \sum_{n_2=1}^{\infty} |\rho_\gamma(\vec{p}_\gamma, \vec{b})|^2 e^{-\bar{n}_\pi - \bar{n}_\gamma} \frac{\bar{n}_\pi^{n_1}}{n_1!} \frac{\bar{n}_\gamma^{n_2-1}}{(n_2-1)!}$$

$$= |\rho_\gamma(\vec{p}_\gamma, \vec{b})|^2 \tag{5}$$

There are no absorption or rescattering corrections to the "Born approximation" for inclusive production of "photons" (or "pions) in this model.[9] Our "photons" and "pions" are unconnected with the leading particles ("protons"). Both pionization and massive lepton pairs are similar to the "pions" and "photons" of our model, so that our result is in agreement with the results of Refs. 1 and 2 on the absence of rescattering corrections for these processes.

It is instructive to examine in some detail how our result occurs, especially given that all exclusive processes are absorbed. The cross section for a "photon" accompanied by n_1 "pions" has a factor

$$e^{-\bar{n}_\pi} \bar{n}_\pi^{n_1} / n_1! \tag{6}$$

If we increase the strength of strong interactions, we increase $\bar{n}_\pi$. Differentiating with respect to $\bar{n}_\pi$ we find

$$\frac{d\sigma_{n_1}}{d\bar{n}_\pi} = \sigma_{n_1}\left(-1 + \frac{n_1}{\bar{n}_\pi}\right) . \tag{7}$$

Thus the low multiplicities ($n_1 < \bar{n}_\pi$) decrease as a result of increased absorption, while the higher multiplicities increase in a way which exactly compensates the decrease of the low multiplicities. The compensation can be seen from Eq. (7) since the average value of n_1 is $\bar{n}_\pi$.

We can compare the actual situation in the model as we have described it to the situation as envisioned by Henyey and Savit.[3] We shall ignore the production of any more than one photon. The amplitude for production of zero pions is $(S_{oo}-1)/2i$, and for production of n pions is $S_{no}/2i$. A loop integration contributes a factor of i. No absorption gives the "Born" amplitude $B = \rho_\gamma(\vec{p}_\gamma, \vec{b})/2i$. Final state absorption gives an amplitude $\frac{(S_{oo}-1)}{2}$ B. To compare with the model, we can use Eq. 3 for S_{oo} . We do not include initial state absorption, which was not discussed in Ref. 3. The pp → ppγ cross section is proportional to

$$\left[B + \left(\frac{S_{oo}-1}{2}\right)B\right]^2 = \frac{S_{oo}^2}{4} B^2 + \frac{S_{oo}}{2} B^2 + \frac{B^2}{4} . \tag{8}$$

The pp→ ppγ + nπ amplitude, for n≠o is $\frac{S_{no}}{2}$ B . Using Eq. 4

$$S_{no} = S_{oo} \prod_{j=1}^{n} \rho_\pi(\vec{p}_{\pi_j}, \vec{b})$$

so that the first term on the right side of Eq. 8 plus the contribution to the cross section from pp → ppγ + nπ, n ≠ 1, is

$$\tfrac{1}{4} S_{oo}^2 \sum_{n=0}^{\infty} \left[\int |\rho_\pi|^2 \frac{d^3 p_\pi}{E_\pi} \right]^n \Big/ n! \quad B^2$$

$$= \tfrac{1}{4} S_{oo}^2 S_{oo}^{-2} B^2 = \frac{B^2}{4} \quad . \tag{9}$$

Adding in the other terms of Eq. 8, we find

$$E_\gamma \frac{d\sigma}{d^3 p_\gamma} \propto B^2 \left(\frac{1+S_{oo}}{2} \right)$$

$$= B^2 - B^2 \left(\frac{1-S_{oo}}{2} \right) \tag{10}$$

which is less than B^2. If we had retained initial state absorption we would have ended up with

$$E_\gamma \frac{d\sigma}{d^3 p_\gamma} = B^2 - \frac{3}{4} B^2 (1-S_{oo}^2) \tag{11}$$

even less than Eq. 10.

By comparing with the model we see that Henyey and Savit get the $pp \to pp\gamma$ amplitude correct (if initial state absorption is included), including all the negative contributions. The positive contributions of $pp \to pp\gamma + n\pi$'s however, come out a factor of 4 too small (See Eq. 9). Thus, Henyey and Savit end up with a negative net contribution. What is left out of cutting up their essentially planar diagrams is the possibility that the n "pions" could be produced in whole or in part before the "photon" was produced. This possibility doubles the size of the amplitude for these processes.

More generally, consider any model in which the amplitude for $pp \to$ "γ"N (where N is some collection of hadrons) can be represented by the factorized form

$$S_\gamma(\underset{\sim}{b},\vec{p}_\gamma)S_N(\underset{\sim}{b},\vec{p}_1,\vec{p}_2,\ldots,\vec{p}_N) \tag{12}$$

where S_N is the strong interaction amplitude for $pp\rightarrow N$. Unitarity for the strong interactions implies $S_N^+S_N = 1$, so obviously the inclusive cross section for "γ" production, $pp\rightarrow$"γ"X, will be proportional to $|\rho_\gamma|^2$, independent of the precise nature of S_N. Whether the strong interaction amplitudes S_N manifest absorption or not is irrelevant, so long as the photon is produced independently, as it is in the parton model. On the other hand, the observation of a hadron in addition to the photon, e.g., $pp\rightarrow$"γ"πX, will have absorptive corrections depending on whether $pp\rightarrow\pi X$ is absorbed or not. ABK argued that $pp\rightarrow\pi X$ is not absorbed in the pionization region which is also true in the model above. However, absorption is not zero for the fragmentation region, so we expect the same for $pp\rightarrow$"γ"πX.[10]

We have not shown that the models considered here are equivalent to the arguments of Ref. 2; indeed, the discussion there seems to be more general. Nevertheless, we feel it is useful to be able to see how it works in a specific model.

We believe that the result of Ref. 2, substantiated by our considerations, is of potentially great significance. On the purely theoretical side, it puts the Drell-Yan formula on firm footing. Recalling that the light cone algebra is inapplicable to heavy lepton-pair production,[11] this result on the absence of absorption seems even more surprising.

From a phenomenological point of view, the result is equally important. Einhorn and Savit[12] showed that, given the data on deep inelastic electron and neutrino scattering, the Drell-Yan formula for the production of a heavy lepton-pair predicts a cross section considerably smaller than observed. Drawing firm conclusions from their result was blocked in part by the theoretical uncertainty surrounding the possible role of strong interactions. Now, however, the disagreement with experiment becomes that much more of a challenge to the parton model.

In this paper we have examined inclusive production in a model which incorporates unitarity and has a similar effect to that of scaling of inclusive production. In this model we find no change in the inclusive production in the central region, in agreement with some earlier results. We have identified the missing part of a result to the contrary.[3] Our results do not extend to the fragmentation region. The absence of rescattering corrections to the inclusive production of massive lepton pairs was shown to be true in any factorizable model. The result is of both theoretical and phenomological importance.

We would like to thank M-S Chen and G. Kane for useful discussions. One of us (MBE) would like to thank E. Rabinovici and S. Ellis for discussions on the subject.

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