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Application of Current Algebra Techniques to Neutral-Current-Induced Threshold Pion Production

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ABSTRACT

We apply current algebra techniques to study threshold pion production induced by the weak neutral current. We find that the Argonne data for threshold $\pi^- p$ production are in strong disagreement with the predictions of both the SU(2) $\otimes$ U(1) gauge model and the "baryon current" model for the neutral current. Indeed, it seems unlikely that any hadronic neutral current formed from the usual vector and axial-vector current nonets can explain the Argonne observations, suggesting that new coupling types may be present.

The initial experiments discovering weak neutral currents in high energy deep inelastic neutrino reactions[1] have now been supplemented with the observation of neutral current effects in low energy neutrino pion production.[2, 3] We focus here on a particular aspect of the new data, the observation at Argonne of a large cross section for threshold pion production in the reaction $\nu_\mu + n \rightarrow \nu_\mu + \pi^- + p$.[2] This surprising feature is an open invitation to apply current-algebra soft-pion techniques, which are capable of giving a good description of threshold pion production processes. In this letter we briefly describe the methods used in making such an analysis, and summarize the results obtained.

We begin by giving a simple analytic treatment of threshold pion production, which although somewhat naive, illustrates the basic ideas which we exploit in our more careful numerical calculations. According to standard soft pion lore,[4] the amplitude for the pion emission process $\mathcal{J} + \alpha \rightarrow \pi^j + \beta$, with α, β hadronic states and $\mathcal{J}$ an external current, is given as the sum of two terms. The first consists of a sum of external line insertions in which the pion π^j is emitted from the external hadronic lines of the pionless process $\mathcal{J} + \alpha \rightarrow \beta$, while the second is an equal time commutator term proportional to the amplitude for the reaction $\mathcal{J}' + \alpha \rightarrow \beta$, with $\mathcal{J}'$ the modified current obtained from the commutator $\mathcal{J}' = [F_j^5, \mathcal{J}]$. In the case of neutral current weak pion production, the current $\mathcal{J}$ is of course the hadronic weak neutral current and the states α, β are each a single free nucleon. For simplicity, let us restrict ourselves for the moment to cases in which the equal time commutator term vanishes, as occurs, for example, if the current $\mathcal{J}$ is an isoscalar V, A structure containing an arbitrary linear combination of $\mathcal{F}_0^\lambda, \mathcal{F}_8^\lambda, \mathcal{F}_0^{5\lambda}, \mathcal{F}_8^{5\lambda}$.[5] The pion emission amplitude then consists entirely of the external line insertion terms. Evaluating these terms at threshold (where the insertion on the outgoing nucleon line vanishes) and neglecting the pion mass in all kinematics, we find the following relation between threshold pion production and neutrino proton elastic scattering,

$$\frac{1}{|\vec{q}|}\,\frac{d\sigma(\nu+N\to\nu+N+\pi^j)}{d(k^2)\,dW}\bigg|_{\text{threshold}} = \frac{a^2}{4\pi^2 M_\pi^2}\left(\frac{g_r M_\pi}{2M_N}\right)^2 \frac{k^2}{M_N^2}\left(1+\frac{k^2}{4M_N^2}\right)$$
$$\times\left(1+\frac{k^2}{2M_N^2}\right)^{-2}\frac{d\sigma(\nu+p\to\nu+p)}{d(k^2)}\ . \qquad (1)$$

Here M_N, M_π are the nucleon and pion mass, W is the mass of the final $\pi^j N$ isobar and $|\vec{q}|$ is the pion momentum in the isobaric rest frame, k^2 is the leptonic squared four-momentum transfer (space-like $k^2 > 0$), $g_r \approx 13.5$ is the pion-nucleon coupling constant, and the isospin matrix element a takes the values $|a| = \sqrt{2}$ for $\pi^j = \pi^\pm$, $|a| = 1$ for $\pi^j = \pi^0$. The significance of Eq. (1) is that it allows one to translate an upper bound on the cross section for $\nu_\mu + p \to \nu_\mu + p$ into an upper bound on the strength of threshold pion production by the weak neutral current.

As we have already suggested, the above derivation is too naive in a number of respects. First of all, the external line insertion terms are rapidly varying pole terms, and so the kinematic approximation of neglecting M_π in calculating them is dangerous. Secondly, by considering only cases in which the equal time commutator term $\mathscr{S}'$ vanishes, we exclude from consideration such processes as π^- production in the $SU(2) \otimes U(1)$ gauge model. And finally, it is important to estimate the leading $O(q)$ corrections to the soft pion approximation, and to calculate the effects in the threshold region of the tail of the (3, 3) resonance. We deal with these problems by using an extended version of a model for weak pion production which we have described in detail elsewhere.[6] In its original form, the model included the rapidly varying pole terms and the resonant (3, 3) multipoles, with no kinematic approximations. The extensions consist of adding subtraction constants (in the dispersion theory sense) to the non-Born terms of the model, which guarantee that it satisfies the relevant soft pion theorems and which include the leading corrections (of first order in the pion four-momentum q and zeroth order in the lepton four-momentum transfer k) to the soft pion limit. These latter corrections are calculated by the method of Low[7] and Adler and Dothan[7]; for the vector current amplitude they vanish, while for the isovector axial-vector amplitude they are related by PCAC to momentum derivatives of the pion-nucleon scattering amplitude at the crossing symmetric point. For an isoscalar axial-vector current the order q corrections cannot be precisely calculated, but an heuristic resonance dominance argument suggests that they should be much smaller than in the isovector axial-vector case.

We give now the results of numerical calculations using the extended model in various cases. (1) Isoscalar vector neutral current. For the form factors in this case we take, for definiteness, a dipole formula with characteristic mass M_N,

$$2M_N F_2^S(k^2) = \mu F_1^S(k^2) , \quad F_1^S(k^2)/F_1^S(0) = (1 + k^2/M_N^2)^{-2}. \qquad (2)$$

Assuming the 95% confidence bound[2]

$$\sigma(\nu_\mu + p \rightarrow \nu_\mu + p) \leq 0.32\,\sigma(\nu_\mu + n \rightarrow \mu^- + p) , \qquad (3)$$

we find that the cross section for $\nu_\mu + n \rightarrow \nu_\mu + \pi^- + p$, with $\pi^- p$ invariant mass W between 1080 and 1120 MeV, is bounded by[8]

$$0.32\sigma(\nu_\mu + n \to \mu^- + p) \times \sigma(\nu_\mu + n \to \nu_\mu + \pi^- + p)/\sigma(\nu_\mu + p \to \nu_\mu + p)$$
$$= 10^{-41}\mathrm{cm}^2 \times (0.0598 + 0.0239\mu + 0.0291\mu^2)/(0.348 + 0.0223\mu + 0.0288\mu^2) \tag{4}$$
$$\leq 1.0 \times 10^{-41}\mathrm{cm}^2 .$$

The Argonne experiment observes 5 events in the same mass range, corresponding to a cross section of $22 \times 10^{-41}\mathrm{cm}^2$. If the bound of Eq. (3) is improved, the discrepancy will of course get proportionately worse. [We find in this case that the naive form of the low energy theorem in Eq. (1) is reasonably good, predicting a bound one-half as large as that of Eq. (4).[9]]

(2) Weinberg-Salam SU(2) ⊗ U(1) Model. In the simplest, one parameter version of this model, the neutral current has the form

$$\mathcal{J}_N^\lambda = \mathcal{F}_3^\lambda - \mathcal{F}_3^{5\lambda} - 2x(\mathcal{F}_3^\lambda + 3^{-\frac{1}{2}}\mathcal{F}_8^\lambda) + \Delta\mathcal{J}^\lambda , \quad x \equiv \sin^2\theta_W , \tag{5}$$

with $\Delta\mathcal{J}^\lambda$ an isoscalar V-A strangeness and "charm" current contribution which is conventionally assumed to couple only weakly to non-strange low mass hadrons. Neglecting $\Delta\mathcal{J}^\lambda$ for the moment, we can make an absolute prediction of the cross section for $\nu_\mu + n \to \nu_\mu + \pi^- + p$. The result, plotted in Fig. 1 together with the Argonne events, is evidently much too small in the region near threshold - the predicted cross section for $\pi^- p$ invariant mass W between 1080 and 1120 MeV is only $0.80 \times 10^{-41}\mathrm{cm}^2$, a factor of 28 smaller than the experimental result. By way of contrast, in Fig. 2 we compare the predictions of our model with the Argonne results for the charged current reaction $\nu_\mu + p \to \mu^- + \pi^+ + p$. The predicted cross section for $\pi^+ p$ invariant mass W between 1080 and 1120 MeV is $7.1 \times 10^{-41}\,\mathrm{cm}^2$, in good agreement with the observed cross section of $9.3 \times 10^{-41}\,\mathrm{cm}^2$.

In certain extensions of the original Weinberg-Salam model, the neutral current has the general form of Eq. (5), but with an adjustable strength parameter in front. We can test this two-parameter form by the method of comparison with the reaction $\nu_\mu + p \to \nu_\mu + p$ used above. Again neglecting $\Delta \mathcal{J}^\lambda$ and assuming Eq. (3), we find that the cross section for $\nu_\mu + n \to \nu_\mu + \pi^- + p$ with $\pi^- p$ invariant mass W between 1080 and 1120 MeV is bounded over the whole range of parameters by[8]

$$10^{-41}\,\mathrm{cm}^2 \times (0.486 - 1.12\,x + 0.920\,x^2)/(0.202 - 0.588\,x + 0.706\,x^2) \qquad (6)$$
$$\leq 2.7 \times 10^{-41}\,\mathrm{cm}^2 ,$$

about a factor of 8 smaller than observed. Any improvements in the bound of Eq. (3) will, as before, increase the discrepancy proportionately. Finally, let us examine whether the neglected isoscalar contribution $\Delta \mathcal{J}^\lambda$ can eliminate the discrepancy. We have already seen that an isoscalar vector current cannot produce strong threshold pion production without violating the upper bound on $\sigma(\nu_\mu + p \to \nu_\mu + p)$, so we need only examine the effect of an isoscalar axial-vector current. Assuming an isoscalar axial-vector form factor given by

$$g_A(k^2)/g_A(0) = (1 + k^2/M_N^2)^{-2} , \qquad (7)$$

and neglecting the order q corrections (which we believe to be small), we find that the isoscalar analog of Eqs. (4) and (6) is a bound[8] of $0.85 \times 10^{-41}\ \mathrm{cm}^2$ for the Argonne cross section in the first two bins. Hence barring the possibility of much larger order q corrections than anticipated,[10] the current $\Delta \mathcal{J}^{\lambda}$ does not seem to be capable of resolving the discrepancy. In the detailed paper describing this work we will present a search of the 4-parameter space obtained by including the simple Weinberg-Salam, isoscalar vector and isoscalar axial-vector currents simultaneously, but given the smallness of the bounds obtained above, we do not anticipate that the inclusion of interference terms will appreciably change things. It appears that the Argonne results, if confirmed in subsequent experiments, are incompatible with any neutral current formed from the usual vector and axial-vector current nonets.

This surprising conclusion suggests that the hadronic weak neutral interaction may involve unusual types of coupling. One possibility is that the interaction is of V, A type, but involves currents outside the usual quark model vector and axial-vector nonets. Another possibility, which has been raised in several recent papers,[11] is that S, P and T type neutral couplings may be present. If we define S, P and T hadronic "currents" $\mathcal{F}_j$, $\mathcal{F}_j^5$, $\mathcal{F}_j^{\lambda\sigma}$ and abstract their commutation relations from the quark model forms

$$\mathcal{F}_j = \bar{q}\,\tfrac{1}{2}\lambda_j q\,, \quad \mathcal{F}_j^5 = \bar{q}\,\tfrac{1}{2}\lambda_j \gamma_5 q\,, \quad \mathcal{F}_j^{\lambda\eta} = \bar{q}\,\tfrac{1}{2}\lambda_j \sigma^{\lambda\eta} q\,, \tag{8}$$

then the commutator term $\mathcal{J}'$ appearing in the soft pion analysis above will have SU(3) D- rather than F-type structure. This will substantially alter the structure of the low energy theorems (for instance, the commutator term no longer will vanish for an isoscalar neutral current), and may permit strong threshold pion production without violating the experimental bounds on neutrino proton elastic scattering.

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2. P. A. Schreiner, Argonne National Laboratory Report ANL/HEP 7436, presented at the XVII International Conference on High Energy Physics, London, July, 1974.

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4. S. L. Adler and R. F. Dashen, Current Algebras (W. A. Benjamin, New York, 1968).

5. Vanishing of the equal time commutator in this case was noted by J. J. Sakurai, in Proceedings of the Fourth International Conference on Neutrino Physics and Astrophysics (1974).

6. S. L. Adler, Ann. Phys. (N. Y.) 50, 189 (1968). [See also S. L. Adler, Phys. Rev. D9, 229 (1974).] The extended model is obtained by adding as subtraction constants Eq. (5A. 21) for $\bar{A}_2^{(-)}|_0$, $\bar{A}_4^{(-)}|_0$ and $\bar{A}_7^{(+)}|_0$,

Eq. (5A. 22) for $\bar{V}_1^{(+)}|_0$, $\bar{V}_1^{(0)}|_0$ and $\bar{V}_6^{(-)}|_0$, Eq. (5A. 9) for $\bar{A}_3^{(+)}|_0$ and Eq. (5A. 30) for $\bar{A}_1^{(-)}|_0$. The order q terms $\bar{A}_3^{(+)}|_0$ and $\bar{A}_1^{(-)}|_0$ were assumed to have k^2 dependence $(1+k^2/M_N^2)^{-2}$; variation of this assumed dependence produced only small changes in the results. We took the axial-vector form factor mass as $M_A = 0.9$ GeV.

7. F. E. Low, Phys. Rev. 110, 974 (1958); S. L. Adler and Y. Dothan, Phys. Rev. 151, 1267 (1966).

8. These bounds are not corrected for the difference in k^2 distributions between the reactions $\nu_\mu + p \to \nu_\mu + p$ and $\nu_\mu + n \to \mu^- + p$. For neutral current form factors which decrease much more slowly than the charged current form factors, the effect of such corrections would be to increase the discrepancies quoted.

9. In the case of $\nu_\mu + N \to \nu_\mu + N + \pi^0$ in the Weinberg-Salam model, where Eq. (1) should formally hold, we find that the order q corrections increase the (greatly suppressed) threshold pion production by an order of magnitude. As a result, the threshold π^0 production becomes comparable to that in $\nu_\mu + n \to \nu_\mu + \pi^- + p$ (where the order q corrections have only an ~15% effect).

10. The ninth axial current anomaly vanishes at $k = 0$ (W. A. Bardeen, in press) and so does not contribute to the order q correction term.

11. B. Kayser, G. T. Garvey, E. Fischbach and S. P. Rosen (to be published); R. L. Kingsley, F. Wilczek and A. Zee (to be published).

FIGURE CAPTIONS

Figure 1. Comparison of the Weinberg-Salam model (for x = 0.35) with the Argonne neutral current data. Each event represents an Argonne flux-averaged cross section of 4.4×10^{-41} cm^2.

Figure 2. Comparison of the extended pion production model with the Argonne charged current data. Each event represents an Argonne flux-averaged cross section of 2.3×10^{-41} cm^2.

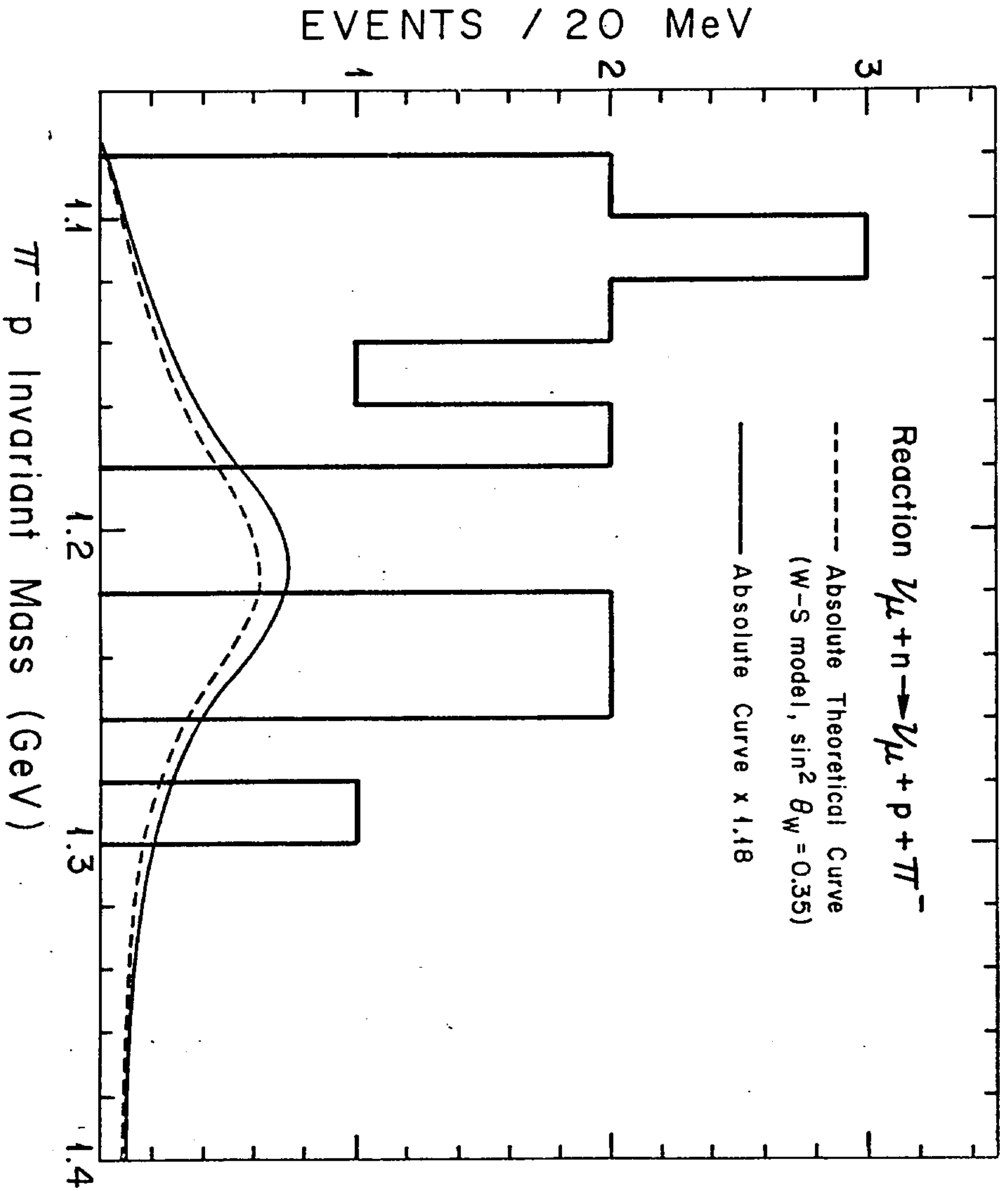
EVENTS / 20 MeV
1
2
3
Reaction $\nu_\mu + n \rightarrow \nu_\mu + p + \pi^-$
Absolute Theoretical Curve
(W–S model, $\sin^2\theta_W = 0.35$)
Absolute Curve x 1.18
1.1
1.2
1.3
1.4
$\pi^- p$ Invariant Mass (GeV)

FIGURE 1.

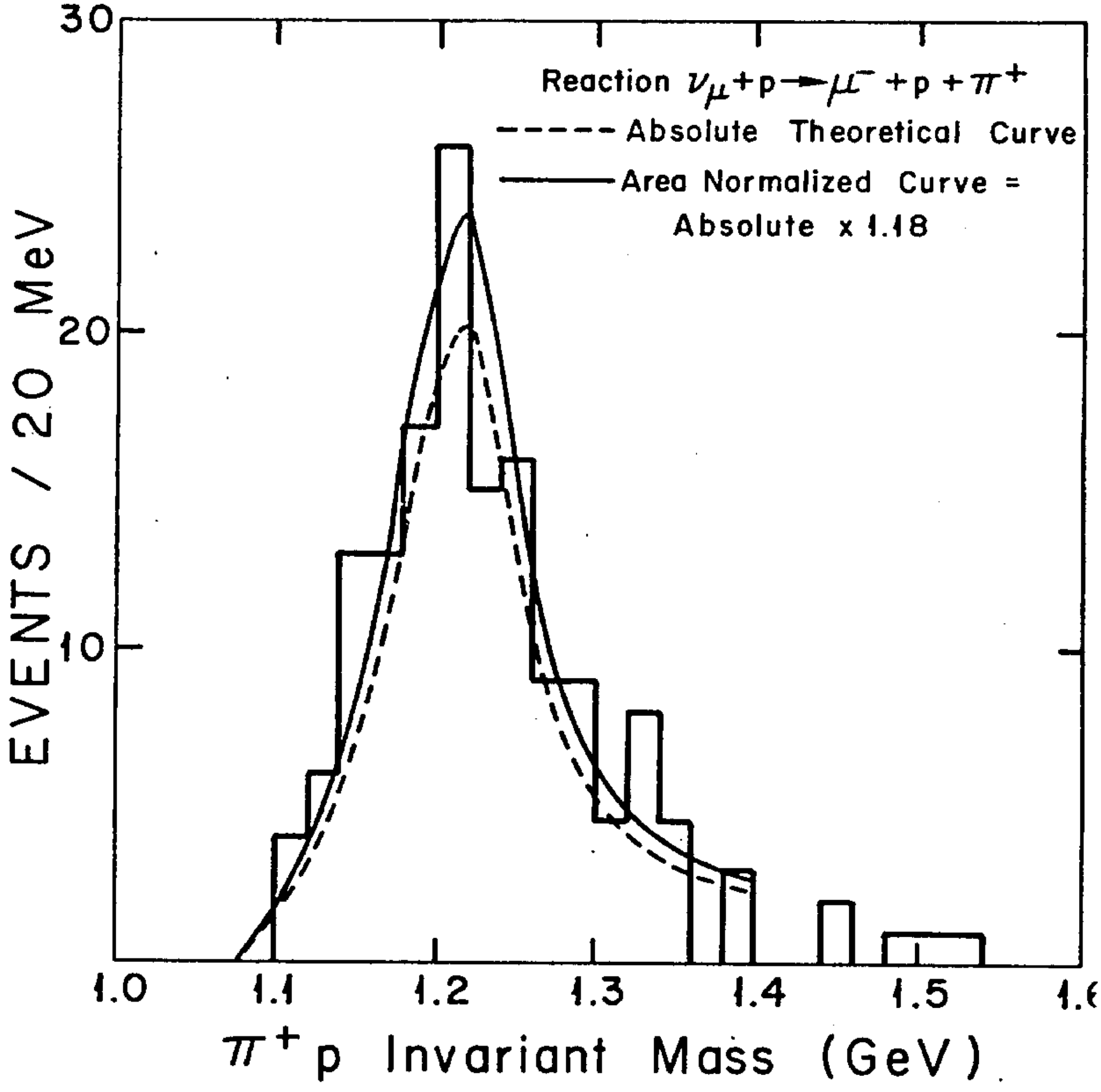
Reaction $\nu_\mu + p \rightarrow \mu^- + p + \pi^+$
Absolute Theoretical Curve
Area Normalized Curve = Absolute x 1.18
EVENTS / 20 MeV
$\pi^+ p$ Invariant Mass (GeV)
30
20
10
1.0
1.1
1.2
1.3
1.4
1.5

FIGURE 2.