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Do Results from e^+e^- Annihilation Experiments Rule Out Partons?

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ABSTRACT

Increase in $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ with the beam energy and non-scaling behavior of structure functions for the reactions $e^+e^- \to$ hadron + anything have been experimentally observed. We point out that these are not evidences against the basic concept of the parton model.

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Exciting information has recently became available from experiments with electron-positron colliding beams.[1] Among these results, we focus our attention to: (a) the rapid increase of $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ with the beam energy, (b) the indication that the structure functions, $M\overline{W}_1$ and $\nu\overline{W}_2$ for $e^+e^- \rightarrow$ hadron + anything scale for $\omega \gtrsim 0.5$ but they do not scale for $\omega \lesssim 0.5$. (The variables are defined in Fig. 1.) Corresponding predictions for the parton model are: (a) $R = \sum_i e_i^2$,[2] (b) $\nu\overline{W}_2$ and $M\overline{W}_1$ scales[3] for all ω. How do we interpret the parton model in view of these unsuccessful predictions for the annihilation processes, especially after it has enjoyed great deal of successes in the lepton nucleon deep inelastic scattering processes?

In previous publications it has been shown[4] that the usual parton model result for electron-proton deep inelastic scattering

$$\lim_{\substack{Q^2,\, \nu \rightarrow \infty \\ x = Q^2/2M\nu \text{ fixed}}} \nu W_2(Q^2, \nu) = \sum_i e_i^2\, x\, f_i(x) \qquad (1)$$

follows from very few assumptions.[5] They are: (i) Currents couple to partons. (ii) The probability for finding a parton inside a proton which carries large transverse momentum relative to the momentum of the proton is very small. (iii) The hadrons produced in the final state have transverse momentum cut off with respect to the virtual photon direction.

(iv) The multiplicity of hadrons $\langle n\rangle \ll \sqrt{\nu/M}$. The assumptions (i) and (ii) are the basic assumptions of any parton model. The assumption (iii) and (iv) have been experimentally verified.[6] Therefore (1) is on a very strong ground. In comparison, predictions for electron-positron annihilation process are much more model dependent. We will show that: (a) $\nu\bar{W}_2$ and $M\bar{W}_1$ scale for large Q^2 and ν only in the region $\omega Q/2 \gg 350$ MeV. (b) in the region where structure functions scale,

$$\sum_h \frac{1}{\sigma_\ell} \int_{\omega_c}^{1} \frac{d\sigma_h}{d\omega}\, d\omega \le 2 \sum_{ij} |e_i e_j| \tag{2}$$

$\omega_c = (1.5 \text{ GeV})\, 2/Q$, $\sigma_\ell = \dfrac{4\pi\alpha^2}{3Q^2}$, h stands for the observed hadron .
(c) the prediction $R = \Sigma\, e_i^2$ is strongly model dependent.

It was shown that the cross section for $e^+e^- \to h$ + anything is given by[7]

$$\frac{d\sigma_h}{d\omega} = \sigma_\ell\, \frac{\beta\omega^2}{2}(\sqrt{3}-\beta)^2 \sum_{ij\,\alpha} e_i e_j\, \frac{(2\pi)^3}{V} \int d^3k\, d^3k' z\, \delta\!\left(\frac{1}{\omega} - z - \sum_{\ell=1}^{n} \frac{m_\ell^2 + \bar{p}_{\ell\perp}^2}{2 y_i M\nu}\, z\right)$$

$$\times \langle j{:}k', q-k' | h(p), \alpha\rangle_{-}\ {}_{-}\langle h(p), \alpha | k, q-k{:}i\rangle \,. \tag{3}$$

Consider the matrix element ${}_{-}\langle h(p), \alpha | k, q-k{:}\, i\rangle$ in the limit Q^2, $\nu \to \infty$ while ω fixed. We make two assumptions on this matrix element. (A) The hadrons in the final state are such that their transverse momenta with respect to either k or q-k are limited. (i.e., $\bar{p}_{i\perp} \ll Q$). (B) The associated multiplicity $\langle n(p)\rangle$ is small compared to Q/M. ($\langle n(p)\rangle$ is a multiplicity of hadrons for events which contain a hadron with momentum p.)

With above assumptions, the direction of $\vec{k}$ and $\vec{q}-\vec{k}$ are fixed. $\vec{q}-\vec{k}$ is approximately parallel to $\vec{p}$.[8] Furthermore we have $\sum_{i=1}^{n} (m^2 + \bar{p}_{i\perp}^{\,2})/2y_i \ll M\nu$. Then

$$\frac{d\sigma_h}{d\omega} = \sigma_\ell \beta^3 \sum_{ij} e_i e_j\, \omega G_{ij}^h(\omega) \tag{4}$$

$$G_{ij}^h(\omega) = \frac{(2\pi)^3}{V} \int dz\, d^2k_\perp\, d^3k'\, \delta\left(\frac{1}{\omega} - z\right) \langle j:k', q-k' | b^+(p) b(p) | k, q-k:i \rangle \tag{5}$$

$b(p)$ is an annihilation operator for a hadron h. The matrix element is small unless $k_z = k_z'$ and $\vec{q}-\vec{k}$ is parallel to $\vec{p}$. Therefore, right hand side is a function of only ω. The lesson we want to stress here is that: In general k and k' are integrated over all phase space. Due to assumption (A) however, directions of k and k' are fixed. With assumption (B) energy conservation δ function becomes $\delta(\frac{1}{\omega} - z)$ and fixes the magnitude of k and k'. With these restrictions the prediction becomes that of the naive parton mo

(a) Scaling at finite value of Q^2 and ν

We point out that the result (4) obtained in the asymptotic limit may not hold in some kinematic region at present energies. In the derivation of (4), it was crucial to fix the direction of $\vec{k}$. If $\vec{p}$ gives a well-defined direction, the direction of $\vec{q}-\vec{k}$ and $\vec{q}-\vec{k}'$ were approximately parallel to $\vec{p}$. Let ϵ be the typical energy of the slow moving hadrons in the laboratory frame. Since slow hadrons are expected to have more or less isotropic distribution, the condition for the detected hadron to give a well defined direction is $p >> \epsilon$. If the detected hadron has a small energy in the laboratory frame $p < \epsilon$, then it does not fix a direction. In such a case, the parton state $|k, q-k; i>$, being an intermediate state, must be integrated over all phase space for $\vec{k}$. This may introduce Q dependence in $G_{ij}^{h}(\omega)$. and the result may deviate from that of the naive parton model.

With the condition $p >> \epsilon$, (4) holds for $\omega = 2p/Q >> 2\epsilon/Q$. In an experimental situation, if $\epsilon \approx 350$ MeV,[9] the detected hadron, with $p > 1.5$ GeV in the laboratory frame fixes the direction.[10] Let ω_c be the value of ω above which (4) should be valid. For energies available now or in the near future.[11]

$$Q/2 = 1.5,\ 2,\ 2.5,\ 4.5 \text{ GeV};$$
$$\omega_c = 1,\ 0.75,\ 0.6,\ 0.33 \tag{6}$$

respectively.

(b) Upper bound for $\frac{d\sigma_h}{d\omega}$.

By rescaling z component of momentum, we obtained a physical interpretation of $G_{ii}^{h}(\omega)$ which is valid to zeroth order in $1/P$ and $1/Q$. Consider a state $|k, q-k:i>$ and let $\vec{k}_i = (\eta Q, 0, \eta^2 P)$, $\vec{k}_{\bar{i}} = \vec{q} - \vec{k}_i = (0, 0, P)$. $G_{ii}(\omega)d\omega$ is a number of hadron h with z component momentum between ωP and $(\omega + d\omega)P$ that can be found in the state $|k, q-k:i>$. Using this physical interpretation,

$$\sum_{h} \int_{\omega_c}^{1} \omega G_{ii}^{h}(\omega) d\omega \leq 2 \tag{7}$$

Factor of 2 comes from the fact that momentum $\vec{k}$ can be assigned to either parton i or $\bar{i}$. Using Schwartz's inequality we obtain (2).

(c) R and its energy dependence.

R can be obtained from

$$R = \frac{3}{2} \int_{2M/Q}^{1} \beta \omega^2 \left[M\bar{W}_1 + \frac{\beta^2 \omega}{6} \nu W_2 \right] \tag{8}$$

It is clear that we cannot make a statement about R without further assumptions since it will require knowledge of $M\bar{W}_1$ and $\nu\bar{W}_2$ for $\omega < \omega_c$. In view of the new data,[1] a question arises whether the prediction $R = \sum_i e_i^2$ still holds if and when R reaches a constant value. It can be shown[4] that if the multiplicity of the final state hadron is much less than Q/M, the prediction follows from the parton model assumptions stated in this paper. If the multiplicity grows as Q/M, then it is quite possible that $R \neq \Sigma e_i^2$ even if it reaches as asymptotic value. Experimentally,[1] the multiplicity of hadrons is consistent with a linear rise with Q/M.

Figure 2 summarizes the difference between e-p deep-inelastic scattering and e^+e^- annihilation. The crucial difference between Fig. 2a and 2b is: In Fig. 2a, $\vec{p}$ and $\vec{q}$ give well defined direction for all partons. In Fig. 2b, direction of $\vec{k}$ is not yet defined. Only detection of a fast hadron (large ω) will fix the direction of $\vec{k}$. If slow hadrons are detected, we need further dynamical assumption. If we <u>assume</u> that the partons <u>always</u> "decay" into a well defined jets then scaling of $\nu\bar{W}_2$ follows even for small ω. One cannot, however, exclude the possibility that two partons "decay" into a fireball of slow hadrons without forming <u>any</u> fast hadrons. This will lead to scale breaking at small ω. In the deep-inelastic ep scattering similar possibility is excluded by experimental observations (i) and (ii) at least at present energies.[6]

We summarize some experimental checks that can be made to test the parton model at present range of Q^2. Values of ω_c for various energies are given in (6).

for $\omega > \omega_c$: (a) $\nu\bar{W}_2$ and $M\bar{W}_1$ scales, (b) $<n(\omega)> << Q/M$, (c) Eq. (2) should hold.

for $\omega < \omega_c$: (a) If R increases faster than or equal to log Q/M, $<n(\omega)>$ increases faster than or equal to $(Q/M)/\log Q/M$.[7] (b) $\frac{1}{\sigma_\ell}\frac{d\sigma_h}{d\omega}$ increases rapidly with Q at some ω.

FIGURE CAPTIONS

Fig. 1 $e^+e^- \rightarrow h$ + anything. A diagrammatic representation of (3).

Fig. 2 Comparison of e^+e^- annihilation and e-p deep inelastic scattering.

REFERENCES

[1] B. Richter, talk presented at APS Meeting, Feb. 4-6, 1974. A. Litke et al., Phys. Rev. Letters 30, 1189 (1973).

[2] R. P. Feynman, p. 163 Photon-Hadron Interactions, W.A. Benjamin, Inc., Reading Massachusetts. e_i is the charge of parton "i" in units of e.

[3] S. D. Drell, D. J. Levy and T.-M. Yan, Phys. Rev. D1, 1617 (1970).

[4] A. I. Sanda, Phys. Rev. D8, 4510 (1973), NAL-Pub-73/75-THY, Phys. Rev. ~~in press.~~ D9 1785, (1974)

[5] $f_i(x)\,dx$ = number of parton "i" with fraction of momentum between x and x + dx.

[6] L. Ahrens et al., Cornell preprint CLNS-255 (1973).

[7] A. I. Sanda, Phys Rev D9, 780(1974). A diagrammatic representation of (3) is shown in Fig. 1. β is the velocity of the detected hadron. Equation (3) is evaluated in the $P \to \infty$ frame where $\vec{p} = (0, 0, P)$, $\vec{p}_i = (p_{i\perp}, y_i P)$, $\vec{k} = (\vec{k}_\perp, \eta^2 zP)$, $\vec{q} = [\eta Q, 0, \frac{\nu\tau}{M} P - \frac{M\nu}{2P}(1-\tau)]$. $\bar{p}_{i\perp}$ = transverse momentum of $\vec{p}$ relative to the direction $\vec{k}$ or $\vec{q}-\vec{k}$. $\tau = 1 - \sqrt{1-(1+\eta^2)Q^2/\nu^2}$. η^2 is a free parameter which defines the boosted direction. To obtain (3) we chose $\eta^2 = 2\beta^2/(\sqrt{3}-\beta)^2$. $|k,q-k:i\rangle$ is a state which consists of parton i and antiparton $\bar{i}$ with momentum $\vec{k}$ and $\vec{q}-\vec{k}$ respectively. ${}_-\langle h(p), \alpha|$ is an out state consisting of the detected hadron with momentum p and collection of hadrons which make up the unobserved states. (3) was derived using the time ordered expansion of the matrix element. The energy conservation is not imposed on the intermediate parton states. The δ function arises from the energy conservation of the final-hadronic state.

[8] This is an arbitrary choice. It would lead to the same result had we chosen $\vec{p}$ to be parallel to $\vec{k}$.

[9] ϵ should be approximately equal to the transverse momentum cut off in hadronic reactions.

[10] The estimate $p \gtrsim 1.5$ GeV is obtained from experimental finding that the transverse momentum distribution of inclusive reaction $pp \to \pi$ + anything seems to deviate from simple exponential fall off starting from $p_\perp \approx 1.5$ GeV. See, for example, D.C. Carey, et al., Phys. Rev. Lett. 32, 24 (1974), F.W. Bussel, et al., Phys. Lett. 46B, 471 (1973). J.W. Cronin et al., Phys. Rev. Lett. 31, 1426 (1973).

[11] These numbers are calculated for pions.

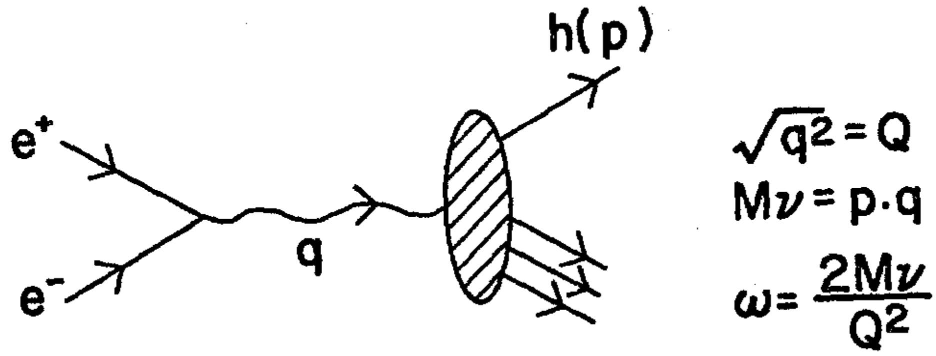


Figure 1

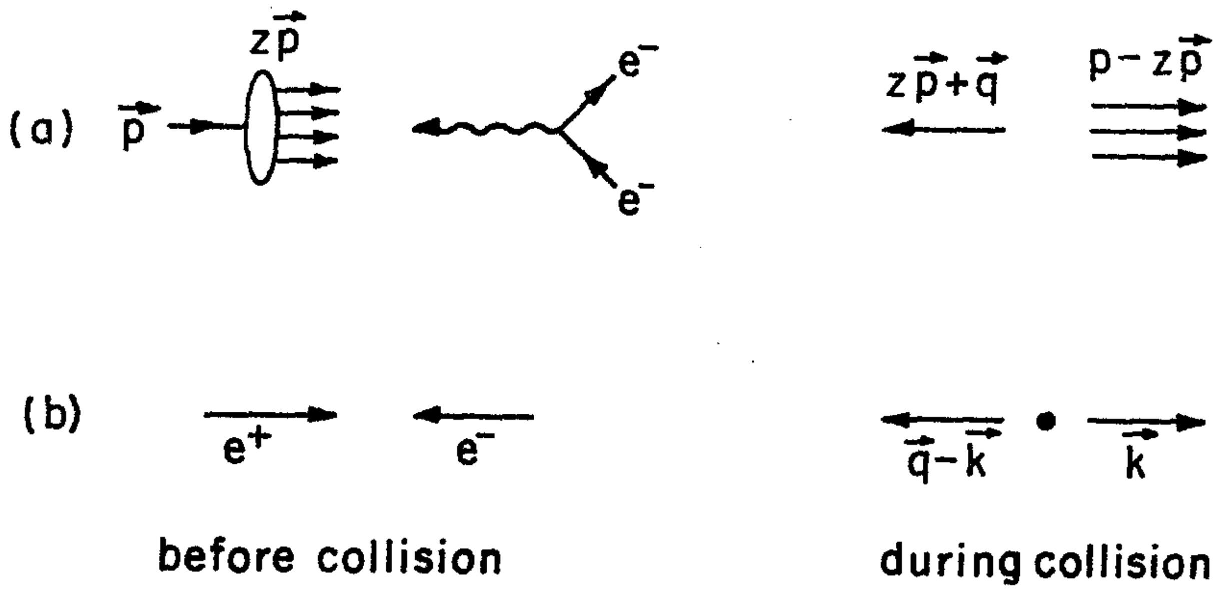
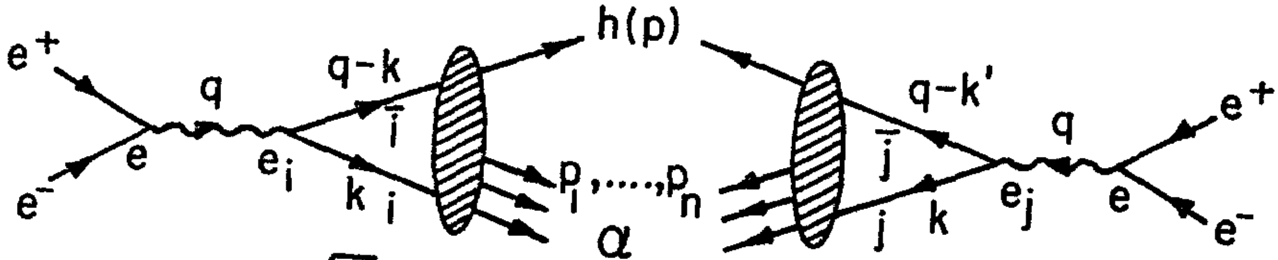

(a)
$\vec{p}$
$z\vec{p}$
e^-
e^-
$z\vec{p}+\vec{q}$
$p-z\vec{p}$
(b)
e^+
e^-
$\vec{q}-\vec{k}$
$\vec{k}$
before collision
during collision

Fig. 2



$$\sqrt{q^2} = Q, \quad M\nu = p\cdot q, \quad \omega = \frac{2M\nu}{Q^2}$$

Fig. 3