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Massive Quarks and The Quark-Parton Model

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ABSTRACT

The parton model result $\nu W_2(x) = \sum_i e_i^2 f_i(x)$ is derived for massive quarks in the case where partons are identified as quarks. A simple model of bound quark system is also given in order to illuminate the ideas behind the derivation.

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I. INTRODUCTION

In the parton model, the scaling of the structure functions for electron-proton deep inelastic scattering is interpreted in terms of point-like constituents of the target proton.[1] In this model,[2]

$$\lim_{\substack{Q^2, \nu \to \infty \\ x = \frac{Q^2}{2M\nu} \text{ fixed}}} \nu W_2(Q^2, \nu) \equiv \nu W_2(x) = \sum_i e_i^2 x f_i(x). \tag{1}$$

An interesting question is: Can partons be quarks? There are numerous predictions once we assume that partons are quarks (Q-P model). While it is too early to say that the experimental results support the Q-P model, it is quite impressive to see that not a single experimental result is inconsistent with the model.[3*] There is however one theoretical paradox in connection with the Q-P model. In the parton model,[1] partons must be seen as free point-like objects with small mass to derive (1). Then there is apriori no reason why the parton which was struck by the photon does not get out. It is our aim to give one possible solution to this paradox. We take the point of view that free quarks exist in Nature but they are very heavy and present accelerators do not have sufficient energies to produce them. Our result is that m_Q, the mass of a quark, plays the role of a new fundamental energy scale and unless the accelerator energies become comparable to m_Q, the physics does not depend on m_Q.[4]

Therefore $\nu W_2(x)$ scales and (1) is valid for $M\nu << m_Q^{\ 2}$. We also show in a potential scattering model, that the above derivation holds and that the effective mass of a quark in a bound system may be quite different from m_Q. Our assumptions are:

(a) There is a Hamiltonian $\mathscr{H} = H + \lambda V$ which governs the strong interaction dynamics. H and λV correspond to the free and the interaction parts of the Hamiltonian respectively. Partons are eigen states of H. Hadrons are eigen states of $\mathscr{H}$.

(b) $\mathscr{H}$ is such that in a collision at high energy, the transverse momenta of the final state hadrons, with respect to the beam direction are limited.

(c) The multiplicity $<n>$ of the hadrons does not increase as fast as Q.

In Sec. II, we derive (1) under the above assumptions in such a way that the derivation is independent of m_Q as long as $M\nu << m_Q^{\ 2}$. In Sec. III we give a potential scattering model which explicitly exhibits all the features mentioned above.

II. Q-P MODEL WITH MASSIVE QUARKS

The absorptive part of the forward Compton amplitude with virtual photon can be written as

$$\mathrm{Im}\, T_{\mu\nu} = (2\pi)^2 \int d^4x e^{-iq\cdot x} \frac{P}{M} \langle p | J_\mu^+(x) J_\nu(0) | p \rangle$$
$$= \frac{1}{M^2} (P_\mu - \frac{p\cdot q}{q^2} q_\mu)(P_\nu - \frac{p\cdot q}{q^2} q_\nu) W_2 - (g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) W_1 \,. \quad (2)$$

The variables are defined in Fig. 1.

The usual parton model results can be obtained if we suppose that the current $J_\mu(x)$ can be approximated by its free form[1]

$$J_\mu(x) = \bar{\psi}(x)\, \gamma_\mu \psi(x) \quad (3)$$

and that the masses of partons are negligible compared to Q. $\psi(x)$ is the field operator for partons. Once the free form of the current is assumed, the matrix element can be calculated by putting in the complete set of parton states in between two currents. Im $T_{\mu\nu}$ is diagrammatically shown in Fig. 2.

In the Q-P model, this diagram certainly suggests that some fractionally charged particles may be seen in the current fragmentation region. In our derivation we avoid (3) but note that at one time say $x_o = 0$, we can define

$$J_\mu(0, \vec{x}) = \bar{\psi}(0, \vec{x}) \gamma_\mu \psi(0, \vec{x}) \,. \quad (4)$$

We do not know the time dependence of the current since we do not know the exact form of $\mathscr{H}$. Using the complete set of hadron states,

$$1 = \sum_n |n \text{ out}\rangle\langle \text{out } n| ,$$

$$\mathrm{Im}\ T_{\mu\nu} = \sum_n (2\pi)^3 \int d^3x e^{i\vec{q}\cdot\vec{x}}\ \delta(p^0 + q^0 - \sum_n p_i^0)$$

$$\times \frac{P}{M} \langle p | J_\mu^+(0,\vec{x}) | n \text{ out}\rangle\langle \text{out } n | J_\nu(0) | p\rangle . \tag{5}$$

p_i are the four momenta of the hadrons in the final states. We evaluate this in the particular frame

$$p = (P + \frac{M^2}{2P}, 0, 0, P)$$

$$q = (\frac{M\nu}{P},\ Q, 0, 0)$$

$$p_i = (y_i P + \frac{M^2}{2y_i P},\ p_{i\perp}, y_i P) \tag{6}$$

P is a parameter which will be taken to be very large. In such a frame,

$$\int d^3x e^{-i\vec{q}\cdot\vec{x}} J_\mu(0,x) = \sum_i \int d^3k\ e_i \sqrt{\frac{m_Q^2}{(k+q)^0 k^0}}\ \bar{u}_i(k+q)\gamma_\mu u_i(k)\ a_i^+(k+q) a_i(k) \tag{7}$$

where i runs over the species of quarks. Introducing this current into (5), we can write (5) diagrammatically as shown in Fig. 3. In the frame specified by (6), the final hadrons are distributed as follows. Taking $k_z = zP$

$$\begin{aligned} &y_i \gg \frac{M}{Q};\ p_{\perp_i} \approx 0 && \text{target fragmentation} \\ &y_i \gg \frac{M}{Q};\ |p_{i_\perp} - \frac{y_i}{z} Q| \equiv \tilde{p}_{i_\perp} \approx 0 && \text{current fragmentation} \\ &y_i \approx \frac{M}{Q};\ p_{\perp_i} \approx 0 && \text{central plateau} \end{aligned} \tag{8}$$

The limits on the transverse momenta follow from assumption (b).

Let $p_1, \ldots, p_\sigma$ be the momenta of the hadrons in the target fragmentation region plus that of some hadrons in the central plateau region, $p_{\sigma+1}, \ldots, p_n$ be the momenta of the hadrons in the current fragmentation region plus that of some hadrons in the central plateau region, then[5]

$$\begin{aligned} \sum_{i=1}^{\sigma} p_i &\simeq p - k + 0\left(\frac{M}{Q}\right) \\ \sum_{i=\sigma+1}^{n} p_i &\simeq q + k - 0\left(\frac{M}{Q}\right) \end{aligned} \tag{9}$$

$$\begin{aligned} \sum_{i=1}^{\sigma} p_i^0 &= (1-z)P + \sum_{i=1}^{\sigma} \frac{M^2 + p_{i_\perp}}{2y_i P} \\ \sum_{i=\sigma+1}^{n} p_i^0 &= zP + \frac{Q^2}{2zP} + \sum_{i=\sigma+1}^{n} \frac{M^2 + \tilde{p}_{i_\perp}}{2y_i P} \; . \end{aligned} \tag{10}$$

W_2 can now be calculated.

$$W_2 = \frac{M}{P} \frac{(2\pi)^3}{V} \sum_n \sum_{ij} \int d^3k\, d^3k'\, e_i e_j \langle p \,|\, a^+(k')a(k'+q) \,|\, n \text{ out} \rangle$$

$$\langle \text{out } n \,|\, a^+(k+q)a(k) \,|\, p \rangle \quad \delta\left(\frac{M^2}{2P} + \frac{M\nu}{P} - \frac{Q^2}{2zP} - \sum_{i=1}^{n} \frac{M^2 + p_{i\perp}'^2}{2y_i P}\right) \tag{11}$$

$$p_{i_\perp}' = p_{i_\perp} \text{ for } i \le \sigma, \quad p_{i_\perp}' = \tilde{p}_{i_\perp} \text{ for } i \ge \sigma + 1.$$

The sum in the δ function can be estimated to be

$$\sum_{i=1}^{n} \frac{M^2 + p_{i\perp}'^2}{2y_i P} \le 0\left(\langle n \rangle \frac{QM}{P}\right). \tag{12}$$

This term is small if the multiplicity of the hadrons satisfies our assumption (c). Then, summing over n,

$$\nu W_2 = \sum_i e_i^2 \int P d z d^2 k_\perp \, \delta\left(z - \frac{Q^2}{2M\nu}\right) z \, \langle p \,|\, a^+(k) a(k) \,|\, p\rangle$$

$$= \sum_i e_i \, x \, f_i(x) \, . \tag{13}$$

Note that the discussion was given in terms of final state hadrons and in particular, that the parton mass never entered into our discussion. Therefore our derivation holds for arbitrary values of the quark mass. Of course, if we start producing quarks, (11) will depend on the quark mass and in particular when $Q \approx m_Q$, the sum can no longer be neglected and our argument fails.

The scaling limit is approached when the sum given in (12) is negligible. That is when $\frac{Q}{M} \gg \langle n \rangle$.

III. A SIMPLE MODEL

A parton bound in a nucleon feels an effective potential arising from the interactions with all the other partons in the nucleon. If there is an interaction between the partons by exchange of a scalar field, $\bar{\psi}\psi\phi$, it is quite natural to assume that the equation of motion for a parton bound in a nucleon is given by a Lagrangian of the form

$$L = \bar{\psi}\left(\not{\partial} - (m_Q - V_S)\right)\psi + \text{other terms} \, . \tag{14}$$

V_S is the scalar potential which originates from the exchanges of ϕ

between the parton and the remaining partons in the nucleon.[7] For simplicity we take V_S to be a square well

$$V_S = m_Q - m \quad \text{for } |x| \leq a$$
$$am >> 1$$
$$V_S = 0 \quad \text{for } |x| > a$$

and neglect other interaction terms. A Dirac equation can be derived from (14) and it can be easily solved. When the energy of the parton is $E << m_Q$,

$$|x| \leq a ; \psi_n(x) = \xi_{1,n} e^{ik_n \cdot x} u(k_n) + \xi_{2,n} e^{-ik_n \cdot x} u(-k_n)$$

$$|x| > a ; \psi_n(x) = 0(e^{-m_Q |x|}) \approx 0 \text{ for } m_Q a >> 1 . \tag{15}$$

When $E > m_Q$, there is a finite probability for detecting the quark. In the bound system, the quark has an effective mass m.

Suppose a photon with four momentum q, satisfying $m << q_o << m_Q$ hits the quark bound in the potential. There are two likely decay modes:

1) The momentum q acquired by the struck quark will be shared more or less evenly with other quarks in the system. This results in excitation of many hadronic levels. Depending on the excited states, this decay mode may result in hadrons with large transverse momenta.

2) When the quark is struck by the photon, the quark will pick

up an antiquark to form a hadronic bound state. The bound system now feels no potential. It will then escape the system carrying almost all of its original momentum $\vec{q}$. In this case, it results in sharp transversal momentum cut off for the final state hadrons. In accordance with assumption (c), we assume that decay mode (2) is dominant over (1). In this simplified potential scattering model, we keep only one term for the current matrix element.

$$\int e^{i\vec{q}\cdot\vec{x}} \left\langle i \left| J_\mu^+(0,\vec{x}) \right| n \right\rangle d^3x = e\int d^3k \sqrt{\frac{m^2}{(k+q)^0 k^0}}\; \bar{u}(k)\gamma\mu\, u(k+q) \left\langle i \left| a^+(k)a(k+q) \right| n \right\rangle \tag{16}$$

The structure functions for state i of the bound state (characterized by $k_i = \frac{(2i+1)\pi}{2a}$) are given by

$$\mathrm{Im}\, T_{\mu\nu} = \frac{1}{m^2}\left(k_{i\mu} - \frac{k_i\cdot q}{q^2} q_\mu\right)\left(k_{i\nu} - \frac{k_i\cdot q}{q^2} q_\nu\right) W_2^i(q^2, k_i\cdot q) - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1^i(q^2, k_i\cdot q)$$

$$= \sum_n (2\pi)^3 \frac{E_i}{m} \int d^3x\, e^{i\vec{q}\cdot\vec{x}}\, \delta(E_i + q^0 - E_n) \left\langle i \left| J_\mu^+(o,\vec{x}) \right| n \right\rangle \left\langle n \left| J_\nu(o) \right| i \right\rangle \tag{17}$$

The energy conserving δ function can be calculated.

$$\delta(E_i + q^0 - E_n) \simeq \delta\left(m + \nu - \sqrt{q^2 + m^2}\right)$$

$$= \frac{1}{m}\,\delta\left(1 - \frac{Q^2}{2m\nu}\right) \tag{18}$$

The δ function does not depend on n and thus

$$\mathrm{Im}\, T_{\mu\nu} = (2\pi)^3 \int d^3x\, e^{i\vec{q}\cdot\vec{x}}\, \frac{1}{m}\, \delta(1 - \frac{Q^2}{2m_\nu}) \left\langle i \left[J^+_\mu(0,\vec{x})\, J_\nu(0) \right| i \right\rangle$$

$$= \frac{1}{8m^2\nu}\, \delta(1 - \frac{Q^2}{2m\nu}) e^2 \int d^3k\, \delta^3(\vec{k})\, \mathrm{Tr}\left[(\not{k}+m)\gamma_\mu(\not{k}+\not{q}+m)\gamma_\nu \right] \tag{19}$$

where we have summed over initial spin states.

Identifying the tensor structure, we obtain

$$\nu W_2 = \frac{Q^2}{2m\nu}\, \delta(1 - \frac{Q^2}{2m\nu})\, e^2$$

$$2mW_1 = \delta(1 - \frac{Q^2}{2m\nu})\, e^2 \tag{20}$$

Thus we have the usual relation

$$\nu W_2 = 2mxW_1 \tag{21}$$

which is a characteristics of spin 1/2 constituent and its small effective mass.

Though the shape of the structure function is quite unreasonable, νW_2 for this model is exactly that expected from the physical interpretation given in (1) derived by the parton model.

ACKNOWLEDGEMENT

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REFERENCES

[1]J. D. Bjorken and E. A. Paschos, 185 1975 (1969), R. F. Feynman, Photon Hadron Interactions, (W. A. Benjamin Inc.) 1972.

[2]$f_i(s)$ is defined as follows: Suppose we observe the target proton in a fast moving reference frame where the proton has a large momentum P. $f_i(x)$ is the probability for finding the proton in various parton states which contains a parton "i" with momentum xP. e_i is the charge of the parton "i".

[3]E. A. Paschos, "Theoretical Interpretation of Neutrino Experiments," NAL-Conf-73/27-THY.

[4]This point has been speculated by many authors. For example, T. D. Lee, "Scaling Properties in Weak and Electromagnetic Processes," talk given at the "Neutrino '72" Conference, Balatonfured, Hungary (1972). K. Wilson Proceedings 1971 International Symposium on Electron and Photon Interactions at High Energies .

[5]For details see A. I. Sanda, "A Field Theory Formulation of the Parton Model," NAL-Pub-73/36-THY (1973), to be published in the Phys. Rev.

[6]We have used the anticommutation relation for a_k and the fact that $\langle p | a^+(k+q)\, a(k+q) | p \rangle$ is small. The probability for a proton to be seen in a parton state which contains a parton with large transversal momentum with respect to the proton momentum is assumed to be small.

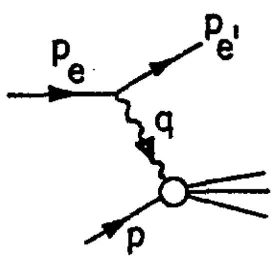
[7] We thank H. J. Lipkin for discussions. We follow H. J. Lipkin, "Quarks for Pedestrians," the Weizmann Institute for Science, Preprint WIS-7318-Ph. We avoid the negative energy solution by taking $am >> 1$.

FIGURE CAPTIONS

Fig. 1 The deep inelastic electron-proton scattering.

Fig. 2 The parton model diagram for the deep inelastic electron-proton scattering.

Fig. 3 The actual diagram for the deep inelastic electron-proton scattering.



$$\nu = \frac{p \cdot q}{M} \quad Q^2 = -q^2$$

$$x = \frac{Q^2}{2M\nu}$$

Figure 1

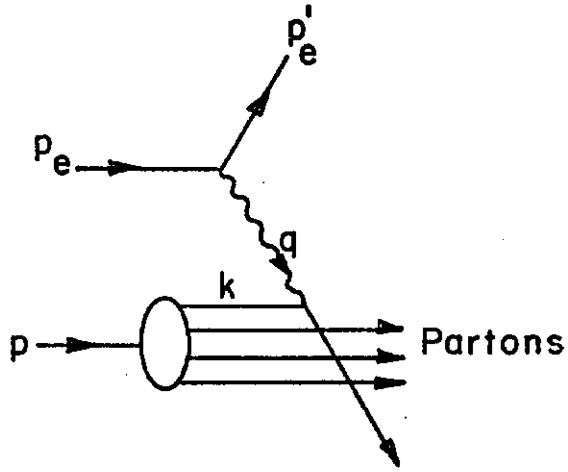


Figure 2

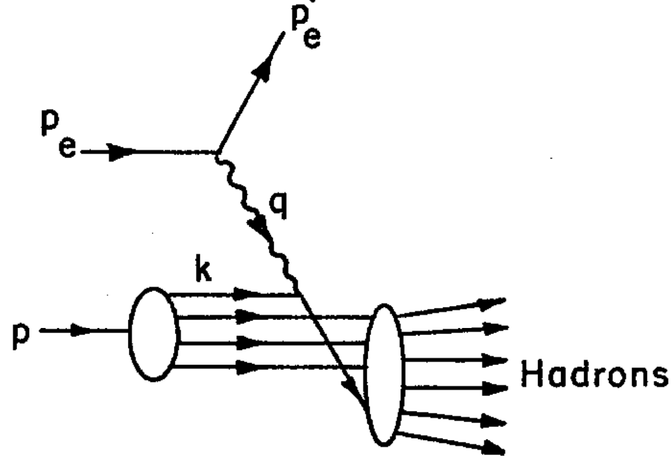


Figure 3