national accelerator laboratory

NAL-Pub-73/36-THY
May 1973

A Field Theory Formulation of the Parton Model

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ABSTRACT

A formulation of field theory given by Van Hove in 1955 is shown to be useful for putting "parton model like ideas" on firm theoretical grounds. This is done by discussing electron-proton deep inelastic scattering, electron-positron annihilation process and proton elastic form factor. A basic requirement that must be imposed on any field theory in order to discuss the concept of hadron constituent is given. To put the formulation on a firm ground (with respect to renormalizability) we assume that the wave function renormalization constants of the theory are finite. This assumption satisfies above mentioned basic requirements, though probably not necessary. Within this framework we prove $\nu W_2(q^2, \nu)$ is equal to that of the parton model. In this formalism, electron-positron annihilation process is quite different in

character compared to that of electron-proton deep inelastic scattering. Constancy of the ratio $\sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ can be derived only if much stronger assumptions than the one mentioned above is adopted. The price paid for simplicity in renormalization procedure is two physically undesirable results: (a) The proton elastic form factor is probably finite at large momentum transfer, (b) νW_2 is finite and non zero at $x = 1$. It is conjectured that these problems can be solved without changing other results by allowing infinity in wave function renormalization constants.

I. INTRODUCTION

Experiments studying the deep inelastic scattering $e + p \to e +$ anything performed by the MIT, SLAC collaboration[1] show scaling behavior for the structure functions νW_2 and W_1. That is, as Q^2, $\nu \to \infty$ with $x = \frac{Q^2}{2M\nu}$ fixed, νW_2 and W_1 are functions of x only. The variables are defined in Fig. 1. At first sight, the experimental result is not so startling. After all, nature has only four choices, either $\nu W_2 \to 0$, ∞, an oscillating function of Q^2, or a finite functions of x.

The result becomes more interesting when we try to understand it theoretically. So far, there are at least two distinct theoretical approaches: (a) The parton model.[2] In this model, the proton is seen as a super position of point like constituents named partons. In the deep inelastic limit it is assumed that the interaction of a parton with the photon can be isolated. Then, due to the point like nature of the partons, the inelastic proton form factor approaches a constant value as $Q^2 \to \infty$, x fixed. The possibility that $\nu W_2 \to \infty$ is avoided by a sharp cut off in the transverse momentum distribution of the partons. In this model, in addition to understanding scaling of νW_2, νW_2 can be related to the probability of finding a certain type of partons in a proton. (b) Field theory.[3] νW_2 is calculated in terms of a perturbative expansion of field theory. It was found that there are infinite sets of Feynman diagrams that violate scaling. That is, in the scaling limit, their contribution to νW_2 goes as $(\log \frac{Q^2}{M^2})^n$. Furthermore, certain sets of diagrams sum up to a term which goes to infinity as some power of $\frac{Q^2}{M^2}$.

The parton model gives the exciting possibility of observing constituents of the hadrons. We are, however, reluctant to accept it at the expense of giving up the field theoretic concept of the hadrons. This is especially true since many of the ideas incorporated in the parton model are borrowed from field theory. Independent of the parton model, if νW_2 continues to scale at higher energies, we must settle the question of whether scale breaking of νW_2 predicted by the perturbation theory is specific to the perturbation approach or it is a general property of the field theoretic approach.

The purpose of this paper is to point out that Van Hove's formulation of field theory coupled with some basic starting assumptions yields powerful tools for studying basic ideas on constituents of hadrons. We illustrate it by specific examples. We discuss the electron proton deep inelastic scattering, the electron-positron annihilation process, and the elastic proton form factor. We start out by discussing the requirements that must be imposed on our theory so that it has at least a fighting chance of becoming a framework in which the parton model can be understood.

The basic assumption of the parton model is the existence of probability function for finding n partons with momenta $k_1, \ldots, k_n$ as an intermediate state of a physical proton.

$$|\langle p|k_1,\dots,k_n\rangle'|^2 \equiv \sum_{m=n}^{\infty}\int \prod_{i=n+1}^{m} d^3k_i\,|\langle p|k_1,\dots,k_n,k_{n+1},\dots k_m\rangle|^2 \quad (1.1)$$

We have ignored the statistical factors. This is not a trivial assumption. Consider for example unrenormalized eigenstate of the full Hamiltonian corresponding to a proton (i.e. $|p\rangle = \frac{1}{\sqrt{N}}\,|p_{ur}\rangle$ where $N = \langle p_{ur}|p_{ur}\rangle$). Since parton states, the eigenstates of free Hamiltonian, form a complete set of states the norm $\langle p_{ur}|p_{ur}\rangle$ of the state is

$$N = \int \prod_{j=1}^{n} d^3k_j \sum_{m=n}^{\infty}\int \prod_{i=n+1}^{m} d^3k_i\,|\langle p_{ur}|k_1,\dots,k_n,k_{n+1},\dots,k_m\rangle|^2 \quad (1.2)$$

Suppose the phase space integral $\prod_{j=1}^{n} d^3k_j$ is divergent. Redefining

$$|\langle p|k_1,\dots,k_n\rangle'|^2 = \frac{1}{N}\sum_{m=n}^{\infty}\int \prod_{i=n+1}^{m} d^3k_i\,|\langle p_{ur}|k_1,\dots,k_m\rangle|^2$$

we see that $\langle p|p\rangle = 1$ but $|\langle p|k_1,\ \dots,\ k_n\rangle'|^2$ is zero for any given configuration.

Convergence of the phase space integral $\prod_{j=1}^{n} d^3k_j$ in (1.2) is a necessary condition which any field theory must satisfy if its elementary fields were to have physical meaning.[5] Even if $\prod_{j=1}^{n} d^3k_j$ integration (1.2) is finite, N could be either finite or infinite. The infinity may come from divergence of

$$N' = \sum_{m=n}^{\infty}\int \prod_{i=n+1}^{m} d^3k_i\,|\langle p_{ur}|k_1,\dots,k_m\rangle|^2 \quad (1.3)$$

In this paper we assume that wave function renormalization constants for any hadronic physical state is finite.[6,7] This assumes much more than convergence of phase space integrals $\prod_{j=1}^{n} d^3k_j$ in (1.2). It has, however, an advantage that it guarantees the renormalization proceedure of the theory to be simple. This is a great advantage since we are mainly interested in the formulation of the theory. Once the theory is seen to be in firm foundation, the starting assumptions can be loosened to obtain phenomenologically more desirable results.

It should be admitted, that our discussion of renormalization is incomplete and renormalizability, at this stage is still a conjecture. With finite wave function renormalization constant, however, this conjecture is a safe one.

We obtain the following results:

(a) In the scaling limit, we obtain[8,9]

$$\nu W_2(q^2, \nu) = \sum_i e_i^2 \, x \, f_i(x)$$

(b) Another application of the parton model is the calculation of the cross section for electron-positron annihilation into hadrons. The model predicts $R = \sigma(e^+e^- \to \text{hadron}) / \sigma(e^+e^- \to \mu^+\mu^-)$ to be constant at high energies.[10] Experimentally R is monotonically increasing with energy and at $s = 16\ \text{GeV}^2$, $R = 4.7 \pm 1.1$.[11] It is shown that our formalism and the starting assumption are not enough to derive this parton model result. This is an indication of the possibility that the parton model is successful in the analysis of νW_2 but not in the analysis of R. We will also give an assumption which enables us to obtain bound R by a constant.

(d) Within this formalism we argue that (i) the proton elastic form factor does not vanish at large momentum transfer, (ii) νW_2 is finite and nonzero at $x = 1$. These can be cured by relaxing the assumption that N is finite and take N', and thus N, to be infinite.

In Section II we point out the defects of perturbation theory which are corrected in Van Hove's formalism. The origin of the scale breaking terms in the perturbative calculation of νW_2 is explained. This is crucial in understanding why we do not have the same scale breaking effect in our formalism. In Sec. III we briefly describe Van Hove's results. In Sec. IV we discuss electron-proton deep inelastic scattering. In Sec. V we discuss the electron-positron annihilation process. In Sec. VI the proton form factor $F_1(q^2)$ is discussed. In Appendix A we supply some crucial ingrediants needed for deriving the parton model result for νW_2. In appendix B we define the proper vertex function needed to study renormalizability.

II. PERTURBATION THEORY

The major success of the quantum field theory is the perturbative approach to electrodynamics. The usual generalization of the theory to strong interaction is to obtain similar perturbation series as one encounters in QED and sum up the series. There is one serious problem to any calcultion along this line. Let us consider a field theory where the total Hamiltonian $\mathscr{H}$ can be decomposed into H and λV the free and interaction parts of the Hamiltonian respectively. At $t = -\infty$, when the projectile particles are traveling toward the target particles, the target and the projectile must form wave packets which are superpositions of eigen states of $\mathscr{H}$. The asymptotic states in the formalism must be constructed with eigen states of $\mathscr{H}$. Consider, for example, a perturbation calculation. When λ is small, the asymptotic state which is an eigen state of H is approximately an eigen state of $\mathscr{H}$ and the calculation can be performed consistantly. When λ is large however, there is nothing in the formalism which guaranties that the asymptotic states are eigen states of $\mathscr{H}$.[12] Therefore an obvious extension of the QED approach to a field theoretical calculation of hadronic effects is inadequate.

In particular, the comments made above on the perturbative calculation apply to the existing calculation of the structure functions νW_2.[3] To understand the perturbative approach, we follow the calculation of Chang and Fishbane.[3] In their Lagrangian, the strong interacting fields are a charged spinor (proton) and a neutral pseudoscalar (pion) coupled with[13]

$$\lambda V(x) = i\lambda \bar{\psi}(x)\gamma_5\psi(x)\phi(x) \tag{2.1}$$

ψ and ϕ corresponds to proton and pion fields respectively. The photon couples with proton and the leptons in the usual way. The simplest diagram which gives a scale breaking effect is shown in Fig. 2. The S matrix is obtained by squaring the contribution and integrating k over the allowed phase space. The phase space integral is of course finite due to the energy conservation. If we ignore the energy conservation between the initial electron and proton state and the final electron, proton and pion state, the phase space integral diverges logarithmically. The allowed phase space volume goes to ∞ as $Q^2 \to \infty$. Thus one obtains a result that the contribution of above diagram contains a term $\log Q^2/M^2$. Take M to be the proton mass. There are infinite sets of other graphs, but the origin of the scale breaking effect is unchanged. For the diagram shown in Fig. 2, the initial proton is an eigen state of H and if λ is large, we do not expect the answer to be reliable. Suppose there is a way to guarantee that the initial proton is an eigen state of $\mathscr{H}$. Then a new graph corresponding to this process is shown in Fig. 3. Note that a wave function for proton to be in a state of a bare pion and a bare proton replaces λ. It is possible that this wave function may have slight damping suppressing the part of phase space where the bare proton and pion carry large transversal momentum with respect to the physical proton momentum. Then the integral for such a process converges even without the energy conservation δ function and its contribution to νW_2 is finite in the limit $Q^2 \to \infty$. What is necessary in investagating the scaling behavior (or other strong interaction phenomena) in terms of the field theory is to formulate the theory in such a way that inital state is always an eigen state of $\mathscr{H}$. This is accomplished by the formalism discussed in the next section.

III. FORMALISM

A. Summary of Van Hove's Formalism

In this section we summarize the results of Ref. 4. The Hamiltonian $H + \lambda V$ contains time independent unperturbed part H and a time-independent perturbation term λV. The state $|\alpha\rangle$ is an eigen state of H

$$H|\alpha\rangle = \epsilon(\alpha)|\alpha\rangle$$

with eigen value $\epsilon(\alpha)$. We call the eigen states of H parton states. α denotes the collection of quantum numbers corresponding to that state. A state $|\alpha\rangle$ with n partons is normalized as

$$\langle\alpha|\alpha'\rangle = \prod_{i=1}^{n} \delta^3(k_i - k_i')\ \delta_{\alpha_i \alpha_i'} \tag{3.1}$$

where α_i denotes other discrete quantum numbers. When the exact form of the normalization is not necessary we write $\langle\alpha|\alpha'\rangle = \delta_{\alpha\alpha'}$ as a short hand to mean (3.1). Without loss of generality we take V to be nondiagonal. The resolvent operator is defined by

$$\begin{aligned} R_\ell &= (H + \lambda V - \ell)^{-1} \\ &= D_\ell^0 \sum_{n=0}^{\infty} (-\lambda V D_\ell^0)^n \end{aligned} \tag{3.2}$$

where $D_\ell^0 = (H - \ell)^{-1}$. It is also convenient to define two operators defined in the Hilbert space of parton states.

$$\begin{aligned} D_\ell &= \{R_\ell\}_d \\ G_\ell &= \{-V \sum_{n=1}^{\infty} (-\lambda D_\ell V)^n\}_{id} \end{aligned} \tag{3.3}$$

where $\{\ \}_d$ and $\{\ \}_{id}$ correspond to the "diagonal part" and "irreducibly diagonal part" of the operator $\{\ \}$ respectively. By diagonal part of an operator, say M, we mean the piece of $<\alpha'|M|\alpha>$ which is proportional to $\delta_{\alpha\alpha'}$. Note that in the S matrix language, $\{M\}_d$ is the "completely disconnected piece" of M. Let us illustrate what id means by an example. Suppose we want to calculate $<\alpha|G_\ell|\alpha> \equiv G_\ell(\alpha)$ the nth term of $G_\ell(\alpha)$ is

$$-\int \prod_{i=1}^{n} d\alpha_i (-\lambda)^n [<\alpha|VD_\ell|\alpha_1><\alpha_1|VD_\ell|\alpha_2>\ldots <\alpha_{n-1}|VD_\ell|\alpha_n><\alpha_n|V|\alpha>]_{id}$$

"id" means that portion of the diagonal part which is obtained when the intermediate states $|\alpha_1>, \ldots |\alpha_n>$ are kept different from each other and the states $|\alpha>$. With these definitions, two identities can be shown.

$$\begin{aligned} D_\ell &= (H-\ell-\lambda^2 G_\ell)^{-1} \\ R_\ell &= D_\ell + D_\ell \left\{ \sum_{n=1}^{\infty} (-\lambda V D_\ell)^n \right\}_{nd} \end{aligned} \tag{3.4}$$

where $\{\ \}_{nd}$ meaning "non diagonal part" is computed between the orthogonal states $<\alpha|$ and $|\alpha'>$ in such a way that the intermediate states $|\alpha_1> \ldots |\alpha_n>$ are kept different from each other or from $|\alpha>$ and $|\alpha'>$. The eigen state of the Hamiltonian can be obtained from the properties of R_ℓ. First note that

$$H + \lambda V = \frac{1}{2\pi} \int_{-\infty}^{\infty} E \,\mathrm{Abs}\, R_E dE \tag{3.6}$$

where

$$\text{Abs } R_E = \lim_{\epsilon \to 0} \left[R_{E+i\epsilon} - R_{E-i\epsilon} \right], \quad \epsilon > 0 \tag{3.7}$$

Using (3.5) it can easily be seen that

$$\begin{aligned} \text{Abs } R_E &= \text{Abs } D_E \\ &+ \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left\{ (D_{E+io} V)^m \text{ Abs } D_E (V D_{E-io})^{n-m} \right\}_{nd} \end{aligned} \tag{3.8}$$

with the notation $D_{E \pm io} = \lim_{\epsilon \to \infty} D_{E \pm i\epsilon}$. Now calculate Abs D_E.

From (3.4)

$$\langle \alpha | D_\ell | \alpha \rangle \equiv D_\ell(\alpha) = [\, \epsilon(\alpha) - \ell - \lambda^2 G_\ell(\alpha) \,]^{-1} . \tag{3.9}$$

Expanding at $\ell = E$

$$D_{E \pm i\epsilon}(\alpha) = \left[\epsilon(\alpha) - \lambda^2 G_E(\alpha) - E + \left(1 + \lambda^2 \left. \frac{\partial G_\ell(\alpha)}{\partial \ell} \right|_{\ell = E} \right) (\pm i\epsilon) \right]^{-1} \tag{3.10}$$

$$\text{Abs } D_E(\alpha) = 2\pi N(\alpha) \delta\left(\epsilon(\alpha) - E - \lambda^2 G_E(\alpha) \right) \tag{3.11}$$

where

$$N(\alpha) = \left(1 + \lambda^2 \left. \frac{\partial G_\ell(\alpha)}{\partial \ell} \right|_{\ell = E(\alpha)} \right)^{-1} \tag{3.12}$$

so,

$$\text{Abs } R_E = 2\pi N(\alpha)\left[1 + \sum_{n=1}^{\infty}\left\{(-\lambda D_{E+io}V)^n\right\}_{nd}\right]\delta(H - E - \lambda^2 G_E)$$

$$\left[1 + \sum_{n=1}^{\infty}\left\{(-\lambda V D_{E-io})^n\right\}_{nd}\right] \tag{3.13}$$

It can be shown that D_{E+io} and D_{E-io} can be replaced by $D_{E\pm io}$ and $D_{E\mp io}$ respectively. Putting in complete set of state and defining

$$|\alpha>_\pm = N(\alpha)^{\frac{1}{2}}\left[1 + \sum_{n=1}^{\infty}\left\{(-\lambda D_{E(\alpha)\pm io}V)^n\right\}_{nd}\right]|\alpha> \tag{3.14}$$
$$\equiv O_{E(\alpha)\pm io}|\alpha>$$

we obtain

$$\text{Abs } R_E = 2\pi\int |\alpha>_{\pm\,\pm}<\alpha|\ \delta(E-E(\alpha))\ d\alpha \tag{3.15}$$

where

$$E(\alpha) = \epsilon(\alpha) - \lambda^2 G_{E(\alpha)}(\alpha). \tag{3.16}$$

Then $H+\lambda V = \int |\alpha>_{\pm\,\pm}<\alpha|\ E(\alpha)\, d\alpha$. Using properties of the operator Abs R_E, the orthogonality of states ${}_\pm<\alpha|\alpha'>_\pm = \delta_{\alpha\alpha'}$, can be shown. Thus using (3.16) we have

$$(H + \lambda V)|\alpha>_\pm = E(\alpha)|\alpha>_\pm. \tag{3.17}$$

Therefore $|\alpha>_+$ ($|\alpha>_-$) form a complete orthogonal eigen states of $\mathscr{H}$.

Define

$$|\alpha>_{as} = N(\alpha)\left[1+\sum_{n=1}^{\infty}\left\{(-\lambda\ Y_{\alpha} D_{E(\alpha)\pm i0}\ V)^{n}\right\}_{nd}\right]|\alpha> \quad (3.18)$$

where Y_{α} is a projection operator which eliminates all intermediate states that are responsible for interactions between partons in state α. Only intermediate states responsible for self interactions of partons survive. Defining

$$\varphi(t) = \int c_{\pm}(\alpha)|\alpha>_{\pm} \exp\ [-it\ E(\alpha)]\ d\alpha \quad (3.19)$$

it is shown that

$$\lim_{t\to\mp\infty}\left[\varphi(t) - \int c_{\pm}(\alpha)\ |\alpha>_{as} \exp\ [-it\ E(\alpha)]\ d\alpha\right] = 0 \quad (3.20)$$

Since $c_{\pm}(\alpha)$ is arbitrary, there is one to one correspondance between $|\alpha>_{\pm}$ and $|\alpha>_{as}$. It is important to note that energy associate with $|\alpha>_{as}$ is $E(\alpha)$, the eigen value of $|\alpha>_{\pm}$.

Suppose $|\alpha> = |k_1, \ldots, k_n>$ then since $E(\alpha)$ is the energy of $|\alpha>_{as}$,

$$E(\alpha) = \sum_{i=1}^{n} E(k_i), \quad \epsilon(\alpha) = \sum_{i=1}^{n} \epsilon(k_i) \quad (3.21)$$

where $E(k_i)$ is an energy of a free physical particle with momentum k_i. $\epsilon(k_i)$ is free energy of bare particle with momentum k_i. Thus we can write

$$G_{E(\alpha)}(\alpha) = \sum_{i=1}^{n} G_{E(k_i)}(k_i)$$
$$E(k_i) = \epsilon(k_i) - \lambda^2 G_{E(k_i)}(k_i). \tag{3.22}$$

For large k_i we obtain

$$\lambda^2 G_{E(k_i)}(k_i) = \frac{m_o^2 - M^2}{2k_i} \tag{3.23}$$

where m_o and M are bare and physical masses respectively. $G_{E(\alpha)}(\alpha)$ is, therefore related to the mass shift of state α.

S-matrix.

Consider the wave packet

$$|\varphi_{1,2}(t)\rangle_{\pm} = \int c_{1,2}(\alpha)\ |\alpha\rangle_{\pm} \exp[-it\,E(\alpha)]\, d\alpha. \tag{3.24}$$

Starting from the definition of S matrix it is shown that

$$\lim_{\substack{t_1 \to \infty \\ t_2 \to -\infty}} {}_{+}\langle \varphi_1(t_1)|S|\varphi_2(t_2)\rangle_{+}$$
$$\equiv \int c_1^*(\alpha)\ {}_{+}\langle\alpha|S|\alpha'\rangle_{+}\ c_2(\alpha')\,d\alpha\, d\alpha' \tag{3.25}$$

where ${}_{+}\langle\alpha|S|\alpha'\rangle_{+} = \delta(\alpha-\alpha') - 2\pi i\lambda\ \delta(E(\alpha)-E(\alpha'))\ [N(\alpha)N(\alpha')]^{\frac{1}{2}}$

$$\langle\alpha|V-\lambda\{VR_{E(\alpha)+io}V\}_{nd}|\alpha'\rangle$$

Once we obtain $<\alpha | S | \alpha'>$ we can obtain the S matrix for the wave packet.

B. Renormalization

Expression for $G_\ell(\alpha)$ is in general divergent and thus $D_\ell(\alpha)$, as it stands, is not well defined. Using (3.20) we write

$$D_\ell(\alpha) = [(E(\alpha)-\ell)(1 + \lambda^2 \frac{G_{E(\alpha)}(\alpha) - G_\ell(\alpha)}{E(\alpha) - \ell})]^{-1} \tag{3.26}$$

Since $N(\alpha)$ is finite by assumption, $G_\ell(\alpha)$ is at most logarithmically divergent and thus (3.26) is well defined. In conventional perturbation expansion, the wave function renormalization constant together with the vertex renormalization constant get absorbed in redefinition of coupling constant. In Appendix B we define the proper vertex function Γ. We also show that the S matrix as well as the wave function of hadronic states can be written in terms of $\lambda\Gamma$ and D. As it is discussed above D does not contain any infinity. Suppose Γ is infinite. In this paper we conjecture that $\Gamma = Z\tilde{\Gamma}$, where $\tilde{\Gamma}$ is well defined. Then infinite Z can be absorbed in redefinition of λ.

C. Definition of the Proton State

For definiteness let us consider bare proton and bare pion fields to be the eigen states of the free Hamiltonian H. The perturbation term is given by (2.1). Let $|p>$ denote the eigen state of H, a bare proton state. The eigen state of $H + \lambda V$ can be obtained by the operation

$$|p>_{as} = |p>_{+} = |p>_{-} = O_{E(p)\pm io} |p> \qquad (3.27)$$

where $O_{E(p)\pm io}$ is defined by (3.14). We define the state $|p>_{\pm}$ to be the proton state. It corresponds to a bare proton surrounded by a cloud of pions and proton antiproton pairs. It should be constructed with a physical proton state in a conventional field theory.

$$\psi(t) = U(t, -\infty)\, \psi_{free}(-\infty). \qquad (3.28)$$

Asymptotically, therefore, the proton becomes bare.[14] From (3.14) we see that

$$_{\pm}<p' | p> = N(p)^{\frac{1}{2}}\ \delta^{3}(p-p') \qquad (3.29)$$

This is the amplitude for finding a bare proton, and nothing else, in a physical proton. This number is zero if N' [defined in (1.2)] is infinite and finite if N' is finite. In this paper, we treat only the case of finite N'.

C. Transversal Momentum Damping

In the introduction, we saw that N must be finite in order for a theory to have any chance of reproducing parton model results. This restricts large transversal momentum behavior of ${}_{\pm}\langle p \mid k_1, \ldots k_n \rangle$ '. When $k_{j\perp} \to \infty$ keeping $\sum_{i=1}^{n} k_i = p$, we expect[15]

$$\left| {}_{\pm}\langle p \mid k_1, \ldots, k_n \rangle \right|^2 \leq O[(\frac{M}{k_{j\perp}})^{2+\epsilon}] . \tag{3.30}$$

IV. ELECTRON PROTON SCATTERING

In this section we calculate the S matrix for e + p → e + anything. As usual we treat the electromagnetic interaction only to order α, but we treat the strong interaction to arbitrary orders in λ. We deonte $|p, e>$ to be the bare proton and electron state. Denoting $E(p, e)$ as the incident energy we have

$$|p, e>_{+} = O_{E(p, e)+io} |p, e> \tag{4.1}$$

as our initial state of a physical proton and an electron. Similarly we denote $|\alpha, e'>$ as a state with bare hadrons and an electron. The state

$${}_{-}<\alpha, e'| = <\alpha, e'|O^{+}_{E(\alpha, e')-io} \tag{4.2}$$

is the physical final state corresponding to the state $|\alpha, e'>$ where $E(\alpha, e')$ is the energy of the final state. The interaction Hamiltonian of the system is

$$V(x) = i\bar{\psi}(x)\gamma_5\psi(x)\phi(x) - \frac{ie}{\lambda}\bar{\psi}(x)\gamma_\mu\psi(x)A^\mu(x) - \frac{ie}{\lambda}\bar{\psi}_e(x)\gamma_\mu\psi_e(x)A^\mu(x) \tag{4.3}$$

where A_μ is the photon field, ψ_e is the electron field, ψ and ϕ are charged spinor and neutral scalar fields respectively. We take these couplings for definiteness but the arguments are not restrictive to these forms of interactions. The S matrix for the reaction e + p → e + anything can be obtained once we know

$$<\alpha, e' | S | pe> = -2\pi i\ [N(p,e)\ N(\alpha, e')]^{\frac{1}{2}}\ \delta(E(p,e) - E(\alpha, e'))$$

$$<\alpha, e' | \lambda V - \lambda^2 \{ VR_{E(p,e)+io} V \}_{nd} | p, e> \qquad (4.5)$$

$$\lambda V - \lambda^2 \{ VR_{\ell} V \}_{nd} = \lambda V - \lambda^2 \{ VD_{\ell} V \}_{nd} + \lambda^3 \{ VD_{\ell} VD_{\ell} V \}_{nd} - \ldots \qquad (4.6)$$

Substituting (4.4) into (4.6) and keeping only the term proportional to e^2, we obtain the single photon exchange contribution.

$$\{ \lambda V - \lambda^2 (VR_{\ell} V)_{nd} \}_{\text{single photon exchange}} \qquad (4.7)$$

$$= \{ (1 + [\sum_{n=1}^{\infty} (-\lambda VD_{\ell})^n]_{nd}) (-eV_{em}) (1 + [\sum_{n=1}^{\infty} (-\lambda D_{\ell} V)^n]_{nd})$$

$$(-eD_{\ell} V_{em}) (1 + [\sum_{n=1}^{\infty} (-\lambda D_{\ell} V)^n]_{nd} \}_{nd}$$

$$<\alpha, e' | S | p, e> = -2\pi i [N(p,e) N(\alpha, e')]^{\frac{1}{2}}\ \delta(E(p,e) - E(\alpha, e'))$$

$$<\alpha, e' | O^{+}_{E(p,e)-io} (-eV_{em}) R_{E(p,e)+io} (-eV_{em}) O_{E(p,e)+io} | p, e> \qquad (4.8)$$

Diagramatically (4.8) corresponds to two terms shown in Fig. 4. With some algebra we obtain

$$<\alpha e' | S | pe> = -2\pi i [N(p,e) N(\alpha, e')]^{\frac{1}{2}}\ \delta(E(p) + E_e - E(\alpha') - E_{e'})$$

$$\{ \int \prod_{i=1}^{n} d^3 k'_i \prod_{j=1}^{m} d^3 k_j d^3 q <\alpha' | O^{+}_{E(p)-io} | k'_1, \ldots, k'_n>$$

$$<k'_1, \ldots, k'_n | (-eV_{em}) | k_1, \ldots, k_m, q> <k_1, \ldots, k_m | R_{E(p)+io} O_{E(p)+io} | p> \qquad (4.9)$$

$$<qe' | (-eV_{em}) | e> + \text{term representing Fig. (4.b)} \}$$

We have used the following points. To the lowest order in e, the incident energy $E(p,e) = E(p) + E_e$, the final energy $E(\alpha, e') = E(\alpha) + E_{e'}$. $q_o = E_e - E_{e'}$, $O_{E(p,e)+io}|p,e\rangle = O_{E(p)+io}|p\rangle|e\rangle$ and $\langle a'e'|O^+_{E(p,e)-io} = \langle\alpha' e'|O^+_{E(\alpha,e')-io} = \langle e'|\langle\alpha'|O^+_{E(\alpha)-io}$. The last two equalities can be checked by explicitly writing down the first few terms of $O_{E(p,e)}|p,e\rangle$ and $\langle\alpha,e'|O^+_{E(\alpha,e')}$. With tedius but trivial algebra we obtain

$$E_{e'}\frac{d^3\sigma}{d^3p_{e'}} = \frac{\alpha^2}{2q^4E_e} \mathrm{Tr}\,(\not p_e\gamma_\mu \not p_{e'}\gamma_\nu)\,\mathrm{Im}\,T\mu\nu \tag{4.11}$$

$$\mathrm{Im}\,T_{\mu\nu} = \int d\alpha(2\pi)^6\delta^4(p+q-p_\alpha)\frac{P}{M}\ {}_+\langle p|J^+_\mu(0)|\alpha\rangle_-\ {}_-\langle\alpha|J_\nu(0)|p\rangle_+$$

where

$$\begin{aligned} {}_-\langle\alpha|\int d^3x\, e^{i\vec{q}\cdot\vec{x}} J_\mu(o,x)|p\rangle_+ &\equiv \int d^3k\, d\gamma\Big(\sum_{i\in\gamma}(e_i)\,\delta_{kk_i}\Big)\sqrt{\frac{m^2}{(k+q)^o k^o}}\;\bar u(k+q)\gamma_\mu u(k) \\ &\qquad {}_-\langle\alpha|a^+(k+q)a(k)|\gamma\rangle\ \langle\gamma|p\rangle_+ \end{aligned} \tag{4.12}$$

p_α is the total four momentum of state $|\alpha\rangle$. e_i is the charge of i th parton in the unites of e, m is the parton mass. Define

$$\mathrm{Im}\,T_{\mu\nu} \equiv \frac{1}{M^2}\Big(p_\mu - \frac{p\cdot q}{q^2}q_\mu\Big)\Big(p_\nu - \frac{p\cdot q}{q^2}q_\nu\Big)W_2 - \Big(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\Big)W_1 \tag{4.13}$$

It is convenient to evaluate W_2 in the $P \to \infty$ frame.

$$p = (P + \frac{M^2}{2P}, \ 0, \ 0, \ P)$$
$$q = (\frac{M\nu}{P}, \qquad \vec{Q}, \ 0) \tag{4.14}$$

In this frame, setting $\nu = \mu = 0$,

$$W_2 = \int d\alpha d\gamma_1 d\gamma_2 d^3k d^3k' (\sum_{i\in\gamma_1} e_i \delta_{k,k_i}) (\sum_{i'\in\gamma_2} e_{i'} \delta_{k',k_{i'}}) \delta(E(p) + q^0 - E(\alpha))$$
$$\frac{M}{P}\frac{(2\pi)^3}{V} {}_{+}\langle p \,|\, \gamma_2\rangle \langle\gamma_2 \,|\, a^+(k')a(k'+q) \,|\, \alpha\rangle_{-} \, {}_{-}\langle\alpha \,|\, a^+(k+q)a(k) \,|\, \gamma_1\rangle \langle\gamma_1 \,|\, p\rangle_{+} \tag{4.15}$$

The matrix element

$$\int d\gamma_1 d\gamma_3 \, {}_{-}\langle\alpha \,|\, \gamma_3\rangle \langle\gamma_3 \,|\, a^+(k+q)a(k) \,|\, \gamma_1\rangle \langle\gamma_1 \,|\, p\rangle_{+} \tag{4.16}$$

is shown diagramatically in Fig. 5. γ_1 and γ_3 are states of free partons and $|p>_+$ and $|\alpha>_-$ are a physical proton state and a physical hadronic state respectively. Figure 5 is very similar to the deep inelastic scattering diagram in the parton model. There is an important difference. In particular, final state interaction is neglected.[17]

To proceed with (4.15) we must understand $E(\alpha\ )$. In Appendix A we show that

$$E(\alpha) = P + \Delta E_{wee} + \frac{Q^2}{2zP} + \sum_{j=1}^{v} \frac{m^2 + \ell_{j\perp}^2}{2y_j P} + \sum_{j=v+1}^{n} \frac{m^2 + \tilde{\ell}_{j\perp}^2}{2y_j P} \qquad (4.17)$$

where we have denoted $|\alpha\ >_- = |\ell_1, \ldots, \ell_n; (\mathrm{wee})>_-$, $\ell_j = (y_j P, \vec{\ell}_{j\perp})$. The specified momenta correspond to the non wee hadron momenta and (wee) corresponds to a collection of wee hadron momenta. $\vec{\tilde{\ell}}_{j\perp} = \vec{\ell}_j - \frac{y_j}{z}\vec{Q}$ Both $\ell_{j\perp}$ for $1 \le j \le v$ and $\tilde{\ell}_{j\perp}$, for $v+1 \le j \le n$ are bounded by some constant ξ.

$$\Delta E_{wee} = E_{wee}(\alpha) - (\Sigma \ell_z)_{wee} \qquad (4.18)$$

This is due to the presence of wee hadrons. Define

$$\vec{\ell}'_{j\perp} = \begin{cases} \vec{\ell}_{j\perp} & \text{for } 1 \le j \le v \\ \vec{\tilde{\ell}}_{j\perp} & \text{for } v+1 \le j \le n \end{cases} \qquad (4.19)$$

$$\delta(E(p)+q^o-E(\alpha)) = \delta(\Delta E_{wee} + \frac{M^2}{2P} + \frac{M\nu}{P} - \frac{Q^2}{2zP} - \sum_{j=1}^{n} \frac{m^2+\ell'^2_{j\perp}}{2y_j P}) \quad (4.20)$$

Note that if $|\alpha>_-$ state contained a wee hadron, $\Delta E_{wee} = O(m)$ the energy conservation does not allow any wee hadrons in the final state. Restricting $|\alpha>_-$ sum to only those states without wee hadrons, we have[18]

$$\nu W_2 = \int_{nw} d\alpha \int d\gamma_1 d\gamma_2 d^3k d^3k' (\sum_{i\epsilon\gamma_1} e_i \, \delta_{kk_i}) (\sum_{i'\epsilon\gamma_2} e_{i'} \delta_{k'k_{i'}}) \, z\,\delta(z-x)$$
$$\frac{(2\pi)^3}{V} \; {}_+\!<p|\gamma_2><\gamma_2|a^+(k')a(k'+q)|\alpha>_- \; {}_-\!<\alpha|a^+(k+q)a(k)|\gamma_1><\gamma_1|p>_+ \quad (4.21)$$

We have droped the last term in the argument of the δ function given in (4.20). The region $y_j = O(m/\nu)$ requires some care. Note that a vector $(\sqrt{m^2+p^2}, \, p\sin\theta, -p\cos\theta)$ in the laboratory frame has a z component in the infinite momentum frame $(-p\cos\theta + \sqrt{p^2+m^2})P/m$. The only way for y_j to be $O(m/\nu)$ is to have $p = O(\nu)$ and $\theta = O(m/\nu)$. Such a vector in the laboratory frame has a trnsversal momentum Q with respect the photon direction. If the differential cross section is smooth in this kinematical region, the region $y_j = O(m/\nu)$ gives negligible contribution to the total cross section. Thus we need not consider the possibility $y_j = O(m/\nu)$. Also, transforming a wee vector to the laboratory frame we can convince our selves that the amplitude may strongly suppress the wee region. Thus nw sign can be droped. Then,

$$\nu W_2 = \int d\gamma \, d^3k \, z\delta(z-x) d\gamma (\sum_{i\epsilon\gamma} e_i^2 \, \delta_{kk_i}) \frac{(2\pi)^3}{V} \; {}_+\!<p|a^+(k)a(k)|\gamma><\gamma|p>_+ \quad (4.22)$$

We have used the commutation relation $\{a(k+q), a^+(k'+q)\} = \delta^3(k-k')$ and neglected a term proportional to $_+\langle p|a^+(k)a(k)a^+(k+q)a(k+q)|p\rangle_+$. In the parton model notation,[8]

$$\nu W_2 = \sum_i e_i^2\, x\, f_i(x) \tag{4.23}$$

This is the parton model result.

V. ELECTRON-POSITRON ANNIHILATION

Treating the electromagnetic interaction to only second order, the S matrix for $e^+e^- \to$ hadrons is given by

$$\langle\alpha|S|e^+e^-\rangle = -2\pi ie\ \delta(E_{e^+} + E_{e^-} - E(\alpha)) \tag{5.1}$$
$$\sum_i \int d^3k_1 d^3k_2\ e_i \langle\alpha|O^+_{E(\alpha)-io}|i;k_1k_2\rangle\ \langle k_1k_2;i|V_{em}R_{E(\alpha)+io}V_{em}|e^+e^-\rangle$$

where E_{e^-} and E_{e^+} are energies of electron and positron, respectively. V_{em} is the usual electromagnetic coupling. $|i;k_1k_2\rangle$ stands for the two parton state with momentum k_1 and k_2 and i denotes the kind of parton with charge e_i. $O_{E(\alpha)-io}|\alpha\rangle$ is the physical hadron state. This S matrix is seen in Fig. 7.

$$\sigma_T(e^+e^-\to \text{hadrons}) = \frac{\alpha^2}{2q^4}\ \mathrm{Tr}(\not{p}_1\gamma_\mu\not{p}_2\gamma_\nu)\ (-q^{\mu\nu} + \frac{q^\mu q^\nu}{q^2})\Pi(q^2) \tag{5.2}$$

p_1 and p_2 are four vectors of electron and positron, respectively.

$$-(q_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})\Pi(q^2) = \int d\alpha(2\pi)^6\ \delta^4(q-p_\alpha)\ \langle 0|J^+_\mu(0)|\alpha\rangle_{-\ -}\langle\alpha|J_\nu(0)|0\rangle \tag{5.3}$$

where

$$_-\langle\alpha|\int d^3x\ e^{iqx}J_\mu(x)|0\rangle = \sum_i \int d^3k\ e_i \sqrt{\frac{m^2}{k^o(q-k)^o}}\ \bar{u}(k)\gamma_\mu v(q-k)\ \ _-\langle\alpha|i;k,q-k\rangle$$

p_α is the sum of four momenta for the state $_-\langle\alpha|$.

Denote $|\alpha>_- = |\ell_1, \ldots, \ell_n; \text{(wee)}>_-$ $\quad \ell_j = (y_j P, \vec{\ell}_{j\perp})$, $\vec{\ell}_j$ denotes non wee hadron momenta and "wee" denotes other wee hadron momenta.

$$
\begin{aligned}
p_1 &= (P + q^2/8P,\ 0,\ \sqrt{q^2}/2,\ P) \\
p_2 &= (P + q^2/8P,\ 0,\ -\sqrt{q^2}/2,\ P) \\
\ell_i &= (y_i P + \frac{m^2 + \ell_{i\perp}^2}{2y_i P},\ \vec{\ell}_{i\perp},\ y_i P) \\
k_i &= (x_i P + \frac{m_e^2 + k^2}{2x_i P},\ \vec{k}_{i\perp},\ x_i P)
\end{aligned}
\tag{5.4}
$$

$$
\Pi(q^2) = \int d\alpha \frac{d^3k d^3k' (2\pi)^3}{P^2 4 \quad V} \delta(E_{e^+} + E_{e^-} - E(\alpha)) \sum_{ij} e_i e_j <k, q-k; i|\alpha>_{-\,-}<\alpha|j; k', q-k'> \tag{5.5}
$$

Finiteness of $N(\alpha)$ yields

$$
|{}_-<\ell_1, \ldots \ell_n; \text{(wee)}|i; k_1 k_2>|^2 \le 0\,[\mathrm{Min}\{(\frac{M}{k_{i\perp} - \frac{x_i}{y_j}\ell_{j\perp}})^{2+\epsilon};\ j=1,\ldots n\}] \tag{5.6}
$$

when $k_i \to \infty$ while $k_1 + k_2 = q$ fixed.

This bound on the transversal momentum behavior is not good enough to obtain any information. Note even the convergence of integral cannot be established from this bound. The physical reason why this process is so different from the deep inelastic electron proton scattering is that there is no rigid reference direction with which transversal momentum damping can be established. Both k_i and k_i' as well as state α are integrated.

The bound that yields information is

$$|{}_-\langle \ell_1, \ldots, \ell_n; (\text{wee})|i; k_1 k_2\rangle|^2 \leq O[\,\mathrm{Min}\{(\frac{M}{\ell_{i\perp} - \frac{y_j}{x_i} k_{i\perp}})^{4+\epsilon} \; ; i = 1, 2\}\,] \quad (5.7)$$

for large $\ell_{j\perp}$ with $\sum_{s=1}^{n} \vec{\ell}_s = \vec{k}_1 + \vec{k}_2$ fixed.

Note that the power fall off in (5.7) is larger than that of (5.6) and that this is a statement of transversal momentum fall off of a hadron state with respect to a parton state. This does not follow from our original assumption. With (5.7) integration over $\ell_1, \ldots, \ell_n$ can be restricted to cones around k_1 and k_2. Let $\ell_1, \ldots \ell_v$ be in the cone around k_1 and $\ell_{v+1}, \ldots \ell_n$ be in the cone around k_2. Then

$$E(\alpha) = 2P + [\, E_{\text{wee}} - (\Sigma \ell_z)_{\text{wee}}] + \frac{k_\perp^2}{2xP} + \frac{k_\perp^2}{2(2-x)P} + \sum_{i=1}^{n} \frac{m^2 + \ell'^2_{i\perp}}{2y_i P}$$

$$\vec{\ell}'_{i\perp} = \begin{cases} \vec{\ell}_{i\perp} - \dfrac{y_i}{x} \vec{k}_1 & \text{for } 1 \leq i \leq v \\ \vec{\ell}_{i\perp} - \dfrac{y_i}{2-x} \vec{k}_2 & \text{for } v+1 \leq i \leq n \end{cases} \quad (5.8)$$

This is the transverse vector for $\vec{\ell}_i$ relative to $\vec{k}_1$ and $\vec{k}_2$ respectively. The energy conserving δ function in (5.5) becomes

$$\delta(\,[\, E_{\text{wee}} - (\Sigma \ell_z)_{\text{wee}}] + \frac{1}{x(2-x)P}\,[\, k_\perp^2 - x(2-x)\frac{q^2}{4} + x(2-x)\, \Sigma \frac{m^2 + \ell'^2_{i\perp}}{2y_i}]\,) \quad (5.9)$$

If wee hadron is present in state $|\alpha\rangle_-$, $E_{\text{wee}} - (\Sigma \ell_z)_{\text{wee}}$ is $O(m)$ and energy conservation will be violated. Thus the sum over states $|\alpha\rangle_-$ is restricted to those states without any wee hadrons.

Then,

$$\Pi(q^2) = \int_{nw} d\alpha \int d^3k\, d^3k' \frac{(2\pi)^3}{4VP} \delta\left(k_\perp^2 - \frac{q^2}{4}x(2-x)\right) x(2-x) \sum_{ij} e_i e_j \langle k', q-k; i|\alpha\rangle_- {}_-\langle\alpha|j; k, q- \tag{5.10}$$

We have droped the last term in the argument of the δ function given in (5.9). The region $y_i = O(m^2/E^2)$ requires some care. A vector $(\sqrt{m^2+p^2}, p\sin\theta, -p\cos\theta)$ in the center of mass frame of the colliding beam has a z component in the infinite momentum frame $(-p\cos\theta + \sqrt{p^2+m^2})P/E$. The only way for y_i to be $O(m^2/E^2)$ is to have $p = O(E)$ and $\theta = O(m/E)$. If the differential cross section is smooth near $\theta = 0$, the contribution from such a kinematical region is negligible. The approximation also fails if the multiplicity of hadrons with $y_i = 0\left(\frac{m}{E}\right)$ behaves as E/m.

$$\Pi(q^2) \leq \sum_i \frac{\pi e_i^2}{2} \int_0^2 dx\, x(2-x) = \sum_i \frac{2\pi e_i^2}{3} \tag{5.11}$$

We have sumed over all α ignoring the nw sign. Due to the positive definiteness of the integrand, it is an upper bound. A wee vector in the infinite momentum frame corresponds to a vector with momentum of $O(P)$ in the center of mass of the colliding beam. We can imargine that the amplitude is small in such an unphysical region. In such a case, the inequality becomes an equality and we obtain the parton model prediction.

$$\sigma(e^+e^- \to \text{hadrons}) = \frac{4\pi\alpha^2}{3q^2} \sum_i e_i^2$$

or $$R = \sum_i e_i^2$$

We remind the reader that this result is obtained with an additional assumptions on the multiplicity and (5.7). This does not follow from our starting principles.

VI. PRICE PAID FOR SIMPLE RENORMALIZATION PROGRAMS

The proton form factor can be obtained from (4.15)

$$F_1(q^2) = \int d\gamma d^3k \left(\sum_{i\epsilon\gamma} e_i \delta_{k,k_i} \right) {}_{-}\langle p+q | a^+(k+q) a(k) | \gamma \rangle \langle \gamma | p \rangle_+ \tag{6.1}$$

making a rotation,

$$F_1(q^2) = \sum_{n=1}^{\infty} \int \prod_{j=1}^{n} d^3k_j \sum_{i=1}^{n} e_i \; {}_{-}\langle p | k_1 - x_1 q_\perp, \ldots, k_{i-1} - x_{2-1} q_\perp,$$

$$k_i + q - x_i q_\perp, \; k_{i+1} - x_{i+1} q_\perp, \ldots k_n - x_n q_\perp \rangle \langle k_1, \ldots, k_n | p \rangle_+$$

Using our bound (3.38) and identity

$$\int_0^\infty d^2k_\perp \frac{1}{(k_\perp + (1-x)Q)^{1+\epsilon}} \frac{1}{k_\perp^{1+\epsilon}} = \frac{2\pi}{Q^{2\epsilon}} \int_0^\infty \frac{dy}{\{y[y+(1-x)]\}^{1+\epsilon}} \tag{6.2}$$

for $x \neq 1$, we can show that the contribution to form factor from the region $(x-1) \gg 1/Q$ vanish. Since $N(p)^{\frac{1}{2}}$ is nonzero there is finite probability of finding bare proton at $x = 1$. This may give non vanishing form factor and, therefore, we cannot prove that $F_1(Q^2) \to 0$ as $Q^2 \to -\infty$. Note also that νW_2 at $x = 1$ is proportional to $N(p) \neq 0$. These problems can be solved if we have $N(p) = 0$. From the discussion following (3.29) if we start with a theory in which $N' = \infty$, we have $N(p) = 0$. We conjecture that relaxing the assumption of finite wave function renormalization constant will not affect other results discussed in Sections IV and V.

VII CONCLUSION

In a field theory formalism, physical field is formed in terms of a cloud of bare fields. For example, in our formalism

$$|\alpha>_{\pm} = N(\alpha)^{\frac{1}{2}} \left[1 + \sum_{n=1}^{\infty} \{ -\lambda D_{E(\alpha)\pm io} V)^{n} \}_{nd} \right] |\alpha>.$$

A question arises whether the probability function for finding a proton as a state of m bare particles with momentum $k_1, \ldots, k_m$, $|<p|k_1, \ldots, k_m>|^2$, is an observable or not. It is less demanding to ask whether $|<p|k_1, \ldots, k_m>'|^2$, defined in (1.1), a probability function for finding m bare particles with momenta $k_1, \ldots, k_m$ plus anything in a proton state, is observable or not. So far, we do not know of any well defined field theory for which these functions are physically observables. In the perturbative formalism of quantum electrodynamics, for example, a probability function for finding a physical electron as a state of a bare electron and m bare photons is not well defined. It is most interesting to find an example of a realistic field theory for which these functions are physically observables. In this paper, we leave this interesting problem aside and assume that there exists a field theory for which the probability function for finding a physical state to be in a certain bare state is an observable.

Van Hove's formulation of field theory is particularly suited for our problem. Within this formulation we assume that (a) we are dealing with a field theory in which wave function renormalization constants for all states $|\alpha>$ are finite. This implies that the phase space integral

$$\int \prod_{j=1}^{n} d^3k_j \, | {}_{+}<\alpha | k_1, \ldots, k_n> |^2$$

is convergent, (b) this formulation is renormalizable. We found that these two assumptions together with the formulation of field theory gave us a basis on which parton model ideas can be examined. The formulation also seemed to be promising as a basis for phenomenological studies. We summarize virtues and possible short comings of our formalism:

(1) It is important to note that structure of hadron, the cloud effect persists in the asymptotic states.

(2) There is one to one correspondance between

$$|\alpha>, \quad |\alpha>_{\pm}, \text{ and } |\alpha>_{as}$$

and the energy of $|\alpha>_{as}$ is equal to the eigen value of $|\alpha>_{\pm}$. This allows us to write the eigen value of $|\alpha>_{\pm}$ trivially in terms of physical masses. (See (3.21).)

(3) After renormalization of $D_{\ell}(\alpha)$, all masses in the formalism are physical masses. We never talk about free energy of partons. In the parton model, bare masses of partons must

be treated as well defined finite quantities.

(4) The crucial step in discussing νW_2 is the expression for $E(\alpha)$ [see (4.17)]. With property (2), all dynamical information necessary to study $E(\alpha)$ is the starting assumption (a).

(5) Property (2) may be a short coming of our formalism. For any given bare state $|k_1, \ldots, k_n>$, there is an asymptotic state $|k_1, \ldots, k_n>_{as}$ where all particles are free except for self interaction. If we, therefore, admit that there is no fractionally charged particle in nature. we cannot have them in the bare states. This can be traced back to the fact that we cannot discuss bound states.

It is important to understand the difference between our formalism and that of Drell, Levy and Yan.[20] In their work, the parton model results are investigated by the use of the perturbation expansion of the field theory. The transversal momentum cutoff is introduced by hand for each Feynman diagram contributing to the calculation of νW_2. The equation which allows them to obtain the parton model result is $E_p - E_{up} \ll \frac{M\nu}{P}$ and $E_n - E_{un} \ll \frac{M\nu}{P}$ where E_{up} and E_{un} are energies of the parton state $|UP>$ and $U|n>$ respectively. In order to obtain this result, there must not be any wee partons contained in the states $|UP>$ and $U|n>$. The justification of this is that each time ordered graph contributing to the calculation of νW_2, gives negligible contribution from the integration region where either $|UP>$ or $U|n>$ contains any wee parton. (This is one of the reasons why, in the analysis of Feynman integrals $P \to \infty$ limit is extremely useful.) If we, therefore, generalize this property of the Feynman graphs as the property of the full amplitude, then their result follows. We were, however, reluctant to make this generalization which prohibits us to have any wee partons in the formalism.

There are two ways to proceed from here. A complete understanding of the renormalization proceedure for our formalism must be obtained. This allows us to discuss the possibility of $N' = \infty$ (see (1.3)). The advantage of this possibility is given in Sec. VI. As it is discussed in Ref. 2, it is interesting to consider the possibility of the partons being the quarks. There is no problem if we find the quarks in nature. Supposing that the quarks are not seen in nature, it is interesting to consider a theory which has the quarks as bare states but such fractionally charged states are absent in the asymptotic states. In Ref. 4, Van Hove makes a distinction between two systems that can be described in terms of field theories. A non disapative system and a disapative system. This distinction is primarily made in solid state systems. The usual field theories being considered in elementary particle physics is of non-disapative type. It is perhaps worthwhile considering, in analogy with some solid state systems, a possibility that some system of elementary particles are described in the disapative type of field theories. This may give some clue as to how we go about eliminating the fractionally charged state from the asymptotic states.

ACKNOWLEDGEMENT

It is a pleasure to thank S. D. Ellis and S. Gasiorowicz for discussions.

APPENDIX A

In Sec. IV we have seen that an upper bound on νW_2 can be placed rather trivially in the scaling limit once we know the energy, $E(\alpha)$, (4.17).

Due to the correspondance between $|\alpha>_-$ and $|\alpha>_{as}$ $E(\alpha)$ can be calculated with minimal knowledge of strong interaction dynamics. All we need to know about the dynamics is that $N(\alpha)$ is non zero for any states $|\alpha>$. What we want to do is to mathematically justify the following simple picture. The proton is seen as a jet of partons traveling along the z axis in a "narrow cone". When it is struck by a photon with large Q^2 two jets are formed. (See Fig. 8.) States $|\alpha>_-$ will be contained in these two cones. In practice we want to show that sum over α in the expression for W_2, (4.15), can be restricted only within the cones shown in Fig. 8. Equation (4.17) then follows by simple kinematics.

The argument is simple but tedious. We first prove few seemingly unrelated points.

Theorem 1. In the expression for W_2, (4.15),only the region $k_\perp, k'_\perp \ll Q$ is important.

Proof. For fixed k_z, place an upper bound on the integrand of (4.15) by ignoring the energy conservation δ function. One easily obtains

$$\leq \int d^2 k_\perp \sum_{i \epsilon \gamma_1} (e_i^2\ \delta_{kk_i}) \frac{x}{\nu} <+p \,|\, a^+(k) a(k) \,|\, \gamma_1> <\gamma_1 \,|\, p+>.$$

Using (3.31) it is seen that $k_\perp$ sum can be cut off at say

$k_\perp \sim \xi \ll Q.$

Theorem 2. In the expression for W_2, (4.15), only the region $|z - z'| < O(\frac{M}{Q})$ is important. Other regions will give vanishing contribution to νW_2 in the scaling limit.

This is very close to what is known in the parton model as the incoherent scattering assumption.

Proof. Expand the expression for W_2, (4.15), using (3.10) and (3.19)

$$W_2 = \int d\beta_1 d\beta_2 d\beta_3 d\gamma_1 d\gamma_2 dk dk' \left(\sum_{i \epsilon \gamma_1} e_i\ \delta_{kk_i}\right)\left(\sum_{i' \epsilon \gamma_2} e_{i'}\ \delta_{k'k_{i'}}\right) \frac{M}{P} \frac{(2\pi)^3}{V}$$

$$\Big\{ N(\beta_2)\, \delta(E(\beta_2) - E(\alpha))\ {}_+<p \,|\, \gamma_2><\gamma_2 \,|\, a^+(k') a(k'+q) \,|\, \beta_2><\beta_2 \,|\, a^+(k+q) a(k) \,|\, \gamma_1><\gamma_1 \,|\, p>_+$$

$$+ \Big({}_+<p \,|\, \gamma_2><\gamma_2 \,|\, a^+(k') a(k'+q) \,|\, \beta_1> \tag{A-1}$$

$$\sum_{n=1}^{\infty} \sum_{m=0}^{n} <\beta_1 \,|\, \Big[(D_{E(\alpha)+io} V)^m \,|\, \beta_2> \delta(E(\beta_2) - E(\alpha)) N(\beta_2) <\beta_2 \,|\, (V\, D_{E(\alpha)-io})^{n-m} \Big]_{nd} \,|\, \beta_3>$$

$$<\beta_3 \,|\, a^+(k+q) a(k) \,|\, \gamma_1><\gamma_1 \,|\, p>_+ \Big) \Big\}$$

Diagrams for first few terms are given in Fig. 9. Suppose $z \neq z'$ or $|k_\perp - k'_\perp| \sim Q$. First term in (A-1) does not contribute. There are two distinct ways to obtain non vanishing matrix element.

(i) $|\beta_3>$ contains a parton with momentum $\vec{p}_1 = (p_{1z}, \vec{p}_{1\perp})$ were $p_{1z} = z' P$ and $|\vec{p}_{1\perp} - (k'+q)_\perp| = O(1)$, and similarly $|\beta_1>$ contains a parton with momentum $\vec{p}_2 = (p_{2z}, \vec{p}_{2\perp})$ where $p_{2z} = zP$ and $|\vec{p}_{2\perp} - (k+q)_\perp| = O(1)$.

(ii) $|\beta_1>$ and $|\beta_3>$ contains no partons with large transversal momentum except for partons with momentum $k'+q$ and $k+q$ respectively.

The first possibility is simple. The matrix element $<+p|a^+(k')a(k'+q)|\beta_1>$ and $<\beta_3|a^+(k+q)a(k)|p>_+$ both vanish in the scaling limit. So it does not affect νW_2 in the scaling limit.

In the second possibility final state interactions generated by $(DV)^m$ and $(DV)^{n-m}$ must be such that both of the state β_1 and β_3 become β_2. The matrix element [ ] in (A-1) can be rewritten as

$$\begin{aligned} I \equiv &<\beta_1|D_{E(\alpha)+io}(V D_{E(\alpha)+io})^{m-1} V|\beta_2>\delta(E(\beta_2)-E(\alpha)) \\ &<\beta_2|V(D_{E(\alpha)-io}V)^{n-m-1} D_{E(\alpha)-io}|\beta_3> \end{aligned} \tag{A-2}$$

Denote $|\beta_1>$ and $|\beta_3>$ as $|p_1', \ldots, p_s', k'+q>$ and $|p_1, \ldots, p_s, k+q>$ respectively. And suppose that the transversal momentum of $p_1, \ldots, p_s$ and $p_1', \ldots, p_s'$ are small compared to Q.

$$\begin{aligned} I = &\frac{2z'P}{Q^2 - 2z'M\nu} <\beta_1|(V D_{E(\alpha)+io})^{m-1} V|\beta_2> \\ &<\beta_2|V(D_{E(\alpha)-io}V)^{n-m-1}|\beta_3> \frac{2zP}{Q^2 - 2zM\nu} \end{aligned} \tag{A-3}$$

We have neglected terms in denominators which are small compared to Q^2. (P^2 is canceled by normalization factors in V.) First and second denominator factors are both small when $z' = z = \frac{Q^2}{2M\nu}$. The region where $|z - z'| = O(\frac{M}{Q})$ is not satisfied gives vanishing contribution in the scaling limit due to large denominator.

In Sec. III we discussed transversal momentum damping implied by finiteness of N(p). We derived $|\langle k_1, \ldots, k_n | p+\rangle|^2 \le O\left(\left(\frac{M}{k_{j\perp}}\right)^{2+\epsilon}\right)$, $\epsilon > 0$, $1 \le j \le n$ for large $k_{j\perp}$ with $\sum_{i=1}^{n} k_i = p$.

Theorem 3. Setting

$$A = \left| {}_{-}\langle(\ell_1, \ldots, \ell_n)| a^+(k+q) a(k) |p\rangle_+ \right|^2$$

$$B = \left| {}_{+}\langle p | a^+(k') a^+(k'+q) | (\ell_1, \ldots, \ell_n)\rangle_{-} \, {}_{-}\langle(\ell_1, \ldots, \ell_n)| a^+(k+q) a(k) | p\rangle_+ \right| \tag{A-4}$$

$$\text{for } |z - z'| < O\left(\frac{M}{Q}\right),$$

$$A, B \le O\left(\mathrm{Min}\left[\left(\frac{M}{\ell_{i\perp}}\right)^{2+\epsilon}, \left(\frac{M}{\tilde{\ell}_{i\perp}}\right)^{2+\epsilon} \right] \right), \tag{A-5}$$

where $\tilde{\ell}_{i\perp} = \ell_{i\perp} - \frac{y_i}{z} Q$, in the limit $\ell_{i\perp}$ becomes **large.**

Proof. By completeness

$$\sum_n \int \prod_{i=1}^{n} d^3 \ell_i \left| {}_{-}\langle(\ell_1, \ldots, \ell_n)| a^+(k+q) a(k) |p\rangle_+ \right|^2 \simeq {}_{+}\langle p | a^+(k) a(k) | p\rangle_+ \tag{A-6}$$

for large Q. For finite z, right hand side of (A-6) is well defined since N(p) is assumed to be finite. Since $k_\perp$ is limited by Theorem 1, there are only two independent directions p and k+q. Consider for example,

ℓ_1 integration with constraint $\sum_{i=1}^{n} \ell_i = p+q$. The integrand must vanish sufficiently fast when $\ell_{1\perp}$ or $\ell_{1\perp} - \frac{y_1}{z} Q$ (the transverse distance between ℓ_1 and k+q) become large. The energy denominators that appear in field theory calculations favor small $\ell_{1\perp}$ or small $\ell_{1\perp} - \frac{y_1}{z} Q$. Therefore, in general, in order for the integral in (A-6) to converge (A-5) must be satisfied for A. The inequality for B follows trivially from that of A.

The bounds given by Theorem 3; enable us to cutt off the integrals of (4.15).

$$W_2 = \int \frac{d^3kz}{k} \frac{d^3k'z}{k} \int_\xi d\alpha \;\; \delta(E(p) + q - E(\alpha)) MP \frac{(2\pi)^3}{V} \tag{A-7}$$
$$\int d\gamma_2 \Big(\sum_{i' \epsilon \gamma_2} e_{i'} \delta_{k_{i'}, k'}\Big) \;{}_+\langle p | a^+(k') a(k'+q) | \gamma_2 \rangle \langle \gamma_2 | \alpha \rangle_-$$
$$\int d\gamma_1 \Big(\sum_{i \epsilon \gamma_1} e_i \delta_{k_i, k}\Big) \;{}_-\langle \alpha | \gamma_1 \rangle \langle \gamma_1 | a^+(k+q) a(k) | p \rangle_+$$

where

$$\int_\xi d\alpha = \sum_n \prod_{j=1}^{n} \frac{1}{(2\pi)^3} \left[\int_{|\ell_{j\perp}| < \xi} d^3\ell_j + \int_{|\ell_{j\perp} - y_j Q/z| < \xi} d^3\ell_j \right] \tag{A-8}$$

where we have denoted $|\alpha\rangle_- = |\ell_1, \ldots, \ell_j, \ldots, \ell_n\rangle_-$. ξ is some parameter chosen so that $M \ll \xi \ll Q$. We have now established that it is a good approximation, in the scaling limit, to limit ourselves with the case where all non wee vectors of $|\alpha\rangle_-$ are contained in two cones defined by (A-8).

Proof of (4.17). Consider $|\alpha>_- = |\ell_1, \ldots, \ell_n, \text{wee}>_-$,

$$\ell_j = (y_j P, \ell_{j\perp}) \tag{A-9}$$

where specified momenta correspond to non wee hadron momenta and (wee) corresponds to collection of wee hadron momenta. Total z component momentum is P. By momentum conservation,

$$\sum_{j=1}^{n} y_j = 1 - \frac{(\Sigma \ell_z)\text{wee}}{P}$$

$\frac{1}{P}$ term comes from z component momenta of wee hadrons.

We divide non wee members of $|\alpha>_-$ into two parts

$$\ell_1, \ldots, \ell_v \in c_1; \qquad |\ell_{j\perp}| < \xi \text{ for } 1 \leq j \leq v$$

$$\ell_{v+1}, \ldots, \ell_n \in c_2; \qquad |\ell_j - \frac{y_j}{z} Q| < \xi \text{ for } v+1 > j > n$$

Then $\sum_{j=1}^{v} y_j = (1-z)$, $\sum_{j=v+1}^{n} y_j = z$

$$E(\alpha) - E_{\text{wee}}(\alpha) = (1 - \frac{(\Sigma \ell\, z)\text{wee}}{P})P + \sum_{j=1}^{n} \frac{m^2 + \ell_{j\perp}^2}{2y_j P}$$

$$E(\alpha) - \Delta E_{\text{wee}}(\alpha) = P + \sum_{j=1}^{v} \frac{m^2 + \ell_{j\perp}^2}{2y_j P} + \sum_{j=v+1}^{n} \frac{m^2 + (\frac{y_j}{z} Q + \tilde{\ell}_j)_{\perp}^2}{2y_j P}$$

where $\tilde{\ell}_j = \ell_{j\perp} - \frac{y_j}{z} Q$, $\Delta E_{\text{wee}}(\alpha) = E_{\text{wee}}(\alpha) - (\Sigma \ell_z)_{\text{wee}}$

$$E(\alpha) - \Delta E_{\text{wee}}(\alpha) = P + \sum_{j=1}^{v} \frac{m^2 + \ell_{j\perp}^2}{2y_j P} + \sum_{j=v+1}^{n} \frac{m^2 + \tilde{\ell}_{j\perp}^2}{2y_j P} + \frac{Q^2}{2zP}$$

APPENDIX B

The formalism in Sec. III has been written in terms of the bare vertex λV. We will show that R_ℓ can be rewritten in terms of a vertex function which contains all vertex corrections.

To illustrate our point, let us consider a theory with coupling $i\lambda\bar{\psi}\gamma_5\psi\phi$ where ψ and ϕ correspond to fields for proton and pion in the theory. Then V contains couplings

$$<p|V|p\pi>,\ <p\bar{p}|V|\pi>,\ <\bar{p}|V|\pi\bar{p}>,\ <0|V|\bar{p}p\pi> \tag{B-1}$$

and hermitian conjugates.

Consider the matrix element

$$<\alpha'|\{V\sum_{n=0}^{\infty}(-\lambda D_\ell V)^n\}_{nd}|\alpha> \tag{B-2}$$

For example take $|\alpha>$ to be two proton state, and $|\alpha'>$ to be two protons and one pion state. Diagramatically first and third term is given in Fig.10. In each order there are set of graphs which contains δ function singularity representing the fact that two protons propagate without any interaction between them. We call these terms most singular term. All of the graphs given in the example are of that nature except the set of graphs in Fig.10c. In general the matrix element (B-2) contains most singular terms, i.e., set of graphs which represents the least number of interaction between particles in state $|\alpha>$ and equivalently particles in state $|\alpha'>$. We represent

$$\Gamma_\ell \equiv \left(V \sum_{n=0}^{\infty} (-\lambda D_\ell V)^n\right)_{nd}^{ms} \tag{B-3}$$

the super and subscripts reminds us that we pick up only the most singular part of the non diagonal matrix element. The matrix element for $<np, \pi | \Gamma_\perp | np>$ is shown in Fig.11. (Note however that such matrix element must be evaluated in accordance with the definition of the non-diagonal matrix element given in Sec. III). In terms of Γ_ℓ we can show that

$$[D_\ell \sum_{n=1}^{\infty} (-\lambda V D_\ell)^n]_{nd} = [D_\ell \sum_{n=1}^{\infty} (-\lambda \Gamma_\ell D_\ell)^n]_{nd} \tag{B-4}$$

Essentially one can understand the identity as replacing the bare vertex by full vertex

Therefore, every V in the expression for S matrix (3.26) can be replaced by Γ_ℓ.

Suppose Γ_ℓ as defined in Eq. (B-3) is divergent, we will assume that $\Gamma_\ell = Z\tilde{\Gamma}_\ell$ where $\tilde{\Gamma}_\ell$ is independent of cut off parameter and Z is an infinite constant. Then Z is absorbed in redefinition of λ.

REFERENCES AND FOOTNOTES

[1] G. Miller et al., Phys. Rev. D5 528 (1973).

[2] R. P. Feynman, Photon Hadron Interactions, W. A. Benjamin, Inc. (1972) and references therein.

[3] S-J. Chang and P. M. Fishbane, Phys. Rev. D2, 1084 (1970). P. M. Fishbane and J. D. Sullivan, Phys. Rev. D4, 2516 (1971).

[4] L. Van Hove, Physica XXI, 901 (1955), Physica XXII 343 (1956).

[5] Perturbative approach to quantum electrodynamics does not satisfy this condition. As a consequence the bare electrons and the bare photon do not influence any measurable quantities.

[6] In quantum electrodynamics the wave function renormalization constant is gauge dependent. In this case we mean in the Feynman gauge where there is no ghost particle. (We thank Professor Y. Nambu for discussion on this point and other related points.)

[7] This does not mean that $N' = \infty$ is uninteresting. On the contrary, in the infinite momentum frame, arbitrary many wee partons can be created without drastic violation of energy conservation. It is, therefore, possible to have $N' = \infty$. Also in this case, we expect probability for finding a bare proton and any finite number of parton to be vanishingly small. And only $|\langle +p | k_1, \ldots k_n \rangle'|^2$ makes sense where we summed

over arbitrary many wee parton states. This case is presently under investigation.

[8] $f_i(x)dx$ = number of parton "i" with fraction of momentum between x and x + dx. e_i is the charge of parton "i".

[9] An additional assumption should be mentioned. In the case of ep scattering, the final state hadron carrying large transversal momentum with respect the photon direction in the laboratory frame is assumed to be suppressed. This assumption is necessary since we are not able to make definite statement about the property a the amplitude when arbitrary many Feynman graphs are sumed.

[10] Ref. 2.

[11] A. Litke et al., Phys. Rev. Letters, 30, 1189 (1973).

[12] Formally this problem is avoided by introducing the mass counter term in the interaction Hamiltonian and expanding the S matrix in terms of the "in" or "out" fields which correspond to the free asymptotic states with physical masses. In practice, however, the contributions from the mass counter terms in the interaction Hamiltonian are either neglected or treated only in a finite order.

[13] V(x) stands for Hamiltonian density and V stands for interaction Hamiltonian in the momentum space.

[14] J. D. Bjorken and S. D. Drell, Relativistic Quantum Fields, McGraw-Hill Book Company (1965), p. 175, note also footnote 5, p. 175.

[15] We have in mind treating the problem in the infinite momentum frame where $p = (P + \frac{M^2}{2P}, 0, 0, P)$, $P \rightarrow \infty$. To leading order in P, longitudinal

momentum integration automatically converges and thus only large transversal momentum behavior is of interest here.

[16] This equation holds only to leading order in P. That is, J_o or J_z can be obtained from this equation. Extreme care must be exercised in order to obtain, for example, $J_o - J_z$. Backward moving partons will contribute for this difference.

[17] As it is seen in Sec. VII, to convert (4.15) into an expression for νW_2 in the parton model we replace the state $|\alpha>_-$ by $|\gamma_3>$ and $E(\alpha)$ by $E(\gamma_3)$. Then the energy conservation δ function reduces to $\delta(\frac{M\nu}{P} - \frac{Q^2}{2xP})$ and since the δ function no longer depends on γ_3, the sum over γ_3 can be performed trivially.

[18] Note that upper bound can be obtained directly from (4.15) by just summing over α ignoring the energy conservation δ function. The difference between such a bound and (4.21) is the presence of $\delta(\Delta z)$. If this δ function is absent, the right hand side of (4.21) will behave like the number of partons in the proton state which may be infinite.

[19] For infrared problems in quantum electrodynamics see D. R. Yennie, S. C. Frautschi and H. Suura, Annals of Physics, 13, 379 (1961). For the study of form factors in the vector gluon model see T. Appelquist and J. R. Primack, Phys. Rev. D4, 2454 (1971).

[20] S. D. Drell, D. J. Levy and T. M. Yan, Phys. Rev. D4, 1035 (1970) and references therein.

FIGURE CAPTIONS

Fig. 1 Deep inelastic electron proton scattering and its variables.

Fig. 2 Simplest perturbation diagram which gives scale breaking effect.

Fig. 3 The bare proton in Fig. 2 is replaced by a physical proton.

Fig. 4 Two time ordered graphs contributing to (4.8)

Fig. 5 Diagramatic representation of deep inelastic electron-proton scattering.

Fig. 6a Example of vertex correction to parton photon vertex.

Fig. 6b Example of other radiative corrections.

Fig. 7 S matrix for $e^+ e^- \rightarrow$ hadrons.

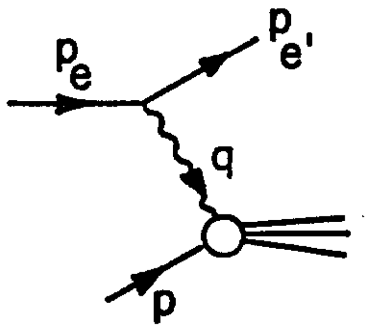
Fig. 8 A schematic diagram for the ep scattering.

Fig. 9 Diagrams for first few terms of (A-7).

Fig. 10 Diagrams representing first and third term of (A-3).

Fig. 11 Diagram for $\langle np, \pi | \Gamma_\ell | np \rangle$.

Figure 1



$$\nu = \frac{p \cdot q}{M} \qquad Q^2 = -q^2$$

$$X = \frac{Q^2}{2M\nu}$$

Figure 2

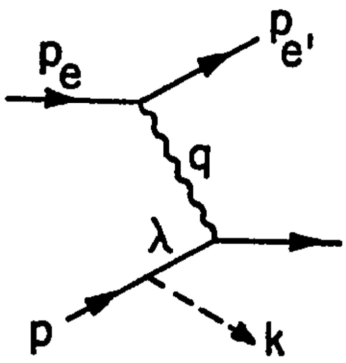
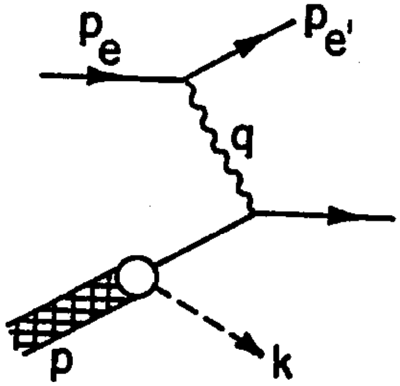
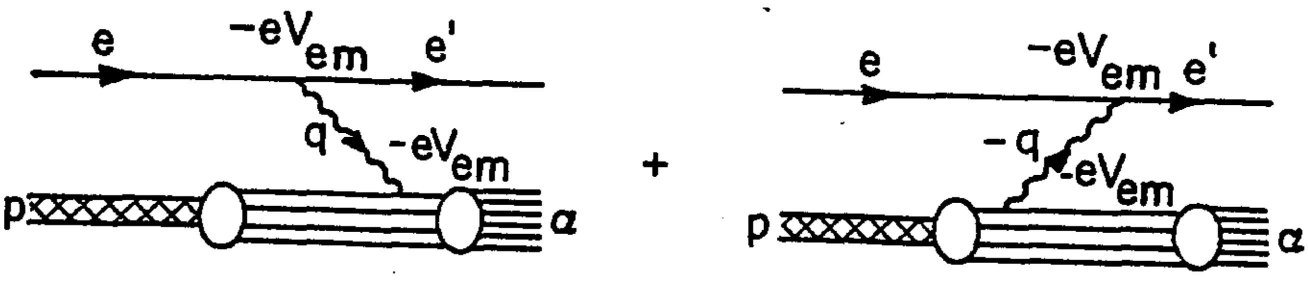


Figure 3





(a) Figure 4 (b)

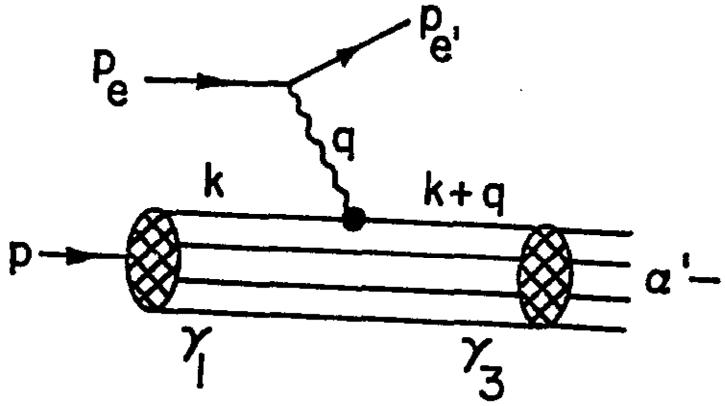
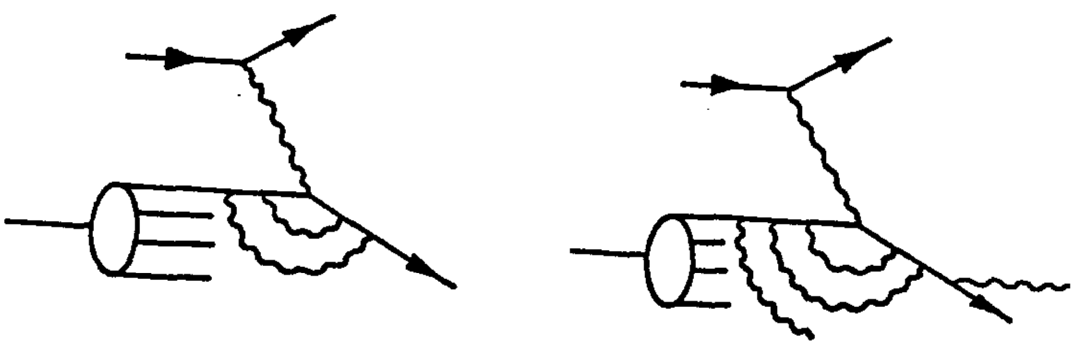


Figure 5



(a) (b)

Figure 6

Figure 7

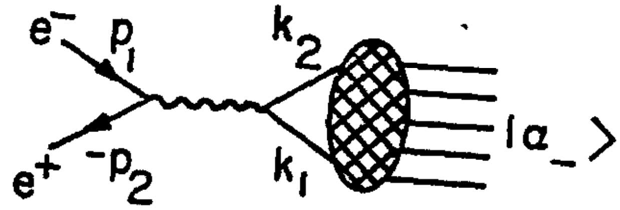
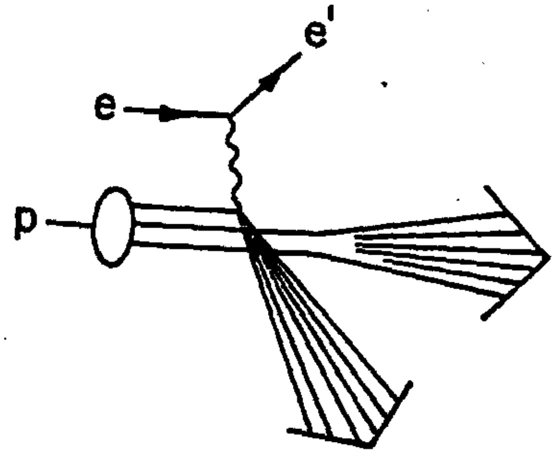
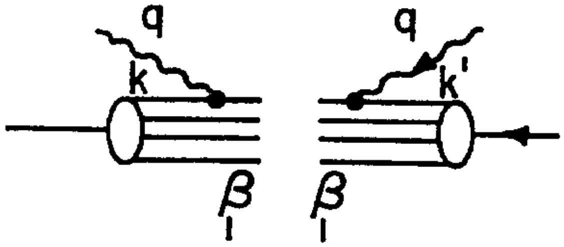


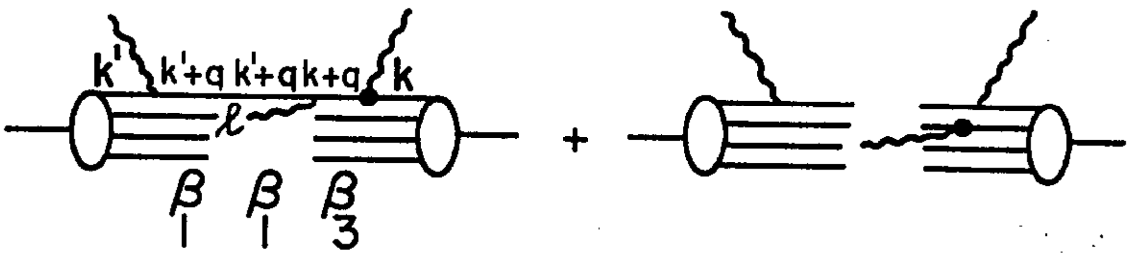
Figure 8



$$ {}_+\langle p|a^+_{k'}a_{k'+q}|\beta_1\rangle\langle\beta_1|a^+_{k+q}a_k|p\rangle_+ \ \delta(E(p)-E(\beta_1))N(\beta_1) $$



$$ {}_+\langle p|a^+_{k'}a_{k'+q}|\beta_1\rangle\delta(E(\beta_1)-E(p))\langle\beta_1|VD_{E(p)-i0}|\beta_3\rangle $$
$$ \langle\beta_3|a^+_{k+q}a_k|p\rangle_+ $$



+ diagrams with gluons attached to other lines in β_3

$$ {}_+\langle p|a^+_{k'}a_{k'+q}|\beta_1\rangle\langle\beta_1|D_{E(p)+i0}V|\beta_2\rangle N(\beta_2)\delta(E(\beta_2)-E(p)) $$
$$ \langle\beta_2|VD_{E(p)-i0}|\beta_3\rangle\langle\beta_3|a^+_{k+q}a_k|p\rangle $$

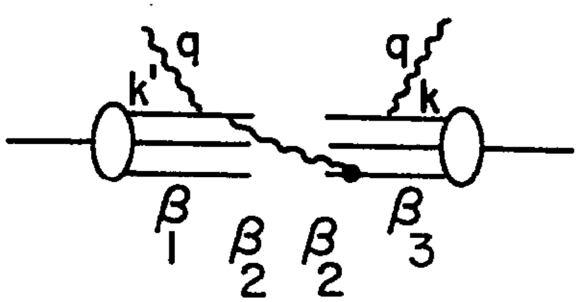


Figure 9

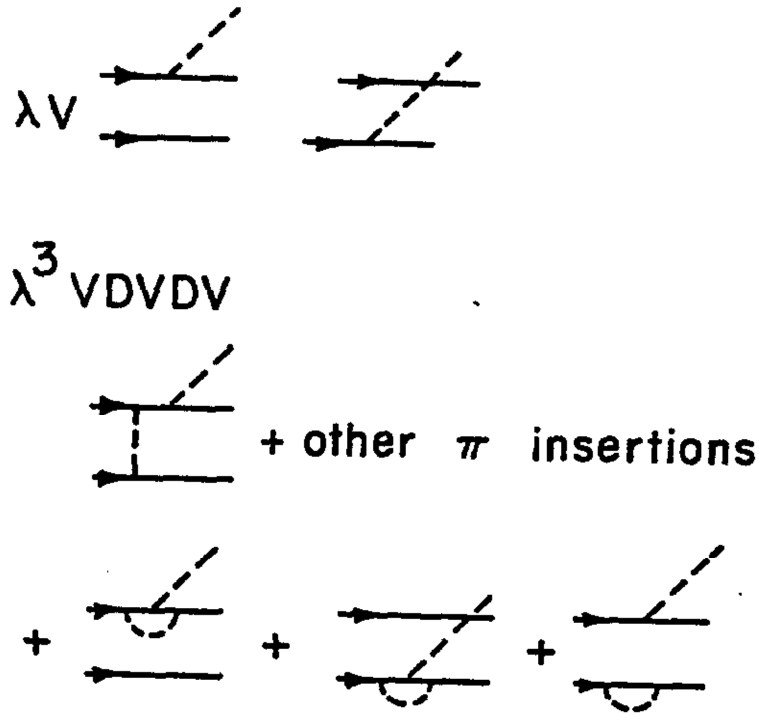


Figure 10

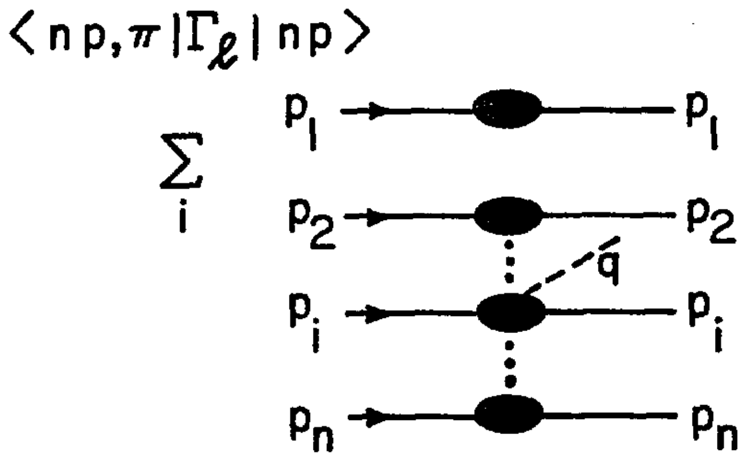


Figure 11