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A Relation Between Regge Trajectory Intercepts and the Asymptotic Behaviour of the Multiplicity Moments f_k

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Assuming that the multiplicity moments $f_k(s)$ formed from the topological cross section $\sigma_n(s)$ or from integrated inclusive correlation functions behave for large s as $f_k(s) \sim c_k \log s$, we discuss a general relation between the c_k and the leading asymptotic behaviour of $\sigma_n(s)$. The relation has been given by Harari, but our arguments demonstrate that it is not connected with any hypothesis concerning the dependence of hadronic parameters on some underlying coupling constants.

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In an interesting article[1] Harari has suggested that the Pomeron trajectory intercept α_P is exactly equal to one for any value of an underlying hadronic coupling constant g. At the same time he argued that the "ordinary" Regge trajectory $\alpha_R(g)$ which, for example, determines the power dependence in s of the n-particle cross section $\sigma_n(s)$, must vary with g and, most strikingly, have the limiting value $\alpha_R(g=0) = 1$. Within the context of a rather general form for $\sigma_n(s)$, he then derived a relation between $\alpha_R(g)$ at the physical value of g and the coefficients of a presumed log s behaviour in the multiplicity moments

$$f_1 = \langle n \rangle, \; f_2 = \langle n(n-1) \rangle - \langle n \rangle^2, \text{ etc.} \tag{1}$$

formed from $\sigma_n(s)$.

We would like to demonstrate that in fact the relation between the value of $\alpha_R(g)$ and the f_k's is entirely independent of any assumption on $\alpha_P(g)$ or $\alpha_R(0)$ but follows directly from the presumed behaviour[2]

$$f_k(s) \underset{s \to \infty}{\sim} c_k \log s + d_k . \tag{2}$$

However interesting the idea that $\alpha_P(g)$ is independent of g and $\alpha_R(0) = 1$ indicating some underlying "vector" field theory, the testable relations presented by Harari to defend that idea have no bearing on the issue.

To proceed, we form the generating function

$$R(z) = \sum_{n=0}^{\infty} z^n \sigma_{n+2}(s) \Big/ \sigma_T(s) \tag{3}$$

$$= \exp \sum_{k=1}^{\infty} (z-1)^k f_k(s)/k! \; . \tag{4}$$

Here $\sigma_T(s)$ is the total cross section. If $f_k(s)$ behaves as in (2), then

$$F(z) = \log R(z) \underset{s\to\infty}{\sim} p(z) \log s + q(z), \tag{5}$$

where

$$p(z) = \sum_{k=1}^{\infty} (z-1)^k c_k / k! , \tag{6}$$

and

$$q(z) = \sum_{k=1}^{\infty} (z-1)^k d_k / k! . \tag{7}$$

Using the fact that for $n \geq 1$,

$$n!\, \sigma_{n+2}(s) / \sigma_T(s) = e^{F(0)} \left[\left(F' + \frac{d}{dz}\right)^{n-1} F'(z) \right]_{z=0} , \tag{8}$$

we learn that

$$n!\, \sigma_{n+2}(s) / \sigma_T(s) \underset{s\to\infty}{\sim} s^{p(0)} \left[\text{Polynomial in } \log s\right]. \tag{9}$$

Suppose now with Harari that there is only one "component" to the production mechanism for $\sigma_{n+2}(s)$, and, up to logarithms, has the high energy behavior

$$\sigma_{n+2}(s) \underset{s\to\infty}{\sim} s^{2\alpha_R - 2} \tag{10}$$

which builds up the behaviour of $\sigma_T(s)$ to be

$$\sigma_T(s) \underset{s\to\infty}{\sim} s^{\alpha_P - 1} . \tag{11}$$

It then follows immediately that from (9) that

$$2\alpha_R - \alpha_P - 1 = p(0) = \sum_{k=1}^{\infty} (-1)^k c_k / k! , \tag{12}$$

which is exactly the relation that Harari employs to defend his interesting ideas about α_P and α_R as functions of g. We now see the independence of the relation (12) from any notion of the behaviour of Regge trajectory intercepts on hadronic coupling constants.

If we pursue this line of thought further and imagine that there are both "multiperipheral" and "diffractive" contributions to production amplitudes, each of which builds up the same power behaviours in σ_T, then we would write for $\sigma_{n+2}(s)$

$$\sigma_{n+2}(s) \underset{s\to\infty}{\sim} h_n^M(s)\, s^{2\alpha_R-2} + h_n^{MD}(s)\, s^{\alpha_P+\alpha_R-2} + h_n^D(s)\, s^{2\alpha_P-2}, \tag{13}$$

where the multiperipheral piece is taken to generate α_R and the diffractive piece involves α_P itself. Now the full $f_k(s)$ no longer behaves as log s, as is well known,[3] but grows as $(\log s)^k$, when the f_k of each component behaves as log s. Indeed, if we imagine that each component of $\sigma_{n+2}(s)$ produces an f_k which grows like log s

$$\underset{s\to\infty}{\sim} c_k^M \log s + d_k^M \tag{14}$$

and similarly for f_k^{MD} and f_k^D, then we find three relations like (12)

$$2\alpha_R - \alpha_P - 1 = \sum_{k=1}^{\infty} (-1)^k c_k^M / k!, \tag{15}$$

$$\alpha_R - 1 = \sum_{k=1}^{\infty} (-1)^k c_k^{MD} / k!, \tag{16}$$

and

$$\alpha_P - 1 = \sum_{k=1}^{\infty} C_k^D \, (-1)^k / k! \quad . \tag{17}$$

Of these, the last is self-inconsistent, if $\alpha_P = 1$. Note that

$$R^D(z) = \sum_{n=0}^{\infty} z^n \sigma_{n+2}^D(s) \Big/ \sum_{n=0}^{\infty} \sigma_{n+2}^D(s) \tag{18}$$

$$= \exp \sum_{k=1}^{\infty} (z-1)^k f_k^D(s) / k! \, , \tag{19}$$

has the property $R^D(0) < 1$ because of the positivity of the σ_n, so $\sum_{k=1}^{\infty} (-1)^k \frac{f_k^D}{k!} < 0$, which implies via (17) that $\alpha_P < 1$. This is a restatement of the result of Le Bellac[4] and in the present context implies $f_k^D(s)$ probably behaves as $(\log s)^k$.

The only ingredient in the key assumption that $f_k(s) \sim \log s$ is that the singularities in the complex j-plane at $t = 0$ be isolated and factorizable. Since this is true in the form of model Harari discusses, it is not surprising that he finds (12). Unfortunately it has no bearing on his other intruiging ideas about the Pomeron intercept being fixed at one.

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