



Electromagnetic Background in the Search for Neutral Weak Currents

$$\text{via } e^+e^- \rightarrow \mu^+\mu^-$$

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We estimate the radiative **corrections** to  $e^+e^- \rightarrow \mu^+\mu^-$  for several experimental situations. In particular, we discuss the asymmetry about  $90^\circ$  of the muon differential cross section and compare this to weak neutral current asymmetry predictions. The differences between this work and previous estimates lie in the inclusion of (1) polarized beam effects, (2) an exact calculation of the two-photon channel, and (3) an improved soft-photon treatment.

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The possibility that the existence of weak neutral currents could be tested in polarized colliding beam experiments has received some attention recently.<sup>1,3</sup> The transverse polarization needed arises naturally by way of spin-flip transitions due to synchrotron radiation; the resulting alignment is parallel (antiparallel) to the magnetic field for the incident positrons (electrons). While muon polarization<sup>2,3</sup> measurements in  $e^+e^- \rightarrow \mu^+\mu^-$  would probe for weak parity-violating effects, a more accessible signal is the asymmetry<sup>3</sup> between  $\theta$  and  $\pi-\theta$  in the muon differential cross section. The asymmetry vanishes in lowest-order electrodynamics; however, there are important higher-order EM contributions.

We present here an estimate of the radiative corrections to  $e^+e^- \rightarrow \mu^+\mu^-$  for polarized beams. Thus we are able to compare the expected EM asymmetry with any weak neutral current predictions.

In the c.m. frame (also the laboratory for colliding beams), we choose axes for our coordinate system such that the electron is incident with energy  $E$  in the positive  $z$  direction. Assume that the  $e^\pm$  have polarization  $p_\pm$  along the  $x$ -axis, the direction of the magnetic field, and that the  $\mu^-$  emerges at angles  $(\theta, \phi)$  in this system with its polarization unobserved. Then the differential cross section from the lowest-order graph (Fig. 1a) is

$$\frac{d\sigma^0}{d\Omega} = \frac{\alpha^2}{16E^2} f(\theta, \phi) \quad (1)$$

where  $f(\theta, \phi)$  is symmetric around  $\theta = \frac{\pi}{2}$  and is given by

$$f(\theta, \phi) = 1 + \cos^2\theta - |p_+ p_-| \sin^2\theta \cos 2\phi. \quad (2)$$

(We consistently neglect terms which vanish in the limit  $\frac{m}{E}, \frac{\mu}{E} \rightarrow 0$  where  $m$  and  $\mu$  are the electron and muon masses, respectively).

The radiative corrections  $-\alpha^3$  terms - arise from the interference of the graphs (b) - (g) with (a) in Fig. 1 plus the lowest order inelastic cross section  $e^+e^- \rightarrow \mu^+\mu^-\gamma$  from graphs (h) - (k). This bremsstrahlung contribution is calculated in a "medium-photon" approximation - an improvement upon the usual soft-photon calculation - following the lead of Meister and Yennie.<sup>4</sup> We will first list and then discuss the various parts of the total correction  $\delta$  defined through

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^0}{d\Omega} (1 + \delta). \quad (3)$$

The result of our calculation of  $\delta$  is the sum of the following terms:

Vertex corrections ( $E \gg \mu, m$ )

$$\delta^V(\mu\mu) = \frac{2\alpha}{\pi} \left[ -1 + \frac{\pi^2}{3} + \frac{3}{2} \ln \frac{2E}{\mu} - \left( \ln \frac{2E}{\mu} \right)^2 + (1 - 2 \ln \frac{2E}{\mu}) \ln \frac{\mu}{\lambda} \right],$$

$$\delta^V(ee) = \frac{2\alpha}{\pi} [\mu \rightarrow m]. \quad (4)$$

Vacuum polarization ( $E \gg \mu, m$ )

$$\delta^P(\mu\mu) = \frac{2\alpha}{\pi} \left[ -\frac{5}{9} + \frac{2}{3} \ln \frac{2E}{\mu} \right],$$

$$\delta^P(ee) = \frac{2\alpha}{\pi} [\mu \rightarrow m]. \quad (5)$$

Two-photon channel ( $E \gg \mu, m; \sin\theta \gg \frac{\mu}{E}, \frac{m}{E}$ )

$$\begin{aligned} \delta^{\gamma\gamma}(\mu e) = & -\frac{8\alpha}{\pi} \ln \tan \frac{\theta}{2} \ln \frac{2E}{\lambda} - \frac{4\alpha}{\pi} \frac{1}{f(\theta, \phi)} \{ \cos\theta [(\ln \sin \frac{\theta}{2})^2 + (\ln \cos \frac{\theta}{2})^2] \\ & + \sin^2 \frac{\theta}{2} \ln \cos \frac{\theta}{2} - \cos^2 \frac{\theta}{2} \ln \sin \frac{\theta}{2} \\ & - |p_+ p_-| \cos 2\phi [ \cos\theta \tan^2 \frac{\theta}{2} (\ln \sin \frac{\theta}{2})^2 + \cos\theta \cot^2 \frac{\theta}{2} (\ln \cos \frac{\theta}{2})^2 \\ & + \cos^2 \frac{\theta}{2} \ln \cos \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \ln \sin \frac{\theta}{2} ] \}. \quad (6) \end{aligned}$$

Bremsstrahlung ( $E \gg \mu, m, \Delta E; \sin \theta \gg \frac{\mu}{E}, \frac{m}{E}, \frac{\Delta E}{E}$ )

$$\delta^B(\mu\mu) = \frac{2\alpha}{\pi} \left[ -\frac{\pi^2}{12} + \frac{1}{2} \ln 2 - \frac{1}{4} (\ln 2)^2 - (1-2 \ln \frac{2E}{\mu}) \ln \frac{2\Delta E}{\lambda} + \frac{1}{2} \text{Li}_2 \left( -\frac{4E\Delta E}{\mu^2} \right) \right],$$

$$\delta^B(ee) = \frac{2\alpha}{\pi} \left[ -\frac{\pi^2}{12} + \left( \ln \frac{2E}{m} \right)^2 - \left( \ln \tan \frac{\theta}{2} \right)^2 + (1-2 \ln \frac{2E}{m}) \ln \left( \frac{\lambda E \sin \theta}{2m\Delta E} \right) \right],$$

$$\delta^B(\mu e) = \frac{8\alpha}{\pi} \ln \tan \frac{\theta}{2} \ln \frac{2\Delta E}{\lambda}. \quad (7)$$

In these expressions,  $\lambda$  is the small photon mass used to resolve the infrared divergence problem. Also, in Eq. (7), we have introduced the

dilogarithm  $\text{Li}_2(x) = -\int_0^x dy \frac{\ln(1-y)}{y}$  and the energy resolution  $\Delta E$ :

the muon is detected in the energy range  $E - \Delta E \rightarrow E$ . Note that the notation  $\delta(\mu\mu)$ , etc., refers to the lepton lines involved in the radiative correction.

Eqs. (4) and (5) for the vertex and vacuum polarization corrections are old results.<sup>5</sup> On the other hand, the two-photon result is new.<sup>6</sup> Due to the difficulty in deriving this result, previous authors have been satisfied with rough estimates but an asymmetry discussion on the 1% level requires its careful evaluation. It is interesting to note that Eq. (6) is an excellent approximation to the exact calculation<sup>7</sup> which includes the full mass dependence. We have found that (6) is good to within 1% even for  $\sin\theta \sim \frac{\mu}{E}$ .

The bremsstrahlung corrections, Eq. (7), suffer from one basic defect. The spin terms<sup>4</sup> (the photon momenta in the numerators of the lepton propagators) have been neglected. To estimate the error involved in neglecting the spin-terms, note that they contribute mostly in the region of phase space where a hard photon is radiated parallel to one of the final leptons. Ref. 4 indicates that spin-terms may contribute at most a logarithm in  $(1 + \frac{4E\Delta E}{\mu^2})$ , the logarithmic term occurring only in  $\delta^B(\mu\mu)$ . Hence, we estimate an error of only a few per cent in our expression for  $\delta_{\text{sym}} \equiv \frac{1}{2}[\delta(\theta) + \delta(\pi-\theta)]$  and we expect even better accuracy for  $\delta^B(\mu e)$ . In contrast to the usual soft-photon approach, our phase space integration of the remaining terms respects fully (1) the angular variation of the photon energy and (2) the possibility that  $\Delta E \gtrsim \mu, m$ . For comparison, we have also calculated  $\delta^B$  in the soft-photon approximation and, as seen in the example given later, the differences for  $\delta^B(\mu\mu)$  and  $\delta^B(ee)$  are striking.<sup>8</sup> When putting  $\mu = m$  our leading-logarithm terms agree with the results of Meister and Yennie.<sup>4</sup>

Since only the interference between the two-photon and the single-photon graphs and between the electron and muon bremsstrahlung graphs are antisymmetric under  $\theta \rightarrow \pi - \theta$ , the asymmetry is

$$A^{EM}(\theta, \phi) \equiv \frac{d\sigma(\theta, \phi) - d\sigma(\pi - \theta, \phi)}{d\sigma(\theta, \phi) + d\sigma(\pi - \theta, \phi)} = \delta^{YY}(\mu e) + \delta^B(\mu e) + O(\alpha^2) \quad (8)$$

Numerical results for  $\delta$  and  $\delta_{\text{sym}}$  are given in Table I for  $p_+ = p_- = 0$ .

In all of our numerical work we have assumed  $\Delta E/E = 1\%$ . For comparison, in the soft-photon approximation  $\delta_{\text{sym}}$  is independent of  $\theta$  and equals  $-32.3\%$  and  $-43.5\%$  for  $E=1.0$  and  $5.5$  GeV, respectively. In Fig. 2 the dashed line shows  $A^{EM}$  for  $p_+ = -p_- = .924$  and we mention that in our approximations only the noninfrared part of  $\delta^{YY}(\mu e)$  depends on the polarization (and  $\phi$ ).

We now consider how the asymmetry is affected by neutral currents. The details depend on the model chosen to describe weak interactions, and we shall use the Weinberg model<sup>9</sup> for illustration. This is a conservative choice, because the asymmetries we discuss could be larger in other models.<sup>3,10</sup> The interference between lowest order electromagnetic and weak amplitudes gives rise to an asymmetry<sup>3</sup>

$$A^{\text{Weak}}(\theta, \phi) = - \frac{1}{f(\theta, \phi)} \frac{\sqrt{2}G^F}{\alpha\pi} E^2 \cos\theta \quad (9)$$

where  $G^F$  is the Fermi coupling constant (we have taken  $E \ll m_Z$ , where  $m_Z$  is the mass of the neutral vector boson in this model). We observe that while polarization of the incident beams increases the magnitude of  $A^{\text{Weak}}$ , it slightly decreases  $A^{EM}$ . In Fig. 2 we plot the sum  $A^{EM} + A^{\text{Weak}}$  for  $E = 2.5, 4,$  and  $5.5$  GeV and  $p_+ = -p_- = .924$ .

The large electromagnetic background will make it difficult to separate weak effects by observing only the asymmetry  $A(\theta, \phi)$ . The lion's share of

$A^{EM}$ , arising from the combination of infrared terms in  $\delta^B(\mu e)$  and  $\delta^{YY}(\mu e)$ , can be eliminated by considering the difference

$$\Delta(\theta, \phi, \phi') = A(\theta, \phi) - A(\theta, \phi') \quad (10)$$

Since  $\Delta^{Weak}$  has the same sign as  $\Delta^{EM}$ , in general, the sum  $\Delta^{EM} + \Delta^{Weak}$  could be large enough to be meaningful for experiments. Table II contains values for their combined contributions. We describe two experiments which can search for such effects.

Exp. 1: In this case one observes one muon and its charge with a momentum uncertainty arising from the finite energy resolution. The terms  $\delta_{sym}$  are seen in Table II to be large  $\sim -35\%$  and for this reason it is important to include estimates of  $\delta_{sym}$  in any asymmetry discussion. In the present case contributions of  $O(\alpha^2)$  to the asymmetries are important and one must also calculate terms of the same order arising from  $\delta^{YY}(\mu e)$  and  $\delta^B(\mu e)$ . Such terms have not been calculated yet.

Exp. 2: A more relevant experiment is one in which the muons are detected back to back with attendant uncertainties in both collinearity and energy. All our calculations again hold, except for the bremsstrahlung terms, which are now quite different. A recent calculation<sup>11</sup> of the bremsstrahlung terms indicates that their contribution to  $|\delta_{sym}|$  is less than 6% for  $.5 \leq E \leq 5.0$  GeV and a maximum acollinearity of  $10^\circ$ . Under such conditions it seems that a calculation of the asymmetries  $A$  and  $\Delta$  to  $O(\alpha)$  is sufficient, and the values of Table II hold. Finally, since every experiment has its own intrinsic uncertainties, with regard to acollinearity and energy resolution, it is necessary to recalculate only the bremsstrahlung terms for each specific case.

We would like to point out that in the absence of weak effects a measurement of the asymmetry would constitute an important check of QED. For hadronic

final states the polarization of the beams is useful in separating the structure functions. Encouraged by a large asymmetry in this process, one may also look for it in hadronic channels, e.g.,  $e^+e^- \rightarrow \pi^+\pi^-$ .

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## REFERENCES AND FOOTNOTES

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- <sup>3</sup>V.K. Cung, A.K. Mann and E.A. Paschos, *Phys. Letters* 41B, 355 (1972); J. Godine and A. Hankey, *Phys. Rev.* (to be published).
- <sup>4</sup>N. Meister and D.R. Yennie, *Phys. Rev.* 130, 1210 (1963).
- <sup>5</sup>See, for example, C. Kallen, *Handbuch der Physik*, Vol. 5, part 1, p. 304 (Springer-Verlag, Berlin 1958).
- <sup>6</sup>An independent calculation by I.B. Khriplovich (Novosibirsk preprint, submitted to the Chicago-Batavia Conference, 1972) of the asymmetry - see Eq. (8) - has been done recently in a soft-photon approximation. After separating out the bremsstrahlung part of his expression, the remainder can be shown to be identical to our formula for  $\delta^{\gamma\gamma}(\mu e)$ , Eq. (6). Note that the function  $g(\theta)$  in this reference can be written in terms of dilogarithms:
- $$g(\theta) = \frac{1}{2} [\text{Li}_2(\sin^2 \frac{\theta}{2}) - \text{Li}_2(\cos^2 \frac{\theta}{2})] .$$
- <sup>7</sup>The complete mass-dependent expression is given by R.W. Brown and K.O. Mikaelian (to be published). This reference deals with the role of the two-photon channel in  $\mu^\pm$  range differences and in small angle lepton nucleus scattering.
- <sup>8</sup>Furlan et al., [G. Furlan, R. Gatto, and C. Longhi, *Phys. Lett.* 12, 262 (1964)] have given estimates for  $\delta$ . The differences between their work and ours lie

in their (1) use of the soft-photon approximation, (2) omission of a term  $2\pi\alpha$ , and (3) rough estimate of the two-photon contribution. Note that the scattering angle in this reference is apparently  $\pi-\theta$ .

- <sup>9</sup>S. Weinberg, Phys: Rev. Letters 19, 1264 (1967) and 27, 1688 (1971).  
<sup>10</sup>T. Kinoshita, J. Pestieau, P. Roy and H. Terezawa, Phys. Rev. D2, 910 (1970).  
<sup>11</sup>F.A. Berends, K.J.F. Gaemers, and R. Gastmans, (CERN preprint - TH 1562).

#### TABLE CAPTIONS

Table I. Radiative corrections (in %) to  $e^+e^- \rightarrow \mu^+\mu^-$  for different beam energies  $E$  as a function of the angle  $\theta$  between  $e^-$  and  $\mu^-$ . We have put  $\frac{\Delta E}{E} = 1\%$  and the polarizations  $|p_-| = |p_+| = 0$  (for polarized beams this is equivalent to detection at  $\phi = \frac{\pi}{4}$ ).

Table II.  $\Delta(\theta, \phi, \phi')$  as a function of  $\phi'$  for  $\theta = 75^\circ$ ,  $\phi = 0$ ,  $E = 4$  GeV, and  $p_+ = -p_- = .924$ .

#### FIGURE CAPTIONS

Fig. 1. (a) Lowest order diagram for  $e^+e^- \rightarrow \mu^+\mu^-$ .  
 (b) - (k) Radiative corrections to  $e^+e^- \rightarrow \mu^+\mu^-$ .

Fig. 2. Asymmetry as a function of  $\theta$ . We have put  $p_+ = -p_- = 0.924$ ,  $\phi = 0$  (or  $\pi$ ), and  $\frac{\Delta E}{E} = 1\%$ . The dashed line corresponds to  $A^{EM}$ , and the solid lines to  $(A_{-i}^{EM} + A^{Weak})$  for  $E = 2.5, 4, \text{ and } 5.5$  GeV.

TABLE I

E (GeV)	1.0		2.5		4.0		5.5	
$\theta$	$\delta_{\text{sym}}$	$\delta$	$\delta_{\text{sym}}$	$\delta$	$\delta_{\text{sym}}$	$\delta$	$\delta_{\text{sym}}$	$\delta$
30°	-20.1	-10.4	-23.5	-13.7	-25.1	-15.4	-26.3	-16.5
60°	-23.4	-19.3	-27.2	-23.1	-29.2	-25.0	-30.5	-26.3
90°	-24.3	-24.3	-28.2	-28.2	-30.2	-30.2	-31.6	-31.6
120°	-23.4	-27.6	-27.2	-31.4	-29.2	-33.3	-30.5	-34.6
150°	-20.1	-29.9	-23.5	-33.2	-25.1	-34.9	-26.3	-36.1

TABLE II

$\phi'$	$\Delta^{\text{EM}}$ in %	$\Delta^{\text{Weak}} + \Delta^{\text{EM}}$ in %
30°	- .4	- 1.0
60°	- .5	- 1.4
90°	- .6	- 1.5