

A Scaling Relation for a Quantity Related To
Particle Production Multiplicities

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ABSTRACT

A scaling relation, $\frac{d}{d \ln s} \sigma_n(s) \rightarrow p(n/\ln s)$, for high energy production of n particles is proposed. This relation is supposed to be valid for large n and large s . An extension to present energies is suggested and compared with experiment.

Various models of high energy production yield limiting (in energy) relations for certain experimental quantities. It is a hope that deviations from these relations are small at present large but finite energies and that we may confront these theoretical ideas with current data. In this note we present a scaling relation for a quantity related to σ_n , the cross section for producing n particles of a certain type. The index n may refer to charged particles, negative particles, pions, etc. We consider a process where these n particles are produced by an incident state whose center of mass energy is \sqrt{s} .

Let

$$Y = a \ln (s/s_0); \quad (1)$$

s_0 and a are at present arbitrary but finite. The limiting relation we propose is

$$\lim_{n, Y \rightarrow \infty; n/Y = \rho} \frac{\partial}{\partial Y} \ln \sigma_n(Y) = p(\rho) \quad (2)$$

If this relation is true then the left hand side which is a priori a function of two variables, n and Y , approaches at high energies a nontrivial function of their ratio.

The assumptions necessary to establish (2) are:

(1) Correlations in inclusive production are taken to be of short range.¹ More generally we assume the existence of the thermodynamic limit of the Feynman Fluid Analog² to multiparticle production.

Specifically if

$$Q(z, Y) = \sum z^n \sigma_n(Y) \quad (3)$$

we assume that the following limit exists.

$$\lim_{Y \rightarrow \infty} \frac{\ln Q(z, Y)}{Y} = p(z), \quad (4)$$

with $p(z)$ some finite function of the parameter z .

(2) We need an assumption about the rate of decrease of $\sigma_n(Y)$ with Y fixed and n increasing. The simplest assumption is that $\sigma_n = 0$ for $n > N(Y)$ where $N(Y)$ is bounded by a power of Y . (The kinematic limit $N = \sqrt{s}/m$ is not sufficient.) The stringent requirement that $\sigma_n \equiv 0$ for $n > N$ could be relaxed to a smooth but rapid decrease. Not wishing to get involved in delicate details we use the above simple assumption.³

In the Feynman Fluid Analog $Q(z, Y)$ corresponds to the grand canonical partition function, $p(z)$ to the pressure as a function of the fugacity z .⁴ In this framework $\sigma_n(Y)$ is the analog of the partition function in the canonical ensemble and (2) is just relation between this partition function and the pressure which is a function of the density ρ . The derivation of the equivalence of the two ways of obtaining the pressure is identical to that in statistical mechanics⁵ and will not be reproduced here.

As mentioned previously (2) is to hold for large n and Y . If we wish to test it with presently available data we must decide on what value to assign to s_0 in (1). For present energies the value of s_0

may be crucial for the test of (2). An appealing suggestion comes from the fluid analog itself. Y is related to the length of the plateau in one particle inclusive production and the average inelastic multiplicity, $\langle n \rangle$, is, in this analog, directly proportional to Y . Thus, it is plausible that a proper continuation of (2) to present energies is to replace Y by $\langle n \rangle$. The scaling hypothesis we propose to test is

$$\frac{\partial}{\partial \langle n \rangle} \ln \sigma_n(\langle n \rangle) = p(n/\langle n \rangle). \quad (5)$$

The data to which relation (5) was applied to was on proton-proton collisions into n negative particles. Prong distributions used were for incident momenta of 28.5,⁶ 50,⁷ 69,⁷ 103,⁸ 205,⁹ and 303¹⁰ GeV/c. A quadratic approximation was used to obtain the derivatives with respect to $\langle n \rangle$ and thus these are available only for the four middle energies. The results are presented in Fig. 1. Due to the sizable error bars we may only conclude that (5) is consistent with present data. The solid line in Fig. 1 is what we would obtain if σ_n would be given by a Poisson distribution in the negative prongs.

Before closing we should contrast (2) or (5) with a different scaling hypothesis for σ_n . Koba, Nielsen and Olesen have suggested that in the same limit as considered in here

$$\langle n \rangle \sigma_n / \sigma \rightarrow F(n/\langle n \rangle) \quad (6)$$

This relation is inconsistent with (2) or (5) as it is obtained from different assumptions. There is support for the consistency of (6)

with experimental data.¹² For both (5) and (6) neither the accuracy of the data nor the differences in $\langle n \rangle$ between the lowest and highest energies permit a definite statement on these relations. It is possible to find phenomenological fits to σ_n satisfying either (5) or (6) and over the present range of $\langle n \rangle$ which appear (within experimental error) to satisfy the other.¹³ We may thus only note the consistency of both relations with present experiment.

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FIGURE CAPTION

Figure 1: Plot on $\frac{d}{d\langle n \rangle} \ln \sigma_n$ vs. $n/\langle n \rangle$ for the reaction $pp \rightarrow n$ negative prongs. $\langle n \rangle$ is the average inelastic number of negative prongs. The derivatives are evaluated for 50, 69, 103 and 205 GeV/c. The data used is from Ref. 6 - 10.

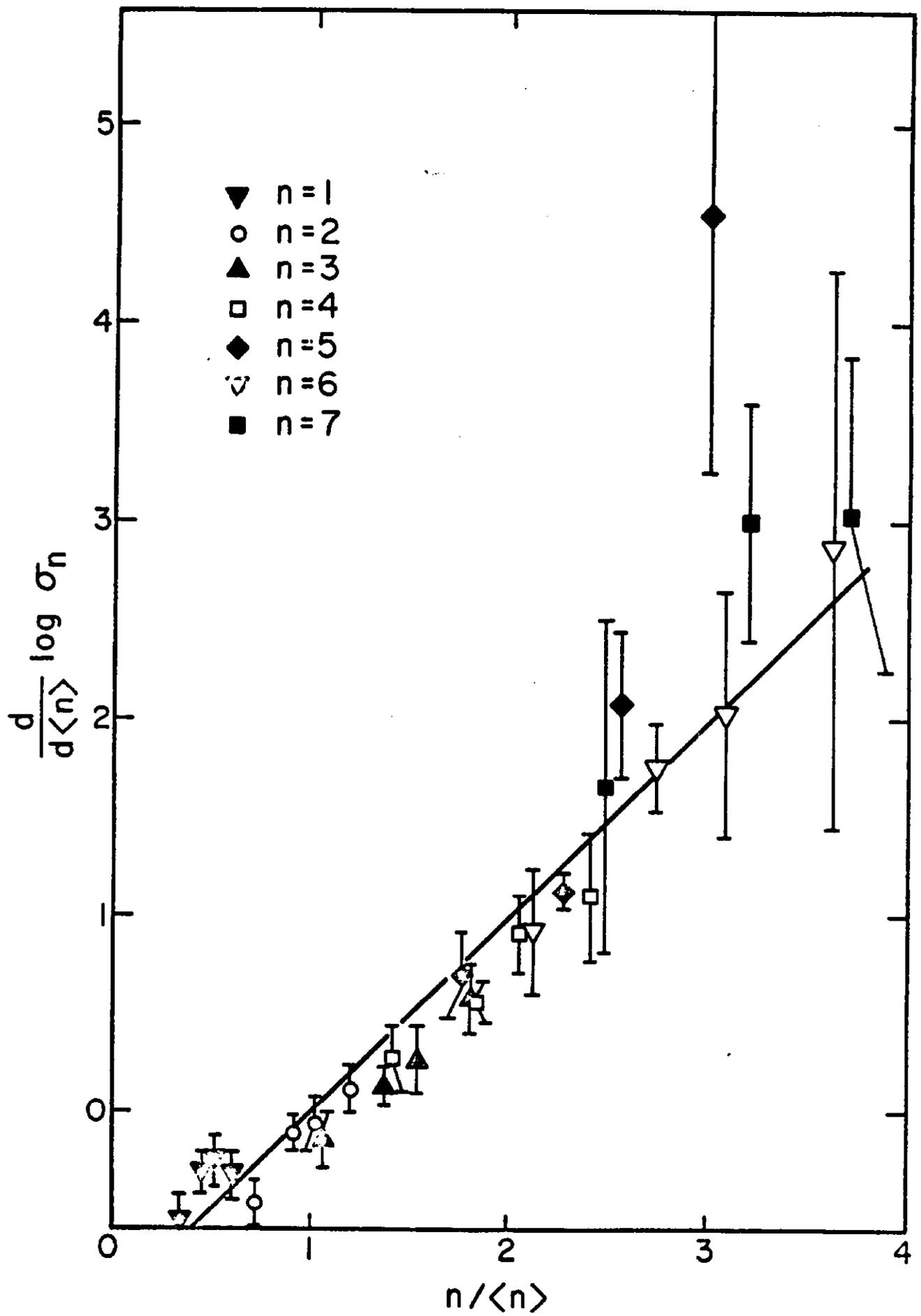


Fig. 1