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Scaling Predictions Independent of Neutrino Spectrum

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ABSTRACT

It is pointed out that the scaling hypothesis and available data imply stringent constraints for the mean energy of the muon and other related quantities. A comparison between electroproduction and neutrino data provides additional support to the proposition that the functional forms of $F_2^{\gamma N}(x)$ and $F_2^{\nu N}(x)$ are similar.

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This letter is motivated by two factors: recent measurements of the total cross sections of neutrinos and antineutrinos by the Gargamelle collaboration[1] and an interest in formulating theoretical predictions in terms of quantities independent of the explicit form of the neutrino specturm. The results of recent experiments[1] indicate that very simple relations may exist between the structure functions describing deep inelastic scattering. This in turn allows the formulation of precise theoretical consequences in terms of quantities which are independent of the neutrino spectrum. Some of the predictions discussed in this letter have already been described in the literature.[2,3] Our aim is to pursue these ideas further and point out that the CERN data imply stringent bounds for the experiments at NAL.

Bounds for the mean muon energy. One of the main physical quantities to be measured in the reaction

$$\nu + N \rightarrow \mu + X \tag{1}$$

is the mean energy of the muon. We remark that within the framework of the scaling hypothesis $\langle E_\mu \rangle$, $\langle Q^2 \rangle$ and other related quantities are bounded. For instance

$$0.50 \leq \left\langle E_\mu / E \right\rangle_\nu \leq 0.75 \tag{2}$$

This result may be obtained readily by following the same line of reasoning as that used in Ref. 4 in order to bound $\sigma^{\bar{\nu}}/\sigma^{\nu}$. Assuming the scaling of all three structure functions, the total cross section can be represented in the form:

$$\sigma^{\nu} = \frac{G^2 ME}{\pi} \int_0^1 dy \int_0^1 dx\, F_2(x) \left\{ (1-y) + y(L) - y(1-y)(R) \right\} \tag{3}$$

where in the standard notation

$$y = \frac{\nu}{E}\,,\quad x = \frac{Q^2}{2M\nu} \quad \text{and} \quad (L) = \frac{\sigma_L}{\sigma_L + \sigma_R + 2\sigma_S}\,,\ (R) = \frac{\sigma_R}{\sigma_L + \sigma_R + 2\sigma_S} \tag{4}$$

where σ_L, σ_R, σ_S are the cross sections for the absorption of a left-handed, right-handed and scalar current, respectively. It also follows that (L) and (R) are functions of the variable x only. The same argument can be repeated for the mean muon energy, where after integration over x and y one obtains:

$$\left\langle E_\mu/E \right\rangle_\nu \equiv \frac{\int \frac{E_\mu}{E} \frac{d\sigma}{dQ^2 d\nu} dQ^2 d\nu}{\sigma_{tot.}} = \frac{\frac{1}{3} + \frac{1}{6}\langle L\rangle - \frac{1}{12}\langle R\rangle}{\frac{1}{2} + \frac{1}{2}\langle L\rangle - \frac{1}{6}\langle R\rangle} \tag{5}$$

The symbol < > stands for the mean values of the quantities. The mean values of the cross section ratios are bounded by

$$0 \leq \langle L\rangle \leq 1\,,\quad 0 \leq \langle R\rangle \leq 1\,,\quad 0 \leq \langle L\rangle + \langle R\rangle \leq 1\,. \tag{6}$$

As a result we find the limits[5] of Eq. (2).

We mention two results[6,7] obtained earlier. Volkov and Folomeshkin[7] calculated $<E_\mu/E>$ assuming scaling and some particular relations between the structure functions. A result close to $<E_\mu/E> = 1/2$ was obtained. Bjorken derived an expression for a muon inelasticity relevant to cosmic ray experiments. Furthermore, the bounds obtained depend explicitly on the initial neutrino spectrum, so than any coincidence is just incidental. The form of the spectrum used in the paper is relevant to experiments with cosmic ray neutrinos but not for experiments on accelerators.

Bounds for the square of the four momentum transfer. It was mentioned first by Myatt and Perkins[3] that $<Q^2>$ should be proportional to 2ME in the scaling region. Existing experimental evidence does not contradict this prediction. We obtain here limits on the coefficient of proportionality. Following the steps of our previous discussion one easily obtains

$$\left\langle \frac{Q^2}{2ME} \right\rangle_\nu = \frac{\int_0^1 x F_2(x)\, dx}{\int_0^1 F_2(x)\, dx} \; \frac{\frac{1}{6} + \frac{1}{3}\langle L \rangle - \frac{1}{12}\langle R \rangle}{\frac{1}{2} + \frac{1}{2}\langle L \rangle - \frac{1}{6}\langle R \rangle} \tag{7}$$

Unfortunately little can be said about the ratio of integrals over x using scaling alone. Thus we come to the following limit

$$0 \leq \left\langle \frac{Q^2}{2ME} \right\rangle_{\nu,\bar{\nu}} \leq \frac{1}{2} \tag{8}$$

where the subscript ν or $\bar{\nu}$ indicates neutrino and antineutrino reactions, respectively. Equation (7) however, is useful for determining the integral $\int xF_2(x)dx$ which provides an independent test of the parton model, to be discussed later on. It is needless to emphasize that the main virtue of the bounds (2) and (8) lies in the fact that they are independent of the neutrino spectrum. It is sufficient to know the energy of the muon and final hadrons.

Bounds and the recent CERN data.

Existing experimental data obtained at CERN do not contradict the hypothesis that scaling is already observed. As mentioned by numerous authors this is a rather surprising fact since the values of ν and Q^2 are not in fact large. Thus, the apparent confirmation of scaling could be just an effect of small statistics. Bounds obtained in the preceeding sections can be checked using all the statistics available. For example, if one assumes that the total cross section rises linearly for $E > 2$ GeV, the ratio $\sigma^{\bar{\nu}}/\sigma^{\nu}$ is known with good accuracy

$$\frac{\sigma^{\bar{\nu}}}{\sigma^{\nu}} = 0.377 \pm 0.023 \tag{9}$$

It is close to the lowest bound allowed by scaling. In view of this result it is reasonable to consider the previous bounds in the case when the ratio of the cross sections is close to 1/3

$$\frac{\sigma^{\bar{\nu}}}{\sigma^{\nu}} = \frac{1}{3}\left(1+\varepsilon\right), \quad \varepsilon \ll 1 \quad . \tag{10}$$

For $\epsilon = 0$ it follows that $\langle L \rangle = 1$ and $\langle R \rangle = 0$. Knowledge of ϵ alone is not sufficient to determine the deviations of $\langle L \rangle$ from one or $\langle R \rangle$ from zero. It provides however the constraint[8] equation

$$\left\langle \sigma_R/\sigma_L \right\rangle + {}^3/_4 \left\langle \sigma_S/\sigma_L \right\rangle = {}^3/_8\, \varepsilon + O\left(\varepsilon^2\right) \tag{11}$$

which leads to the following inequalities:

$$\frac{1}{2} \leq \left\langle {}^{E_\mu}/_E \right\rangle_\nu \leq \frac{1}{2} + \frac{1}{12}\varepsilon \tag{12}$$

$$\frac{3}{4} - \frac{9}{32}\varepsilon \leq \left\langle {}^{E_\mu}/_E \right\rangle_{\bar{\nu}} \leq \frac{3}{4} \tag{13}$$

$$\frac{2}{3} \leq \left\langle {}^{E_\mu}/_E \right\rangle_\nu \Big/ \left\langle {}^{E_\mu}/_E \right\rangle_{\bar{\nu}} \leq \frac{2}{3} + \frac{13}{36}\varepsilon \tag{14}$$

$$2 - \frac{19}{8}\varepsilon \leq \left\langle {}^{Q^2}/_{2ME} \right\rangle_\nu \Big/ \left\langle {}^{Q^2}/_{2ME} \right\rangle_{\bar{\nu}} \leq 2 \tag{15}$$

These inequalities are very restrictive, considering the small value of ϵ ($\sim .132 \pm .023$). Relation (14) seems particularly useful, since for identical neutrino and antineutrino beams it can be tested by measuring only the energies of μ^+ and μ^-'s. In case that the agreement of Eq. (22) with the bounds for the ratio $\sigma^{\bar{\nu}}/\sigma^{\nu}$ is just fortuitous due to small statistics or the inclusion of isobar production,[9] then one would expect violations of bounds (12) - (15).

The same constraint equation also bounds the ratio in Eq. (7)

$$\frac{1}{2} - \frac{1}{12}\varepsilon \leqslant \frac{\frac{1}{6} + \frac{1}{3}\langle L\rangle - \frac{1}{12}\langle R\rangle}{\frac{1}{2} + \frac{1}{2}\langle L\rangle - \frac{1}{6}\langle R\rangle} \leqslant \frac{1}{2} \tag{16}$$

so that for all practical purposes* it is sufficient to assume $\langle R\rangle = 0$ and $\langle L\rangle = 1$, which gives a ratio of 1/2. Equation (7) now leads to

$$\int_0^1 x\, F_2(x)\, dx \approx 2 \left\langle \frac{Q^2}{2ME} \right\rangle_2 \int_0^1 F_2(x)\, dx \approx .11 \tag{17}$$

where we appeal to the recent CERN[2] result

$$\int F_2(x)\, dx = 0.49 \pm 0.07 \tag{18}$$

and an older evaluation[3] of

$$\left\langle \frac{Q^2}{2ME} \right\rangle \approx \frac{1}{9} \quad . \tag{19}$$

We compare the value in Eq. (17) with the prediction of two standard hypotheses: (1) the parton (light-cone) prediction $W_2(V) = W_2(A)$ (2) the isoscalar contribution to $F_2^{\gamma p} + F_2^{\gamma n}$ is ~ 10%. The SLAC-MIT data for electroproduction[10] yield:

$$\int_0^1 x\, F_2(x)\, dx \approx 0.12 \tag{20}$$

The agreement between Eq. (17) and (20) is remarkable. The same comparison[2] in terms of the $\int F_2(x)dx$ also established remarkable

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*Strictly speaking the quantity ϵ occuring in (16) could be slightly smaller than that in Eq. (10).

agreement. The present test is, of course, independent and provides additional support to the proposition that the functional forms of $F_2^{\gamma N}(x)$ and $F_2^{\nu N}(x)$ are very similar.

Intermediate boson and heavy lepton effects. There are two effects which will modify the bounds discussed: the presence of either a W-boson or of a heavy lepton. In the case of a W-boson we make the substitution

$$G^2 \longrightarrow \frac{G^2}{\left(1+\frac{s}{M_W^2}xy\right)^2}, \quad s = 2ME + M^2$$

and proceed as before by setting $\langle R \rangle = 0$ and $\langle L \rangle = 1$. Figure (1) shows $\langle E_\mu / E \rangle$ as a function of s/M_W^2. In calculating the integrals we again appeal to the electroproduction data by following the two assumptions stated after Eq. (19). We notice in the figure that the deviations from a straight line become noticeable for $s/M_W^2 \sim 1 - 2$.

The presence of a heavy lepton produces just the opposite effect. The process now proceeds through the steps

$$\nu + p \rightarrow \ell^* + X$$
$$\hookrightarrow \ell + Y$$

In such a reaction ℓ^* carries, in the mean, half of the neutrino energy. In the subsequent decay ℓ carries only a fraction of ℓ^*'s energy so that $\langle E_\mu / E \rangle$ should be lower than 1/2.

REFERENCES

[1] Preliminary results of the Gargamelle Collaboration were presented by Ph. Heusse at the XVI International Conference on High Energy Physics, Chicago-Batavia, 1972.

[2] D. H. Perkins, Invited talk at the XVI International Conference on High Energy Physics, Chicago-Batavia, 1972.

[3] G. Myatt and D. H. Perkins, Phys. Letters 34B, 542 (1971).

[4] J. D. Bjorken and E. A. Paschos, Phys. Rev. D1, 3151, 1970.

[5] It is worth mentioning that if only νW_2 scales then the ratio of total cross sections lies again between the same two bounds. The bounds for the mean muon energy are weakened being 1/4 and 1.

[6] J. D. Bjorken, Nuovo Cimento LXV III, 569 (1970). We wish to thank Professor J. D. Bjorken for a valuable discussion concerning his work.

[7] A. A. Volkov and V. N. Folomeshkin, IHEP-(70-100), Serpukhov preprint, 1970.

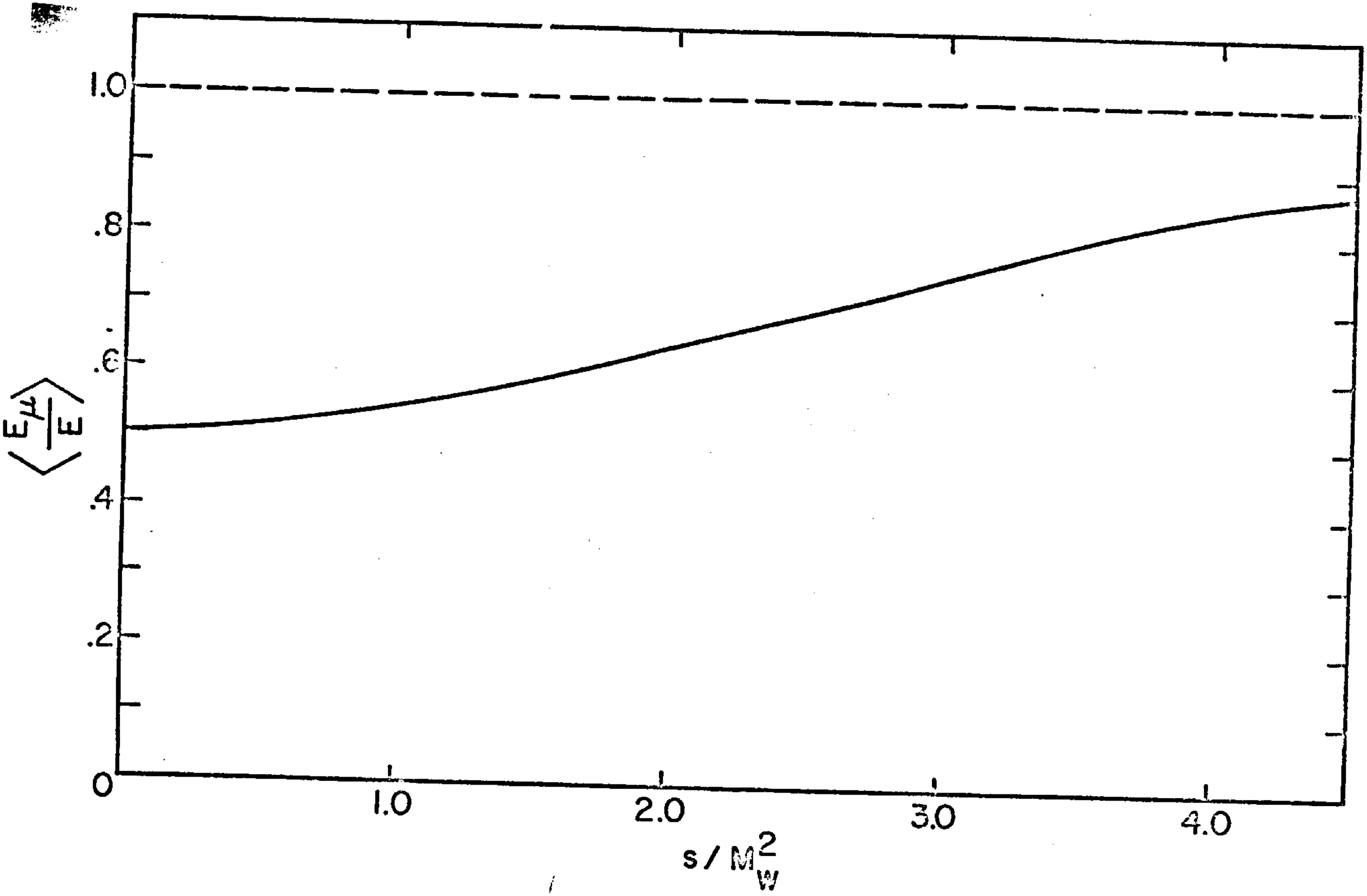
[8] The constraint equation gives $<\sigma_S/\sigma_L> \leq \frac{1}{2}\,\epsilon \sim 0.012$ in good agreement with the corresponding ratio in electroproduction and the parton prediction.

[9] Isobar production seems to constitute a large fraction of the neutrino and antineutrino interactions and if subtracted from σ_{tot}, the ratio of the remaining cross sections seems to violate the bound.

[10] G. Miller et al., Phys. Rev. D5, 528 (1972). A. Bodek et al., paper presented by M. R. Sogard to the XVI International Conference on High Energy Physics, Chicago-Batavia, 1972.

FIGURE CAPTION

The mean energy of the muon in the presence of an intermediate vector boson as a function of s/M_w^2.



1.0
.8
.6
.4
.2
0
1.0
2.0
3.0
4.0
$\left\langle \frac{E_\mu}{E} \right\rangle$
s / M_W^2