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Calculations in and Constraints on a Model of Weak Interactions*

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CALCULATIONS IN AND CONSTRAINTS ON A MODEL OF WEAK INTERACTIONS

In this lecture I will discuss results of some calculations in the spontaneously broken O_3 gauge model of weak and electromagnetic interactions due to Howard Georgi and Shelly Glashow. I will begin by explaining how this model compares with other models by showing how it fits into several different classification schemes for models of this sort. I will then describe several phenomena in this model - anomalous magnetic moments of μ and e, scattering, strangeness-changing decays - and discuss constraints on the model that follow from them.

I. SOME CLASSIFICATIONS OF MODELS

The following table summarizes four distinct but related ways of classifying models, using as examples the first Weinberg model[2] (W), the models of Lee[3] and Prentki and Zumino[4] (LPZ, PZ2), and the Georgi-Glashow model[1] (GG). One can classify by

A. Sorts of neutral leptonic currents present

1. neutral $\bar{\nu}\nu$ currents, no heavy leptons needed (W)
2. no neutral $\bar{\nu}\nu$ currents, heavy leptons needed
   a. $\bar{e}e$ weak current (LPZ)
   b. $\bar{\nu}_e X^0$ as well as $\bar{e}e$ (PZ2) (X^0 is a neutral heavy lepton)
3. no weak neutral current at all, heavy leptons needed (GG)

B. Gauge group used

1. $(SU_2)_L \times (U_1)_R$, hence 4 vector mesons = $W^{\pm}$, Z, γ. The various models (W, L, PZ) differ in the representations used for fermions and scalars.
2. O_3, hence 3 vector mesons: $W^{\pm}$, γ (GG)
3. Big groups: SU_3, $SU_3 \times SU_3$, etc.

C. Method of construction

1. Model constructed by tensoring together two (or more) gauge groups; e.g., $SU_2 \times U_1$. The neutral vector bosons Z and γ are a sum of the vector fields that transform under SU_2 and U_1. The effective Fermi coupling $G/\sqrt{2} = e^2/8M_W^2 \sin^2\theta_W$, so

M_W is bounded below. The charged vectors fields and currents transform under SU_2; weak universality is automatic

2. Model constructed with a simple gauge group, by mixing the left-handed parts of ν_e, ν_μ and heavy neutral leptons X^o, Y^o. The effective Fermi coupling between ordinary (not heavy) leptons is $G/\sqrt{2} = e^2 \sin^2 \beta / 4M_W{}^2$, so M_W is bounded above. Weak universality is forced for the ordinary leptons. The heavy leptons can be coupled much more strongly (no $\sin \beta$ suppression).

D. Method of avoidance of triangle anomaly[5]

1. Cancellation of left- and right-handed anomalies against one another: $(d_{abc})_L = (d_{abc})_R$.

   a. "By arrangement" (anomalies of different fermion fields cancel; e.g., hadronic fermions or heavy leptons needed to cancel anomaly from e, ν_e, and M, ν_μ in the Weinberg model)

   b. By "vector-like" coupling: $(\bar{\psi}\, \underset{\sim}{T}\, \gamma_\mu\, \psi) \cdot \underset{\sim}{W}^\mu$ (no explicit γ_5) (GG)

2. Right and left handed anomalies separately vanish: $(d_{abc})_{L,R} = 0$. "Safe" groups or representations.[5] (No such models in the literature.)

II. PHYSICS IN THE GEORGI-GLASHOW MODEL: LEPTONS

The special features of the Georgi-Glashow O_3 model are thus: (1) no neutral current except the electromagnetic current, (2) vector-like coupling and no anomalies, (3) $M_W = \sin\beta\,\sqrt{2}$ (37 GeV) = $\sin\beta$ (53 GeV), (4) heavy leptons X^o, X^+ in e multiplet, Y^o, Y^+ in μ multiplet - for example

$$\psi_e = \begin{pmatrix} X^+ \\ (\nu_e \sin\beta + X^\circ \cos\beta)_L + X^\circ_R \\ e^- \end{pmatrix},$$

where $(X)_{\substack{L\\R}} = P_\pm X$, $P_\pm = \frac{1}{2}(1\pm\gamma_5)$ (Bjorken and Drell metric).

Explicitly, the terms in the interaction Lagrangian of interest to us are

$$\begin{aligned}
-\mathcal{L}_{int} = \; & e A_\alpha(\bar{X}^+\gamma^\alpha X^+ - \bar{e}^-\gamma^\alpha e^-) \\
& + e W^+_\alpha(\sin\beta\; \bar{\nu}\gamma^\alpha P_- e^- + \cos\beta\; \bar{X}^\circ\gamma^\alpha P_- e^- + \bar{X}^\circ\gamma^\alpha P_+ e^-) + h.c. \\
& + e W^+_\alpha(\sin\beta\; \bar{X}^+\gamma^\alpha P_-\nu + \cos\beta\; \bar{X}^+\gamma^\alpha P_- X^\circ + \bar{X}^+\gamma^\alpha P_+ X^\circ) + h.c. \\
& + e\, \frac{m_{X^+} - m_e}{2M_W}\, \phi\,(\bar{X}^+ X^+ - \bar{e}^- e^-) \\
& + e\, \frac{m_{X^+} + m_e}{2M_W} \sin\beta \left[\tan\beta\; \phi\, \bar{X}^\circ X^\circ + \phi\,(\bar{X}^\circ P_-\nu + \bar{\nu} P_+ X^\circ)\right] \\
& + \{e\to\mu,\; X\to Y,\; \text{etc.}\} + \cdots
\end{aligned}$$

The unusual features, as compared for example with the Weinberg model,[2] are

1. Although the ordinary weak coupling $W_\alpha^+\ \bar{\nu}\ \gamma^\alpha P_- e$ is suppressed by a possibly small factor of $\sin\beta$, the couplings to heavy leptons are not.

2. The coupling of the Higgs scalar meson ϕ to leptons is of order em_{Y^+}/M_W. The masses m_{X^+} and m_{Y^+} are not necessarily small compared to M_W.

Both of these features can cause higher order contributions in the GG model to be considerably larger than in the W or LPZ models, if the W or ϕ masses are small and/or the heavy lepton masses are large in the GG model. For this reason, I regard the GG model as "optimistic". It does not shy away from confrontation with experiment. By comparison the L and PZ models are more "pessimistic".

g-2

The close agreement between the measured values of g_e and g_μ and the predictions of quantum electrodynamics implies that any weak contribution to the lepton magnetic moments must be very small. The experimental values are[6] $[a \equiv \frac{1}{2}(g-2)]$

$$(a_\mu)_{\text{expt}} = (11661.6 \pm 3.1) \times 10^{-7}$$
$$(a_e)_{\text{expt}} = (1159657.8 \pm 3.5) \times 10^{-9}.$$

The contributions of qed are[6]

$$(a_\mu)_{\text{qed}} = (11658.1 \pm 0.2) \times 10^{-7}$$
$$(a_e)_{\text{qed}} = (1159655.0 \pm 3.1) \times 10^{-9}.$$

The strong corrections to the qed calculations are estimated to be $(0.65 \pm 0.1) \times 10^{-7}$ for a_μ, and negligibly small for a_e; so

$$(a_\mu)_{\text{expt}} - (a_\mu)_{\text{qed}} - (a_\mu)_{\text{strong corrections}} = (2.8 \pm 3.1) \times 10^{-7}$$
$$(a_e)_{\text{expt}} - (a_e)_{\text{qed}} = (2.8 \pm 4.0) \times 10^{-9}.$$

It thus seems reasonable to conclude that

$$-3 \times 10^{-7} \le (a_\mu)_{\text{weak}} \le 9 \times 10^{-7}$$
$$-5 \times 10^{-9} \le (a_e)_{\text{weak}} \le 11 \times 10^{-9},$$

allowing for a discrepancy of two standard deviations.

The four Feynman graphs which contribute to muon g-2 in lowest order in the GG model are drawn in Fig. 1. The first graph (a) is just the familiar $\alpha/2\pi$ electromagnetic correction. The graph (b) has been calculated by several authors.[7] Since $g_W = 2$ in models where the vector mesons have Yang-Mills couplings, there is no divergence and the graph gives

$$(a_\mu)_b = \frac{\alpha \sin^2\beta}{8\pi} \frac{m_\mu^2}{M_W^2} \frac{10}{5} \approx 3 \times 10^{-9} ,$$

which is negligible.

The remaining two graphs give the important contributions. Graph (c) requires careful treatment, a fact of which I was insufficiently aware initially. We[8] initially calculated this graph without regularization and found that the contribution to $(g_\mu - 2)$ proportional to $m_{Y^0} m_\mu$ was finite and independent of the routing of the internal momentum. However, a more careful calculation using the 't Hooft-Veltman regularization gives an additional contribution (the final term in the bracket, +1).

$$(a_\mu)_c = -\frac{\alpha}{8\pi} \frac{m_\mu m_{Y^0}}{M_W^2} \left[\frac{3}{(1-r)^2} \left(1 - 3r - \frac{2r^2}{1-r} \ln r \right) + 1 \right] , \quad r \equiv \left(\frac{m_{Y^0}}{M_W} \right)^2$$

We did this regulated calculation after B. W. Lee told us that he and collaborators[9] had obtained the above result by using another technique (their R_ξ formalism). It is satisfying that these two different calculational procedures give the same answer.

The final graph (d) was evaluated by Jackiw and Weinberg[10] in the Weinberg[2] model and found to be $\sim 10^{-9}$. For the GG model it is considerable larger:

$$(a_\mu)_d = \frac{\alpha}{8\pi}\,\frac{m_{Y^+}^2}{M_W^2}\,\frac{m_\mu^2}{\phi^2}\int_0^1 dz\,\frac{2z^2-z^3}{z^2\rho-z+1}\ , \quad \rho\equiv\left(\frac{m_\mu}{M_\phi}\right)^2 .$$

The contributions of graphs (c) and (d) to electron g-2 are smaller by factors of m_e/m_μ and $(m_e/m_\mu)^2$, respectively.

If we assume that m_{Y^+} and m_{Y^0} $[= (m_{Y^+} - m_\mu)/2\cos\beta]$ are both larger than 1/2 GeV, consistent with the nonobservation of these heavy leptons in K decay or at accelerators, then from the lower limit on $(a_e)_{weak}$ given above, we conclude that

$$M_W > 10\ \mathrm{Gev}\ .$$

We could get a stronger lower bound on M_W from a_μ, if it were not for the fact that the ϕ graph (d) contribution is comparable to that of the Y^0 graph (c) and opposite in sign. If we can from some other considerations[11] deduce that $m_{Y^+}\, m_\mu/m_\phi^{\ 2} \ll 1$, so that the ϕ graph contribution is negligible, then we can conclude that

$$M_W > 20\ \mathrm{Gev}\ .$$

It also follows in this case that the mass of the charged heavy lepton Y^+ is effectively bounded above by

$$m_{Y^+} < 7\ \mathrm{Gev}\ ,$$

for $M_W < 50$ GeV. (Although the actual upper limit on possible values of M_W in this model is 53 GeV, this is a singular limit of the model.)

Clearly, although the experimental data is already astonishingly accurate, any improvement in the determination of g_μ or g_e could be very helpful in constraining theoretical models of weak interactions.

PRODUCTION AND DECAY OF HIGGS SCALAR AND LEPTONS

Scalar meson (ϕ). The ϕ will decay into e^+e^- and $\mu^+\mu^-$ in roughly equal proportions, if $m_{X^+} \approx m_{Y^+}$. The partial decay rates are

$$\Gamma(\phi \to e^+e^-) = \frac{\alpha}{8}\left(\frac{m_{X^+}}{M_W}\right)^2 M_\phi$$

$$\Gamma(\phi \to \mu^+\mu^-) = \frac{\alpha}{8}\left(\frac{m_{Y^+}}{M_W}\right)^2 M_\phi \,.$$

Thus the ϕ is relatively short-lived: a ϕ of mass 500 MeV/c^2 would have a half-life of less than 1.5×10^{-17} sec. If it is sufficiently massive, which it presumably is, the ϕ will also have hadronic decay modes.

The ϕ can be produced by "bremsstrahlung" in sufficiently energetic scattering events. It can also be exchanged virtually in many scattering processes, and may contribute significantly to certain processes which are regarded as tests of qed. In particular, it can be searched for in e^+e^- colliding beam experiments, where it would produce a large but narrow peak. For example, in $e^+e^- \to \mu^+\mu^-$, for a bin of width $\Delta s = \eta M_\phi^2$ centered at M_ϕ^2, one would see an enormous cross-section enhancement

$$\frac{\sigma}{\sigma_{qed}} = 1 + \frac{3\pi}{8\alpha\eta}\left(\frac{m}{M_W}\right)^2 ,$$

where m equals the smaller of m_{X^+}, m_{Y^+}; and $\alpha = 1/137$.

Since the ϕ has not been seen in K decay or at colliding beam machines, its mass evidently exceeds 500 MeV. One does not obtain a better lower limit on M_ϕ from muon atom data.[8]

Massive Leptons.[12] The heavy μ-type lepton Y^+ will decay weakly, analogously to μ decay, except that it is sufficiently massive that the decay will be much faster ($\propto m_Y^5$); also, hadrons can appear in the final state. The Y^o has similar decay modes, for example $Y^o \rightarrow \mu^- e^+ \nu_e$ or $Y^o \rightarrow \mu^-$ + (hadrons). Such decays would obviously be very striking to observe. These remarks also apply, *mutatis mutandis*, to the e-type leptons X^+, X^o.

These heavy leptons should be produced in reactions of the sort $\nu_\mu p \rightarrow Y^+$ + (hadrons) at rates comparable to that of $\nu_\mu p \rightarrow \mu^-$ + (hadrons) as soon as sufficient energy is available. It should thus be possible at NAL either to find these particles, or else to put rather high lower limits on their mass -- which would also increase the lower limit on the W mass in the Georgi-Glashow SO(3) model (cf. $(a_\mu)_c$). The charged leptons will also be pair-produced electromagnetically as soon as suffieient energy is available in colliding beam machines. There is obviously a rich variety of ways to seek these hypothetical particles.

III. PHYSICS IN THE GEORGI-GLASHOW MODEL: HADRONS

During the summer of 1972, two groups[13] have calculated the amplitude for processes of the sort $e + \nu_\mu \to e + \nu_\mu$ or $p + \nu \to p + \nu$ in the GG and LPZ models. These models have been designed so that there is no lowest-order vector exchange contribution to such processes, as I have mentioned already. As in other calculations -- for example, the μ decay calculation by the Harvard group[14] reported on at this meeting by Tom Appelquist -- the one-loop graphs give results of order $G\alpha$ (times logarithms). Thus the cross sections for these processes are greatly suppressed (by a factor of α^2 times logarithms) compared to charged-current processes. Experimental observation of these processes with larger cross sections will therefore be of tremendous importance in delimiting acceptable theories: this is an important challenge to experimentalists.

In contrast to elastic neutrino scattering, in strangeness-changing neutral processes an amplitude of order $G\alpha$ is very large indeed. The branching ratio for $K_L \to \bar{\mu}\mu$ and the $K_1 - K_2$ mass difference are so small that they completely rule out the Georgi-Glashow "5-quark" model.[1] For example, in this model the branching ratio is

$$R = \frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \text{all})} \approx 3 \times 10^{-4},$$

independent of M_W or other adjustable parameters.[11] This is to be compared with the "unitarity limit" for R of 6×10^{-9}. (The experimental situation is uncertain, but R evidently does not differ from this value by more than a factor of three.) Georgi and Glashow[1] were aware of the danger, but they speculated that this trouble might be avoided for sufficiently small M_W; this does not turn out to be true.

Glashow, Iliopoulos, and Maiani[15] gave a method for suppressing such unwanted higher-order strangeness-changing effects. Their method consists in introducing a "charmed" p' quark with a p', $\lambda_c = -n \sin\theta_c + \lambda\cos\theta_c$ term in the charged current just like that of p with $n_c = n\cos\theta_c + \lambda\sin\theta_c$. In the limit in which $\Delta m = m_{p'} - m_p$ vanishes, $\Delta S \neq 0$ neutral processes are then completely suppressed; thus such amplitudes are proportional to the p-p' symmetry-breaking parameter Δm. An explicit calculation[11] of the $K_1 - K_2$ mass difference in the Georgi-Glashow "8 quark" model gives $\Delta m \lesssim 1/2$ GeV. Because of approximations and uncertainties in the calculation, this constraint should be regarded as approximate; nevertheless, it shows that this model may be in trouble. Similar constraints also apply to the LPZ and other models of this sort.

IV. A COMMENT

Despite the obviously attractive features of the spontaneously broken gauge approach toward constructing a theory of weak interactions, the models of this sort which have been constructed thus far possess evident shortcomings. In the absence of strong theoretical as well as experimental constraints, these models all appear rather artificial and ad hoc. Restricting our attention to models of the leptons and neglecting strong interactions, we would like our models to give at least a little insight into the role of the muon and the origin of the μ-e mass difference. Instead, additional massive leptons are introduced, and they possess an even more puzzling mass spectrum. Particularly curious in the GG model considered here is the introduction of leptons with masses exceeding 500 MeV/c^2 in the same multiplet as the electron. The large mass of the W, and the consequent weakness of the weak interactions, also remains unexplained.

The spontaneously broken gauge symmetry method will doubtless be a permanent addition to the theorist's repertoire. But additional ideas appear to be necessary in constructing the right theory of weak and electromagnetic interactions of leptons -- not to mention hadrons!

*This talk mostly reports work by Prof. Helen Quinn and myself (see Ref.8, below). I have taken the liberty to correct and update my remarks at the Marseille meeting, and to mention some relevant work done by myself and others during the summer.

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[6]See, for example, the review by S. J. Brodsky, Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energy (Cornell, 1972).

[7]T. Burnett and M. J. Levine, Phys. Letters 24B, 467 (1967). S. J. Brodsky and J. D. Sullivan, Phys. Rev. 156, 1644 (1967). The value $g = 2$ is the "natural" value for a particle of any spin, in two respects: (1) For $g = 2$, the spin-flip Compton amplitude vanishes as $\omega \to 0$ faster

ω^1; consequently the Drell-Hearn integral [Phys. Rev. Letters 16, 908 (1966)] vanishes, (2) For g = 2, the magnetic precession and the Thomas precession of a spinning particle moving in an electromagnetic field exactly cancel; indeed it is this fact which is used in high-precision determinations of g-2. [For details and references, see e.g., S. J. Brodsky and J. R. Primack, Annals of Phys. 52, 315 (1969), expecially pp. 360-362.]

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[11] B. W. Lee, J. R. Primack and S. B. Treiman, NAL preprint, August 1972, show that in the Georgi-Glashow "5-quark" model, the existence of a substantial $\phi\bar{\lambda}n$ coupling and the experimental absence of $K^+ \rightarrow \pi^+\bar{l}l$ imply just such a constraint.

[12] J. D. Bjorken and C. H. Llewellyn Smith, SLAC preprint; August 1972, have examined the various modes of production and decay of massive leptons of the sort that populate these models of weak interactions. I would like to thank them for helpful conversations.

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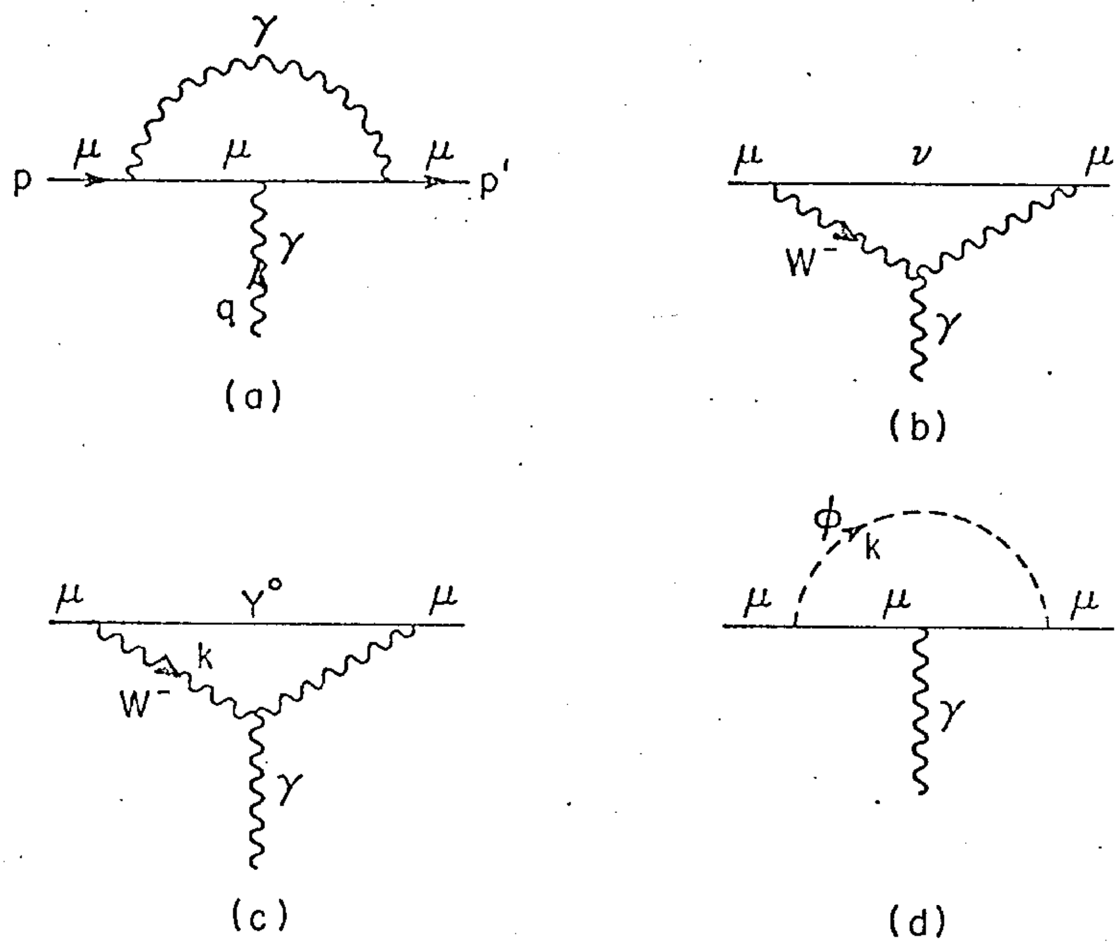

FIG. 1 THE FEYNMAN GRAPHS WHICH CONTRIBUTE TO (g−2) IN ORDER e^2.