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Production of Coherent Mixtures of Resonances in the Quark Model*

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The production of a coherent mixture of the f^o and a degenerate s-wave resonance is described in the quark model by classifying the two resonances in the J = 2 and J = 0 states of the same ^3P configuration. The model predicts experimentally observed decay angular distributions, including the cancellation of the peak in the center of the d-wave by s-wave interference.

Recent experiments indicate an s-wave resonance[1] under the f^o. The peak expected from a pure d-wave in the equatorial region of the π-π angular distribution is not observed. However, there has been no simple explanation of why the two resonances should conspire to exactly cancel the equatorial peak. The following quark-model description gives a natural explanation of this angular distribution.

Consider $\pi\pi$ scattering in the neighborhood of the f^o. The f^o in the quark model is a quark-antiquark pair in the $^3P_2$ state, i.e., it has the quantum numbers L = 1, S = 1, J = 2. The quark model also has $^3P_1$ and $^3P_0$ states from the same configuration, formed by coupling L = 1 and S = 1 to J = 1 and J = 0, respectively. We identify the observed s-wave resonance with the $^3P_0$ state. This disagrees with the common classification of the ϵ^0 as the $^3P_0$. We disregard this point here and return to it later on.

In the simplest quark model description of the production and decay of resonances by pion absorption or emission, the pion is absorbed or emitted by a single quark and the transition matrix element is given by the operator[2]

$$H_{int} \propto (\vec{\sigma} \cdot \vec{k})\, e^{i\vec{k} \cdot \vec{r}}, \qquad (1)$$

where $\vec{\sigma}$ is the spin and $\vec{r}$ is the coordinate of the active quark and $\vec{k}$ is the momentum of the emitted or absorbed pion.

Let us choose our z axis in the direction of the incident pion momenta in the center-of-mass system and consider the production of a p-wave quark-antiquark resonance via the interaction (1)—i.e., the interaction in which one of the pions is absorbed by a quark or antiquark in the other pion. Then $\vec{k}$ is in the z direction and the interaction (1) conserves both L_z and S_z individually. Since the initial two-pion state has $L_z = S_z = 0$, a $q\bar{q}$ state produced with L = 1 and S = 0 must have $L_z = S_z = 0$. This state does not have a definite J but is a mixture of the $^3P_0$ and $^3P_2$ states having J = 0 and J = 2, respectively.

Let us assume that the interaction (1) also describes the decay of this state. A pion is emitted by a quark or antiquark in the $q\bar{q}$ system that makes a transition from the L = 1, L = 0, S = 1, S = 0 state to a final pion state with L = 0, S = 0. The transition matrix element is then

$$M = \langle L=0,\ S=0 | \vec{\sigma} \cdot \vec{k}\, e^{i\vec{k} \cdot \vec{r}} | L=1,\ L_z=0;\ S=1,\ S_z=0 \rangle. \qquad (2)$$

The only component of $\vec{\sigma}$ that can contribute to a transition from (S = 1, S_z = 0) to (S = 0) is σ_z. Similarly, if $e^{i\vec{k} \cdot \vec{r}}$ is expanded in spherical harmonics, the only term that can contribute to an (L = 1, L_z = 0) → (L = 0) transition is the Y_0^1 term, which is proportional to $k_z z$. Thus the matrix element (2) reduces to the form

$$M = \langle S=0 | \sigma_z k_z | S=1,\ S_z=0 \rangle \langle L=0 | k_z z g(k^2 r^2) | L=1,\ L_z=0 \rangle, \qquad (3)$$

where $f(k^2 r^2)$ is the coefficient in the expansion of the exponential

$e^{i\vec{k}\cdot\vec{r}}$. The dependence of M on the direction of the momentum $\vec{k}$ of the outgoing pion is then given by

$$M \propto k_z^2 \propto \cos^2\theta . \qquad (4)$$

The angular distribution of the outgoing pions is then given by

$$|M|^2 \propto \cos^4\theta. \qquad (5)$$

This distribution seems to agree with experiment and has no peak at $\theta = 0$.

This argument is easily generalized to apply to arbitrary L waves in the $\pi\pi$ system, e.g., to the 3^- meson which would have a 1^- background. For the case of a natural-parity resonance with spin J, described in the quark model by a $q\bar{q}$ system with $L = J - 1$, there is another state with $J' = L - 1 = J - 2$. If these two states are degenerate, the angular distribution of $\pi\pi$ scattering coming from the coherent partial waves with angular momenta J and J - 2 is given by

$$|M_J|^2 \propto \cos^2\theta\,[P_{J-1}(\cos\theta)]^2, \qquad (6)$$

where the first factor comes from the spin transition, which is always $(S = 1) \rightarrow (S = 0)$, and the second from the orbital transition, which is $(L = J - 1) \rightarrow (L = 0)$.

The same analysis applies to $K\pi$ and $K\bar{K}$ scattering. Similar s-wave contributions might be expected for other members of the 2^+ nonet, namely the $K^*(1420)$, the A2, and the f^*. However, these cases are not as clear as that of the f^0 because of the existence of competing vector-pseudoscalar decay couplings. These are appreciable for the K^* and f^* and dominate over the two-pseudoscalar mode in the case of the A2.

For the $K^*(1420)$, the effect might be seen in the $K\pi$ decay mode if the effects of $K\rho$, $K\omega$, and $K^*\pi$ couplings can be suppressed. This is most easily achieved by producing the K^* via pion exchange with kaon beams in a kinematic region in which ρ and ω exchange can be neglected and observing the angular distribution of the $K\pi$ decay.[3]

For the A2, the dominant $\rho\pi$ coupling must be suppressed as well as the $K^*\bar{K}$ coupling, and the $K\bar{K}$ or $\eta\pi$ couplings are to be observed. This is most easily achieved by observing $K\bar{K}$ to $\eta\pi$ decays of A2's produced by kaon exchange—i. e., by kaon beams in a kinematic region in which kaon exchange is dominant and K^* exchange can be neglected.

For the f^*, production by kaon exchange is required in a region in which K^* exchange is negligible and the $K\bar{K}$ decay is to be observed.

In this discussion we have assumed that the $^3P_2$ and $^3P_0$ quark-model states are degenerate. This does not correspond to the commonly accepted quark-model classification of the experimentally observed states. In fact, the $I = 0$ $^3P_0$ state has been assumed to be the ϵ^0 identified with the s-wave background under the ρ^0. The quark model has no simple description for <u>two</u> s-wave resonances, one under the ρ and the other under the f^0. The second 0^+ state would be a radially excited p wave, and it would seem extremely unlikely for such a state to be degenerate with the f^0. On the other hand, both 0^+ states arise naturally in dual-resonance models having parallel daughter trajectories.[4]

Under these circumstances it is of interest to check whether there is experimental evidence for s-wave resonances under the other 2^+ mesons and to investigate further the spectroscopy of the 0^+ mesons.

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