



TESTS FOR NEUTRAL CURRENTS IN NEUTRINO REACTIONS

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ABSTRACT

Neutral currents predicted by weak interaction models of the type discussed by Weinberg may be detected in neutrino reactions. Limits on the ratio R of $\sigma(\nu + N \rightarrow \nu + x)$ to $\sigma(\nu + N \rightarrow \mu^- + x)$ are obtained independent of any dynamical assumption. For the total cross-section for high energy neutrinos, we find $R \geq 0.18$, provided the Weinberg mixing angle satisfies $\sin^2 \theta_w \leq 0.33$. For the production of a single π^0 we find $R' \geq 0.50$ contrasted with the experimental result $R' \leq 0.14$ using only the assumption of $(3, 3)$ resonance dominance. Applications are also given to anti-neutrino reactions.

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GENERAL RESULTS

Weak interaction models of the type recently discussed by Weinberg^{1, 2} involve neutral lepton currents which may be coupled to hadrons. There is much interest in testing for such currents in neutrino reactions. In most of the tests that have been discussed,^{2, 3} special dynamical models or approximations have been made. In this note we present tests that do not depend on any such model assumptions. Such tests have also recently been discussed by Pais and Treiman.⁴ Our results are improvements on theirs arising from new estimates of the axial contributions leading to (α) a 25 percent increase of the bound for the total cross sections under zero dynamical assumptions and (β) a derivation of the bound stated in equations (23) and (24) from a single assumption: the scaling of electro- and neutrino-induced-production data in the deep inelastic region.

The effective Lagrangian for strangeness conserving semi-leptonic processes involving neutrinos in models of the Weinberg type is given by

$$\mathcal{L} = \frac{G}{\sqrt{2}} \left\{ \bar{\nu}_\mu \gamma_\alpha (1 + \gamma_5) \nu_\mu \left(J_1^\alpha + i J_2^\alpha \right) + h.c. + \bar{\nu} \gamma_\alpha (1 + \gamma_5) \nu \left(A_3^\alpha + x V_3^\alpha + y J_s^\alpha \right) \right\} \quad (1)$$

where $J_i = V_i + A_i$ is one of the isospin components of the usual V-A currents and J_s is an isoscalar current. In the simple Weinberg⁵ form

$$x = 1 - 2 \sin^2 \theta_w \quad \text{and} \quad y = -2 \sin^2 \theta_w \quad (2)$$

with $J_s = \sqrt{3} V_8$ where θ_w is the mixing angle of the Weinberg model.

We consider in such models the cross-section ratio

$$R = \frac{\frac{1}{2} [\sigma(\nu+p \rightarrow \nu+X_1) + \sigma(\nu+n \rightarrow \nu+X_2)]}{\frac{1}{2} [\sigma(\nu+p \rightarrow \mu^-+X_3) + \sigma(\nu+n \rightarrow \mu^-+X_4)]} \equiv \frac{\sigma_0}{\sigma_-} \quad (3)$$

The final states X_i may be chosen as all possible final states in which case we are considering total cross-section ratios. Alternatively, we may consider some limited kinematic range of the standard variables Q^2 and ν . Our considerations also hold if the X_i 's are limited to a particular class of final states (such as those involving a single pion) provided we sum over all possible charge states of the final particles for each of the reactions. For the denominator in R we write

$$\sigma_- = A + I + V \quad (4)$$

where V comes from the vector current alone, A from the axial current alone and I is the interference term. It then follows by means of an isotopic spin rotation, if we set $m_\mu = 0$, that

$$\sigma_0 = \frac{1}{2} (A + xI + x^2V + y^2S) \quad (5)$$

where S is the contribution of the isoscalar current. The averaging over proton and neutron targets, or equivalently the use of an isospin-zero target, is essential to eliminate the interference term between isoscalar and isovector currents. Since $y^2 S \geq 0$

$$R \geq \frac{1}{2} \frac{A + xI + x^2V}{A + I + V} \quad (6a)$$

Furthermore, Schwarz's inequality implies

$$4AV \geq I^2 \quad (6b)$$

Combining these two inequalities we obtain

$$R \geq \frac{1}{2} \left\{ 1 - (1-x) \left(\frac{V}{A+I+V} \right)^{1/2} \right\}^2 \quad (7)$$

The term V can be deduced from a knowledge of the isovector contribution to the electroproduction cross section $\frac{1}{2} [\sigma(e+p \rightarrow e+x_1) + \sigma(e+n \rightarrow e+x_2)] \equiv \sigma_{em}$ where as before, a sum over all possible charge states of the final particles in each channel is assumed so that there is no isoscalar-isovector interference. Not knowing the isoscalar contribution, we write $V \leq \frac{G^2}{\pi} \frac{Q^4}{4\pi\alpha^2} 2\sigma_{em} \equiv V_{em}$. Combining this with Eq. (7)

$$R \geq \frac{1}{2} \left\{ 1 - (1-x) \left(\frac{V_{em}}{\sigma_-} \right)^{1/2} \right\}^2 \quad (8a)$$

$$R \geq \frac{1}{2} \left\{ 1 - 2 \sin^2 \theta_w \left(\frac{V_{em}}{\sigma_-} \right)^{1/2} \right\}^2 \quad (8b)$$

The right-hand side is now expressed in terms of experimental quantities

$$\frac{V_{em}}{\sigma_-} = \frac{G^2}{\pi} \frac{1}{4\pi\alpha^2} \frac{\int Q^4 (d\sigma/dQ^2 dv d\Gamma)_{em}}{\int (d\sigma/dQ^2 dv d\Gamma)_{\nu \rightarrow \mu^-}} \quad (9)$$

where $d\Gamma$ is a hadronic phase space factor. It should be obvious that

Eq.(8) is completely model-independent and holds for any set of final hadrons and any region of phase space.

Equation(8b) may be contrasted with the limit given by Pais and Treiman.⁴

$$R \geq \frac{1}{2} \left(1 - 2 \sin^2 \theta_w \right) \left(1 - 2 \sin^2 \theta_w \frac{V_{em}}{\sigma_-} \right) \quad (10)$$

Our Eq. 8b) gives a limit which is larger by a term

$$\sin^2 \theta_w \left(1 - \left(\frac{V_{em}}{\sigma_-} \right)^{1/2} \right)^2 \text{ and is valid for values of } \frac{V_{em}}{\sigma_-} \text{ less than } (1-x)^{-2}.$$

In spite of its transparency, this bound may not be the best one. A bound which may be better, is derived by finding a lower limit (designated (1/F)) for the expression

$$\frac{A+V}{I} = \frac{\cos^2 \frac{\theta}{2} W_2 + 2 \sin^2 \frac{\theta}{2} W_1}{\frac{E+E'}{M} \sin^2 \frac{\theta}{2} W_3} = \frac{\left(1 - \frac{Q^2}{4EE'} \right) + \frac{\nu^2 + Q^2}{2EE'} [(L) + (R)]}{(\nu^2 + Q^2)^{1/2} \frac{E+E'}{2EE'} [(L) - (R)]} \quad (11)$$

where the notation is standard in the literature.^{6,7} This is minimized

for (R) = 0, (L) = 1, giving

$$\frac{A+V}{I} \geq \frac{1}{F} = \frac{2EE' + \nu^2 + \frac{1}{2} Q^2}{(E+E')(\nu^2 + Q^2)^{1/2}} = \frac{1 - \mathcal{E}}{(1 - 2\mathcal{E})^{1/2}} \quad (12)$$

with $\mathcal{E} = 2EE' \cos^2 \frac{\theta}{2} / (E+E')^2$. The alternative bound now follows

trivially:

$$R \geq \frac{1}{2} \left[x + (1-x) \frac{1}{1+F} - (1-x^2) \frac{V_{em}}{\sigma_-} \right] \quad (13)$$

Equation (13) becomes an equality if the isoscalar contributions to σ_{em}

and σ_0 are zero⁸ and if, as a result of hadron dynamics, $\sigma_R = \sigma_S = 0$. In

applying Eq. (13) over a large range of Q^2 and ν , one must be careful

to use the smallest value of F compatible with the data; in general, this

involves separately integrating numerator and denominator in Eq. (11).

Recently, data involving anti-neutrinos have also been reported.⁹

Our general results Eq. (8) and (13) apply equally well to the ratio

$$\bar{R} = \frac{\frac{1}{2} [\sigma(\bar{\nu} + p \rightarrow \bar{\nu} + X_1) + \sigma(\bar{\nu} + n \rightarrow \bar{\nu} + X_2)]}{\frac{1}{2} [\sigma(\bar{\nu} + p \rightarrow \mu^+ + X_3) + \sigma(\bar{\nu} + n \rightarrow \mu^+ + X_4)]} \quad (14)$$

provided σ_- is replaced by σ_+ . New results may be obtained by combining ν and $\bar{\nu}$ cross-sections. The Eqs. (4) and (5) for σ_- and σ_0 are changed into equations for σ_+ and $\bar{\sigma}_0$, respectively, by changing the sign of I. Defining

$$D = \frac{\sigma_0 - \bar{\sigma}_0}{\sigma_- - \sigma_+} \quad (15)$$

we find

$$D = \frac{1}{2} x = \frac{1}{2} (1 - 2s \sin^2 \theta_w) \quad (16)$$

Thus limits on D may be used to bound $\sin^2 \theta_w$ in the neighborhood of 1/2. If we use the limit set by electron-anti-neutrino scattering¹⁰ of $\sin^2 \theta_w \leq 0.33$ then $D \geq 0.17$. Alternatively we can add neutrino and anti-neutrino cross-sections giving

$$\bar{R}' = \frac{\sigma_0 + \bar{\sigma}_0}{\sigma_- + \sigma_+} = \frac{1}{2} \frac{A + x^2 V + y^2 S}{A + V} \quad (17)$$

It then follows simply that

$$\bar{R}' \geq \frac{1}{2} \left\{ 1 - 2(1 - x^2) \frac{V_{em}}{\sigma_- + \sigma_+} \right\} \quad (18a)$$

or

$$\bar{R}' \geq \frac{1}{2} \left\{ 1 - 8 \sin^2 \theta_w (1 - \sin^2 \theta_w) \frac{V_{em}}{\sigma_- + \sigma_+} \right\} \quad (18b)$$

Equation (18) is the best possible limit since it becomes an equality if the isoscalar contributions are all zero.

Even if the anti-neutrino cross-sections are not well known, they may still be useful in improving the limits for neutrino events. If it is known experimentally that the ratio

$$\frac{\sigma_{-}}{\sigma_{+}} \leq B \quad (19)$$

then $\frac{I}{A+V} \leq \frac{B-1}{B+1}$ and from (13)

$$R \geq \frac{1}{2} \left[x + (1-x) \frac{B+1}{2B} - (1-x^2) \frac{V_{em}}{\sigma_{-}} \right] \quad (20)$$

Equation 20 may be applied to \bar{R} if σ_{-} and σ_{+} are interchanged in Eq. 19 and in the last term of Eq. 20.

TOTAL CROSS-SECTIONS

To calculate the bounds given by Eqs. (8) all that one needs are the cross-section for electro- and neutrino- induced-production. For the total cross-section of electroproduction, we make use of the scaling property from which

$$V_{em} = \frac{G^2}{\pi} \int \frac{Q^4}{4\pi\alpha^2} \left(\frac{d\sigma}{dQ^2 d\nu} \right)_{em} dQ^2 d\nu \leq \frac{G^2}{\pi} \frac{4}{3} ME \int F_2(\omega) d\omega \quad (21)$$

where $F_2(\omega) = \nu W_2$ is the standard scaling function. The above inequality holds for any value of $\sigma_\ell / \sigma_\tau$; in case that the ratio is zero, as seems to be borne out by the data for deep inelastic scattering, it becomes an equality. Using Eq. (21), we can substitute for V_{em} / σ_- in Eq. (8) the quantity

$$\frac{\int F_2(\omega) d\omega}{\sigma_-} \frac{G^2}{\pi} \frac{4}{3} ME = 0.36 \quad (22)$$

where we have¹¹ used $\int F_2(\omega) d\omega = .14 \pm .02$ and for the total neutrino cross-section¹² $\sigma_- = \frac{G^2}{\pi} ME (.52 \pm .13)$. Requiring $\sin^2 \theta \leq .33$, we find from Eq.(10) (Pais and Treiman) $R \geq 0.14$ whereas from our Eq. (8) we obtain $R \geq 0.18$.

One can use equation (13) to obtain better bounds by comparing electro- and neutrino-induced-production processes point by point. Such detailed data are not available and we discuss again the deep inelastic data. In this case

we must, however, assume in addition to the scaling of electro-production, the scaling of the neutrino-induced production data for which the experimental evidence is still rudimentary. Integrating numerator and denominator in Eq. (11), we find¹³ the limit $(1/F) = 2$. This leads to

$$R \geq \frac{1}{2} \left[\frac{2}{3} + \frac{1}{3} x - (1-x^2) \frac{V_{em}}{\sigma_-} \right] \geq 0.23 \quad (23)$$

It is interesting to compare this Eq. (23) with the equation

$$R \geq \frac{1}{6} (1+x+x^2) \geq 0.24 \quad (24)$$

derived by Pais and Treiman (their Eq. 27) using the additional dynamical assumption of $V = A$. If $V_{em} / \sigma_- = 1/3$, which is approximately correct experimentally, then Eq. (23) and (24) are identical and the additional assumption is unnecessary. The reason for the identity is that if $V / \sigma_- = 1/3$, then our condition $(A + V)/I \geq 2$ leads to $A/(A + I + V) \geq 1/3$. The two equations $V/\sigma_- = 1/3$ and $A/\sigma_- \geq 1/3$ are equivalent, in the sense of inequalities,¹⁴ to $V/\sigma_- = 1/3$ and $A = V$.

Equation (23) can also be derived from Eq. (20) using the theoretical result⁶ that $\sigma_- / \sigma_+ \leq 3$ for the scaling region. Recent experimental results⁹ seem to indicate that the ratio σ_- / σ_+ of the neutrino to the anti-neutrino cross-section is approximately equal to this limiting value of 3, so that it is not possible to use Eq. (23) to improve on Eq. (20). On the other hand, if the experiments are interpreted as giving $\sigma_+ / \sigma_- \geq 1/3$, then Eq. (23) follows without the scaling assumption. The discussion above

provides some insight to this experimental value of σ_-/σ_+ . If it is true that $V/\sigma_- \approx 1/3$, then in any model for which $V = A$, it follows that $V=A=I$, and the ratio of 3 to 1 for σ_-/σ_+ follows.

If we use the approximate experimental results $V_{em}/\sigma_- = 1/3$ and $\sigma_-/\sigma_+ = 3$, then in addition to Eq. (24), we obtain for anti-neutrino experiments

$$\bar{R} \geq \frac{1}{2} (1 - x + x^2) \geq 0.39 \quad (25)$$

and combining neutrino and anti-neutrino experiments

$$\bar{R}' \geq \frac{1}{4} (1 + x^2) \geq 0.28 \quad (26)$$

ISOBAR PRODUCTION

The general results Eqs. (8) and (13) hold when we limit ourselves to the nucleon isobar Δ as the final state. In this case

$$R = \frac{\frac{1}{2} [\sigma(\nu + p \rightarrow \nu + \Delta^+) + \sigma(\nu + n \rightarrow \nu + \Delta^0)]}{\frac{1}{2} [\sigma(\nu + p \rightarrow \mu^- + \Delta^{++}) + \sigma(\nu + n \rightarrow \mu^- + \Delta^+)]} \equiv \frac{\sigma_0}{\sigma_-} \quad (27)$$

Experimental data has been given by W. Lee¹⁵ on a related ratio

$$R' = \frac{\frac{1}{2} [\sigma(\nu + p \rightarrow \nu + p + \pi^0) + \sigma(\nu + n \rightarrow \nu + n + \pi^0)]}{\sigma(\nu + n \rightarrow \mu^- + p + \pi^0)} \equiv \frac{\sigma_0'}{2\sigma_-'} \quad (28)$$

If we assume that these events are all Δ events, then it follows from the isovector character of the weak currents (the isoscalar does not contribute) that

$$\sigma_0' = \frac{2}{3} \sigma_0, \quad \sigma_-' = \frac{1}{6} \sigma_-$$

$$R' = 2R$$

and from Eq. (8)

$$R' \geq \left\{ 1 - (1-x) \left(\frac{V_{em}}{\sigma_-} \right)^{1/2} \right\}^2 \quad (29a)$$

$$R' \geq \left\{ 1 - 2 \sin^2 \theta_w \left(\frac{V_{em}}{\sigma_-} \right)^{1/2} \right\}^2 \quad (29b)$$

In order to determine V_{em} we use the narrow width approximation and the data of Galster,¹⁶ et.al. As it is indicated by their analysis, the non-resonance background is at most 25%. For the corresponding neutrino cross section we use the data of Budagov, et.al.¹⁷ Most of the errors come from the neutrino data, for which new results will soon become available. The value obtained for V_{em}/σ_- is $.20 \pm .05$, which is consistent with the value of .19 used by B. Lee.³ The corresponding bound¹⁸ for R' is

$$R' \geq 0.50$$

This is to be compared with the model dependent limit of 0.62 from the analysis of B. Lee. The only assumption we have made is that all the events observed are to be associated with the Δ . The experimental limit¹⁵ given for R' is $R' \leq 0.14$. Our result strengthens the conclusions of

B. Lee that this experimental result disagrees with the Weinberg model unless there is a great admixture of $I = 1/2$ final states in σ'_- . In the case of Δ production, since there is no isoscalar contribution, it is not necessary to sum over protons and neutrons. Thus we may consider

$$R'' = \frac{\sigma(\nu + p \rightarrow \nu + \Delta^+)}{\sigma(\nu + p \rightarrow \mu^- + \Delta^{++})} \quad (30)$$

We find

$$R'' \geq \frac{1}{3} \left\{ 1 - (1-x) \left(\frac{V_{em}}{\sigma_-} \right)^{1/2} \right\}^2 = 0.17 \quad (31)$$

In case that both the neutrino and anti-neutrino cross-sections are known, we can obtain an equality relating R'' to experimentally measurable quantities by using Eq. (11), (19) and (20).

CONCLUSIONS

In this paper we have established lower limits on the ratio R (\bar{R}) of the cross-sections for $\begin{pmatrix} \nu \\ \nu \end{pmatrix} + \text{nucleon} \rightarrow \begin{pmatrix} \nu \\ \nu \end{pmatrix} + X$ to that of $\begin{pmatrix} \nu \\ \nu \end{pmatrix} + \text{nucleon} \rightarrow \begin{pmatrix} \mu^- \\ \mu \end{pmatrix} + X$ as a function of the parameter $x = 1 - 2 \sin^2 \theta_w$ in Hamiltonians of the form of Eq. 1. The major results are the following:

1. A general result applicable to any ν ($\bar{\nu}$) process wherein the target can be considered as isospin zero is given by Eq. (8). It is important to remember in the definition of R that all charge states must be summed over in the final product X . Combining neutrino and anti-

neutrino cross-sections, we obtain the general results Eqs. (16), (18), and (20).

2. For the same processes a better lower limit may be given by Eq. (13) and (12), in which the result depends on the kinematic variables Q^2 and ν or E' and θ .

3. For the total cross-section of neutrinos, the model-independent result gives $R \geq 0.18$. Assuming scaling for the high-energy neutrino cross-section, we obtain Eq. (23), which raises this limit to 0.23.

4. Experiments suggest that the vector contribution V to the total ν cross-section is approximately $1/3$ and that the ratio of ν to $\bar{\nu}$ cross-section is approximately $1/3$. These two relations together imply that the axial contribution $A = V$, as predicted by most parton models. In this case, the inequalities take the simple form of Eqs. (24) - (26).

5. For the experiment of W. Lee involving single π^0 production, we find a ratio $R' \geq 0.50$ to be compared with an experimental result $R' \leq 0.14$. The only assumption is that the final state is pure $I = \frac{3}{2}$.

6. A general result for Δ production using proton targets alone is given by Eq. (31).

All numerical limits above are subject to considerable uncertainty because of the inaccuracies in the experimental numbers inserted into the equations including the limit $\sin^2 \theta_W \leq 0.33$. Thus they must be considered illustrative rather than definitive.

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REFERENCES

- ¹S. Weinberg, Phys. Rev. Letters 27, 1688 (1971).
- ²S. Weinberg, Phys. Rev. D5, 1412 (1972).
- ³B. W. Lee, NAL preprint, May 1972.
- ⁴A. Pais and S. Treiman, Rockefeller University preprint, (June 1972). (To be published in Phys. Rev.)
- ⁵In Eq. (1) the Cabibbo angle was set equal to zero. In the model of Glashow, Iliopoulos and Maiani [Phys. Rev. D2, 1285 (1970)] the isoscalar current has both a vector and an axial part. All our results also hold for such models.
- ⁶J. D. Bjorken and E. A. Paschos, Phys. Rev. D, 3151 (1970). Our conventions are the same as this reference. The quantity $(R) = \frac{\sigma_R}{(\sigma_R + \sigma_L + 2\sigma_S)}$ should not be confused with the ration R defined by Eq. (3).
- ⁷A. Pais, Ann. of Phys. 63, 361 (1971).
- ⁸Small improvements to Eq. 13 can be made providing lower bounds for the isoscalar contribution. For instance, Schwartz's inequality again implies

$$s \geq \frac{|I_{vs}|^2}{4v} \geq \frac{|I_{vs}|^2}{4(v+s)} = \lambda (\sigma_{em}^p + \sigma_{em}^n)$$

where I_{vs} is the isovector-isoscalar interference contribution to σ_{em} ,
 v is the isovector contribution and

λ can be expressed in terms of proton and neutron cross sections,

$$\lambda = \frac{|\sigma_{em}^p - \sigma_{em}^n|^2}{4(\sigma_{em}^p + \sigma_{em}^n)}$$

The bound given in Eq. 13 can be improved by the addition of the term

$$\lambda(1-x^2) \frac{V_{em}}{\sigma_-}$$

For deep inelastic scattering or total cross sections at high energies, the averaging over protons and neutrons may not be essential, because if we assume that the current scatters incoherently from (Z) protons and (A-Z) neutrons within a stable nucleus then

$$R^N = \frac{Z \sigma(\nu + p \rightarrow \nu + X_1) + (A-Z) \sigma(\nu + n \rightarrow \nu + X_2)}{Z \sigma(\nu + p \rightarrow \mu^- + X_3) + (A-Z) \sigma(\nu + n \rightarrow \mu^- + X_4)} \geq \frac{Z}{A-Z} R$$

⁹D. C. Cundy, report at Brookhaven Conference on Weak Interactions, June 1972.

¹⁰H.S. Gurr, F. Reines, and H. W. Sobel, Phys. Rev. Letters 28, 1406 (1972); H. H. Chen and B. W. Lee, Phys. Rev. D5, 1879 (1972).

¹¹H. W. Kendall, Proc. 1971 International Conference on Electron and Photon Interactions at High Energies (Cornell Univ. 1971), p. 248.

¹²B. Myatt and D. H. Perkins, Phys. Letters 34B, 524 (1971).

¹³This result can be seen directly from Eq. (3.13) of Ref. 6 where

σ_{tot} is proportional to $[1/2 + 1/6 (\langle L \rangle + \langle R \rangle) + 1/3 (\langle L \rangle - \langle R \rangle)]$

Clearly, $(V + A) = 1/2 + 1/6 (\langle L \rangle + \langle R \rangle)$ and $I = 1/3 (\langle L \rangle - \langle R \rangle)$.

The lower limit of $(V + A)/I$ is obtained by setting $\langle R \rangle = 0$, $\langle L \rangle = 1$.

¹⁴For the simple Weinberg model the additional dynamical assumption of

$V = A$ leads to an equality obtained in Ref. 4 Eq. 28.

¹⁵W. Lee, Columbia University preprint 1972 .

¹⁶S. Galster, et. al., Phys. Rev. D5, 519 (1972).

¹⁷I. Budagov, et. al., Phys. Letters 29B, 524 (1969).

¹⁸This bound is not the most conservative one. Ambiguities arise from

(a) the extrapolation of the form factor of the (3, 3) resonance $G_M^*(Q^2)$ for $Q^2 \geq 2.5 (\text{GeV}/c)^2$, and (b) from the admixture of an $I = 1/2$ component.

If we assume that σ_{em} for π production contains a 25% incoherent

background of $I = 1/2$ and that σ_- also contains 25% background of $I = 1/2$ then

$$\frac{V_{em}^{(3/2)}}{\sigma_-^{(3/2)}} = \frac{4}{3} \frac{V_{em}^{(3/2)}}{\sigma_-} = \frac{4}{3} \cdot (0.20)$$

and

$$R' \geq 0.70 \left\{ 1 - 2 \sin^2 \theta_w \left(\frac{V_{em}^{3/2}}{\sigma_-} \right)^{1/2} \right\}^2$$