NAL/THY-62

Yale preprint 3075-19

SYSTEMATICS OF SINGLE-PARTICLE SPECTRA IN PROTON-PROTON COLLISIONS*

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ABSTRACT

The shapes and scaling characteristics of the $\pi^{\pm}$, $K^{\pm}$, p, n, $\bar{p}$, and Λ spectra in proton-proton collisions may be easily understood in terms of the clustering effects observed in hadron production at present machine energies. This is the general basis for the success of the nova model. Good quantitative agreement between the model distributions and the experimental data is found.

*Research (Yale Report No. 3075-19) is supported in part by the U.S. Atomic Energy Commission under contract AT(11-1)3075

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I. INTRODUCTION

Many inclusive cross sections have now been measured, including the momentum distributions of $\pi^{\pm}$, $K^{\pm}$, p, $\bar{p}$, and Λ in proton-proton collisions for different incident proton energies.[1-10]. The two aspects of the systematics of these inclusive single-particle distributions which have attracted the most attention are their shapes and scaling characteristics. Although at first sight the variety of experimental observations appears puzzling, this variation results from a common type of production mechanism. The observed differences in distribution shapes and scaling properties reflect the nature (mass and quantum numbers) of the observed particle more than the details of the production process. Our purpose is to show that the differences can be easily understood in terms of kinematical rather than dynamical properties, once some general properties of production processes are accepted.

The variables we shall use for describing the inclusive distributions are the center-of-mass energy $W = \sqrt{s}$, the Feynman scaling variable $x = 2k_L/\sqrt{s}$ (where k_L is the center-of-mass longitudinal momentum of the observed particle), and the transverse momentum squared k_T^2 [11]. The Lorentz-invariant inclusive distribution is written as

$$f(x,k_T^2,s) = \frac{2\omega_k}{\pi\sqrt{s}} \frac{d^2\sigma}{dx dk_T^2} \qquad (1)$$

where ω_k is the center-of-mass energy of the selected secondary. Only proton-proton collisions are considered in this paper.

The observed shapes of $f(x,k_T^2,s)$ in x for fixed k_T^2 and s vary considerably[9]. For example, in proton-proton collisions, we may contrast the pion distributions[1-5], which fall almost exponentially in x (or x^2 with an average value of x as low as 0.1), to the Λ distribution, which rises gently from x = 0 to x = 0.5, then falls off rapidly[6]. The proton distribution rises in x almost up to the maximum value of x[1,3,4,12]. These different trends are seen in Fig. 1, where several distributions are shown along with the results of a model calculation to be described later.

The scaling properties of the inclusive distributions are equally remarkable[9]. The scaling hypothesis asserts that $f(x, k_T^2, s)$ at fixed k_T^2 and x is independent of s for large s. Since scaling is an asymptotic property of some theoretical models[11,13], finding it present at relatively low energies, at least for some cases, was surprising. For example, results on π^+ and π^- production in the 10 to 30 GeV range in p-p collisions[2,5 agree extremely well with the ISR (CERN Intersecting Storage Rings) measurements[7], over a range of x values extending from 0.1 to 0.3. However, this does not establish scaling as a general property at intermediate energies. In particular, recent data indicate that higher energies are needed ($E_{Lab} \gtrsim$ 100 GeV or more) before the pion distribution approximately scales very near x = 0[14]. Furthermore, even though the pion distributions appear to scale at low energy in an x range which is readily associated with proton fragmentation, early scaling does not hold there for some other secondaries such as K^- or $\bar{p}$. The

$\bar{p}/\pi^-$ ratio at $x = 0.35$ increases by a factor of 4 between 20 and 60 GeV, where the π^- distribution is practically energy independent[8]. The K^-/π^-ratio shows a similar behavior, increasing by a factor of 2 between 20 and 60 GeV/c. At the same time, the Λ and K^+ distributions at average x values ($x \lesssim 0.2$) appear to have scaled by 30 GeV[6,3,4].

This great variety in distribution shapes and scaling properties may, however, be understood in a very simple way, in terms of a few general properties of inelastic hadron collisions plus kinematics. Any phenomenological model which incorporates these general properties should provide a fair account of the systematics just described.

Inelastic collisions tend to be "weakly inelastic": oftentimes one of the initial particles retains a large fraction of the center-of-mass energy[15]. As a result only a fraction of the available center-of-mass energy is shared by the remaining secondaries. These secondaries form a cluster to the extent that they show a relatively small spread in rapidity (±1) around a mean rapidity. This clustering effect is observed in the exclusive analyses of specific final states where, for many events, there is a gap in the rapidity distributions of secondaries between the quasielastically scattered particle and the remaining secondaries which are clearly associated with one cluster.[16]. It is less directly manifested in the single- and two-pion rapidity distributions[17], which are appreciably narrower than required by overall phase-space considerations alone. Although it is not required by the data to associate these clusters with the decay products of resonant-like states, the bumpy structure of the quasielastic peaks encourages this interpretation[1-3]. Some duality arguments also point to this interpretation of the cluster formation, where the cluster is the result of a resonance decay sequence[18,19]. These clusters have been called hadronic novas to emphasize the picture of an excited hadron emitting a bright flare of pions and other hadrons in returning to its ground state[19]. Since the nova decays are similar to resonance decays, we expect the multiplicity of the resulting decay particles to grow rapidly with mass, perhaps linearly[20].

Thus, weak inelasticity suggests focusing attention on the excitation of novas, and on the distributions of their decay fragments. From this point of view, the extremely general k_T^2 behavior observed in many reactions simply reflects the generality of the momentum distribution within the cluster. Therefore, one might expect a statistical treatment of the decay distribution to be approximately valid, which should lead to an isotropic pion-momentum distribution in the cluster rest frame. Approximate isotropy has been observed for low multiplicities where this statistical picture could be most dubious[16]. This assumption then provides a relation between the k_T^2 distribution and the x distributions in terms of the excitation spectrum. Only general features of the excitation spectrum are then needed to obtain good agreement with experiment.

This picture has immediate consequences for the inclusive distributions. For high-multiplicity events the available rapidity interval becomes "packed" and the secondaries approach a phase-space like distribution regardless of the production mechanism. These secondaries must contribute to low x merely by energy-momentum conservation. Therefore, the low-mass novas and the low-multiplicity events to which they correspond determine the shape of the inclusive distribution, at least the way it falls in x away from x = 0 (except, of course, for leading particle contributions). This behavior is largely determined by the nova masses which can give the observed particle, and the mass of the observed particle itself. For example, heavy secondaries can share a sizable fraction of the longitudinal momentum of the nova, and therefore tend to larger x values than lighter secondaries from novas of the same mass. But higher-mass novas give secondaries closer to x = 0.

The rapid approach to scaling of the pion distributions at intermediate x indicates that the low-mass novas are mainly produced in an energy-independent manner. The charge ratios in the fragmentation region[7] also suggest that there is little quantum number exchange. This and other evidence[21] point to the diffractive excitation (energy independent and no quantum number exchange) of the low-mass novas[22]. Therefore, the less massive the novas which can give the observed particle, the lower the energy at which scaling appears in the fragmentation region. This mass is lower for pions and K^+ than for K^- or $\bar{p}$ in proton fragmentation. Since the production of a $p\,\bar{p}$ pair requires a large nova mass, increasing the incident energy can cause a large fractional increase in the probability for forming such clusters, even at very high energies. Thus, the invariant distribution should approach a scaled limit very slowly.

Moreover, the proton from which the cluster is excited and the $p\,\bar{p}$ pair should all have small rapidity. This also suggests a natural explanation of the large ratio of the number of $\bar{p}$'s to p's ($\bar{p}/p \simeq 1/2$) at 90° observed at the ISR[23]. These threshold effects should be considered independently of ordinary Reggeon-exchange energy dependence.

The production of high-mass novas must be handled on less direct phenomenological grounds. However, if the nova excitation spectrum and the fragmentation picture is extended all the way to the limit of phase space, then both asymptotically constant cross sections and a slow logarithmic increase of multiplicity follow. This pure fragmentation picture has been very successful in accounting for the single- and two-particle distributions at all x values[19,24], even though the most explicit checks on this mechanism of particle production are for low-mass novas ($M \leq 3$GeV). This extended fragmentation picture does predict that the pion distributions at $x = 0$ will not near their scaling limits until around $E_{Lab} \approx 100$ GeV and from below as experimentally observed. In other words, for high multiplicity events this parameterization of phase space is as good as any other. Even so, the most solid predictions specific to the model are for the lower mass excitations, which control the scaling and shape of the inclusive distributions at larger x values.

In Sec. II the formulas of the nova model are briefly reviewed, and in Sec. III the results of the model calculations are discussed.

II. NOVA MODEL OF INCLUSIVE DISTRIBUTIONS IN PROTON-PROTON COLLISIONS

Since the model is already extensively described[19], its construction is only briefly reviewed here. The nova excitation spectrum and decay distribution determine the inclusive cross section. For those cases where the observed secondary results from nova decay, the distribution is given by

$$f(x,k_T^2,s) = \int_{m_{th}}^{\sqrt{s}-m_p} dM \int_0^{p_T{}^2{}_{max}} dp_T^2 \int_0^{\pi} \frac{d\phi}{\pi}\, \rho(M,p_T^2)n(M)\omega_q \frac{dD(\vec{q}(\vec{k}))}{d^3q} \qquad (2)$$

where $\rho(M,p_T^2)$ is the differential cross section for producing a mass M nova with transverse momentum p_T, $n(M)$ is the mean number of particles of the observed type resulting from a proton nova of mass M, ω_q is the energy of the secondary in the nova rest frame, and dD/d^3q is the normalized decay distribution in the nova frame. By statistical arguments dD/d^3q is approximately isotropic, so it depends only on ω_q, which is related to the center-of-mass variables, ω_k, k_T, and k_L by

$$\omega_q = [(Q_N^2 + M^2)^{\frac{1}{2}}\omega_k - Q_N(k_L\cos\theta_N + k_T\sin\phi\sin\theta_N)] / M \quad , \qquad (3)$$

where Q_N and θ_N are the momentum and scattering angle of the nova in the center-of-mass system, and ϕ is the angle between the normal to the nova production plane and the transverse direction of the observed secondary in the center-of-mass frame. Since light secondaries share only a small fraction of the transverse momentum of the nova, the dependence on θ_N is weak and the p_T^2 and ϕ integrals can be ignored for pions.

In view of the averaging in Eq. (2), rough estimates of ρ, n, and dD/d^3q are adequate for reproducing the observed distributions. This weak sensitivity to details merely reemphasizes that the clustering effects described in the introduction largely determine the distribution. Only the proton spectrum, which in many kinematical regions is dominated by quasi-elastically scattered protons, requires a more careful analysis of ρ[25]. Otherwise it is adequate to use the rough analytic form,

$$\rho(M,\ p_T^2) = C\beta B\exp[-\beta/(M-m_p)-Bp_T^2]\ /\ (M-m_p)^2 \quad . \qquad (4)$$

The model calculations of this paper were done with $B = 6(\mathrm{GeV}/c)^{-2}$, $\beta = 2$ GeV (which locates the maximum of the excitation function at $M = 2$ GeV), and $c = 22$ mb. The results are not very sensitive to these choices of B and β[25], whereas c determines the overall normalization of the distribution. Besides being a simple analytic form, Eq. (4) agrees with Regge asymptotics[26]. As previously stressed, the extension of Eq. (4) to the edge of phase space without, for example, t_{min} cutoffs is motivated only by its phenomenoligical success. The choice of $c = 22$ mb normalizes ρ to 30 mb at 20 GeV/c incident proton momentum. At first sight, this appears to be a commitment to a single-excitation picture. However, Eq. (2) is not sensitive to the mass of the "spectator." Thus, the approximately 40% of double excitation (some of it associated with charge exchange) will give a spectrum similar to Eq. (2), an average over the mass of the spectator excitations being unnecessary. Equation (4) would then overestimate the number of quasielastic protons by about 50% and the yield of protons by 10%, underestimate the yield of neutrons by 25%, and consequently overestimate the proton/neutron ratio by about 40%. The other predictions of Eq. (2) are insensitive to the single-excitation approximation. We therefore retain Eq. (2) for its simplicity, but, of course, this entails no commitment to a pure single-excitation picture of hadron production.

The total flux of secondaries from each nova must be partitioned among the various particles produced in the decay. Although the flux of pions is roughly proportional to the excitation mass (2.1 per GeV), the fluxes of other secondaries have prominent threshold constraints which are largely determined by conservation laws. The yields, n(M), used in the calculations for Fig. 1 are listed in Table 1. The approach to scaling is largely determined by n(M).

The decay distribution of the nova is motivated by statistical considerations[27]. As a first approximation, we used a normalized isotropic gaussian distribution in the nova rest frame,

$$dD/d^3q = N \exp\left[-(q_L{}^2 + q_T{}^2) / K^2\right] \quad , \tag{5}$$

which is cutoff when energy-momentum conservation is manifestly violated. For pions the mean multiplicity of 2.1 particles per GeV of excitation mass requires $K \simeq 350$ MeV/c. The model distributions are sensitive to K, but, besides the multiplicity growth, this same value of K also successfully determines the k_T and x distributions. In a more refined treatment, one would expect a slight dependence of K on the nova mass and on the observed secondary. For example, a sequential decay picture suggests an increase in K with nova mass for protons, but a constant K for pions[19]. Similarities in the k_T^2 distributions for various secondaries show that these effects are small[9]. Such refinements do not add to the general understanding of the systematics. In particular, the problem of defining the distribution of K's, $\bar{p}$'s, or Λ's in proton nova decays is greatly simplified by the observation that their k_T^2 distributions are similar to those for pions[9]. For the sake of simplicity, we set K = 350 MeV/c for pions and K = 400 MeV/c for all other distributions computed in this paper.

III. MODEL RESULTS

As emphasized in the introduction, the nova model incorporates the weak inelasticity, and energy-independent clustering effects observed in hadron production, which are sufficient conditions for qualitative agreement with the data. In this section, we give the results of explicit calculations of the single-particle distribution using Eqs. (2)-(4), Table I, and the parameters given in Sec. II. Figure 1 shows the results of the model calculations of the invariant distributions for $p\,p \to X +$ anything for $X = \pi^{\pm}$, $K^{\pm}$, p, n, $\bar{p}$, Λ. Comparison with some available data is included[1-10]. There is little freedom to adjust any of the parameters of the $\pi^{\pm}$, p, and n distributions, since these are normalized to the total inelastic cross section. The overall normalization of the $K^{\pm}$, Λ, and $\bar{p}$ distributions are determined by n(M) (Table 1), which are consistent with the measured branching ratios of N^* resonances into these particles. However, the shapes are not very sensitive to the parameterization of n(M). As listed in Table 1, n(M) is normalized to a single excitation approximation, parameterized according to Eq. (4). Since this n(M) is normalized to the observed flux, it will tend to overestimate the branching ratio of a proton nova of mass M into strange particles or $\bar{p}$. For example, in p p collisions the single-excitation approximation would limit Λ production to one per event, where double excitations could give 2 Λ's. The ratios of the n(M) are more meaningful than the actual values. The distribution shape and normalization are insensitive enough to the actual parameterization of n(M) that information on correlations and semi-inclusive reactions are necessary to pin down these parameters. A similar point applies to the neutron/proton ratio, which might appear to give a sensitive test of the ratio of single to double excitation. Since this ratio could typically vary between 1/4 and 1/2, the analysis of branching ratios of proton novas into into protons or neutrons would have to be done with a good deal of care, as would the experimental measurements.

The shape systematics of Fig. 1 may be qualitatively rederived from an approximate form of the equations of Sec. II.

First we note that for a nova of mass M, Eq. (4) is maximum at

$$x_o(M) \simeq M_x/M \qquad (6)$$

The approximate shape of the distribution is gaussian on either side of the maximum. For $x > x_o(M)$, the shape of the distribution for a nova of mass M is approximately[19],

$$f_M(x) \simeq |x| \exp[-x^2M^2/4K^2] \qquad (7)$$

Equations (6) and (7) already indicate the behavior observed in Fig. 1. For example, $M_x/M_{Threshold}$ for the Λ distribution is 0.65, so that the low mass novas (M = 1.6 to 3 GeV) contribute Λ's in the range $0.3 \lesssim x \lesssim 0.6$. Similar discussions for the other distributions are easily supplied by the reader.

In Fig. 2 we have resolved $f(x, k_T^2, s)$ into its contribution from novas of different masses for the Λ spectrum (Fig 2a) and the K^+ spectrum (Fig. 2b) at 19.2 GeV/c. The approximations of Eqs. (6) and (7) agree quite well with the more precise computations shown in Fig. 2. It is also typical of the systematics that the high-mass contribution to the Λ spectrum is essentially the same shape as the $\bar{p}$ distribution.

We now turn to the scaling properties. The threshold constraints on n(M) have a strong effect on the rate of approach to the scaling limit in the fragmentation region. (Quantum number exchange is discussed below.) The rapid approach of the pion distributions to the scaling limit at intermediate x has already been emphasized, and is implied by Eq. (7)[19]. The proton distribution[19] is strongly affected by the leading particle effect.

The protons from a nova decay scale at a low energy, but the contribution of quasielastic protons from a given missing mass contribute to higher values of x as s increases. Thus, for fixed x away from x = 1, the proton distribution decreases to its asymptotic limit. At x = 0.7, the asymptotic limit is essentially reached by E_{Lab}= 200 GeV. The neutron spectrum scales very early, since neutrons are heavy and can be produced by low-mass proton novas. (This statement ignores charge exchange contributions.) Similarly, Λ's and K^+'s can be produced by low-mass novas, which leads to early scaling at intermediate x. On the other hand, it is more difficult for an object with the quantum numbers of the proton to decay into a K^- or $\bar{p}$. Not only does the probability of a $\bar{p}$ grow with nova mass, but an increase in energy represents a large increase in the available phase space for these decays. The parameterization of Table 1 gives an increase of a factor of 3 at x = 0.2 in the $\bar{p}$ distribution between 30 and 100 GeV, with the distribution falling off more steeply in x as the energy is increased.

The approach to scaling near x = 0 is slower because high-mass novas contribute secondaries there. Although the use of a fragmentation model here is not firmly based, the mere fact that the nova gives the observed multiplicity growth is enough for it to account correctly for the scaling properties around x = 0. For example, higher-mass novas, which contribute π^- nearer x = 0, may be produced as the energy is increased, but with smaller cross section. The approach to the limit is rather slow: something like $f_\infty(1-2\text{ GeV}/\sqrt{s})$ for large $\sqrt{s}$. Thus, to within a couple percent, the scaled limit, f_∞, is reached at all ISR energies, but there should be a sizable increase (about a factor of 2) between conventional and ISR accelerator energies at $x = 0$.

A more quantitative analysis of scaling would entail a closer analysis of n(M), a better understanding of the high-mass novas (if indeed a fragmentation model is appropriate), and some understanding of the quantum number exchange contributions. Although the general trends reported are quite unavoidable, these points can affect the precise determination of <u>changes</u> in $f(x, k_T^2, s)$ with s.

The choice of proton-proton collisions was intended to minimize quantum number exchange effects. Thus, we have emphasized purely kinematical threshold effects which can lead to appreciable energy dependence[29]. For quantum-number exchange the Mueller analysis[30] is applicable, and in the fragmentation region, it suggests a power-law approach.

Several rules have been proposed for a more rapid approach to scaling when a sufficient number of channels have exotic quantum numbers[31]. These rules are generalizations of the Harari-Freund rules for energy dependence of total cross sections, but do not give an energy scale against which the approach to the scaling limit should be measured. At present these rules provide necessary more than sufficient conditions. In any event, dual models provide the specific conditions that all relevant channels should be exotic before the energy-dependent terms are absent. Such a case is $p + p \to \bar{p}$ + anything, where all the relevant channels are exotic. The strong energy dependence in the fragmentation region at present machine energy was discussed above. However, we should note that this energy dependence does not violate the exoticity conditions for rapid scaling, since the energy dependence of the inclusive distrubution could be dominated by threshold effects and still not show any Regge behavior. Nevertheless, this shows that at present machine energies that the threshold effects considered here may be the important ones when considering rates of approach to scaling.

Our simplified analysis in which the non-scaling terms associated with quantum number exchange are ignored compliments the Regge analysis, which one should also include following Mueller's approach. A specific fragmentation model is very useful in displaying threshold effects in the approach to scaling which, as stressed, are of extreme importance in cases like the $\bar{p}$ spectrum, while of little relevance in others like the pion spectra, Nevertheless, quantum number exchange notwithstanding, the approach to scaling in the pion distribution is observed to be very fast.

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21. Let $\langle x\rangle_{n,i}$ be the average value of x for π_i's in proton-proton collisions for n-prong events. For $n \geq 4$, $\langle x\rangle_{n,\pi^+} = \langle x\rangle_{n,\pi^-}$ with errors, and $\langle x\rangle_4 = 0.13$, $\langle x\rangle_6 = 0.10$, and $\langle x\rangle_8 = 0.08$, independent of energy from 10 to 30 GeV/c. These numbers come from the data of Ref. 2. Also see Ref. 19.

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25. Only the quasielastic proton spectrum is sensitive to these choices, and there, Eq. (4) gives only the crudest account of the data (See Fig. 1). This problem is more carefully analyzed in Ref. 12.

26. The M^{-2} decreases of ρ agrees with a triple Regge model

where the leading particle effect is due to the Pomeranchon-Pomeranchon-Reggeon diagram. Of course, one must be careful in associating triple Regge diagrams with diffraction. A triple Regge analysis of secondaries from the nova decay does not correspond to this diagram, although the production mechanism is diffractive. The first three moments of Eq. (4) agree with experiment, which suggests that the M^{-2} decreases of ρ is a reasonable estimate. Nevertheless the major motivation for extending this to the edge of phase space remains phenomenological simplicity.

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29. Our choice of proton-proton collisions helps minimize the effect of nondiffractive mechanisms. In nonexotic initial states, there exist mechanisms which, although smaller in cross section than the dominant diffractive mechanisms, may be much more efficient at producing one of the final particles of interest. For example, forward Λ's are efficiently produced through baryon exchange in K^-p interactions, and less efficiently from the decay of excited K^- states.

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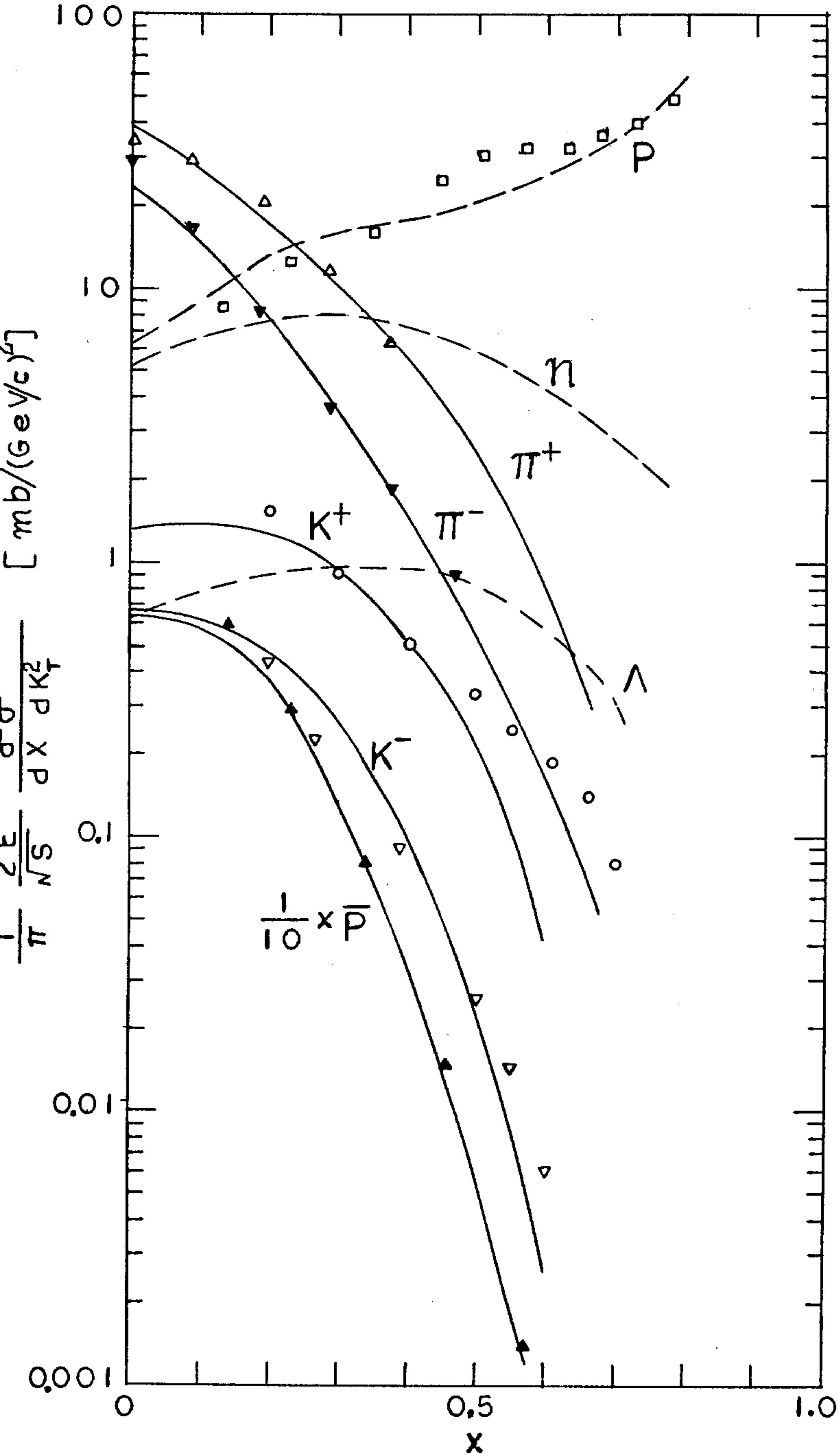
X	M_{th}(GeV)	a_X	b_X	$\langle n_X \rangle$
π^+	1.08	0.67	0.33	2
π^-	1.22	-0.33	0.33	1
K^+	1.61	0.05	0.0066	0.072
K^-	1.92	0.006	0.006	0.027
p	1.08	0.6	-0.01	1.56
n	1.08	0.4	0.01	0.44
Λ	1.61	0.05	0.0025	0.056
$\bar{p}$	2.82	-0.007	0.0018	0.0017

Table 1: Parameterization of $n_X(M) = a_X+b_XN$ for Eq. (2). (For proton and neutron with M>5 GeV, n_X is set to be 0.5). These values were used in the calculations reported in the figures. The calculation of the mean multiplicity of X,$\langle n_X \rangle$, in the single excitation approximation, at 19.2 GeV/c using Eq. (4) and this parameterization of $n_X(M)$ is also given. The significance of and corrections to these quantities are discussed in Sec. III.

Figure Captions:

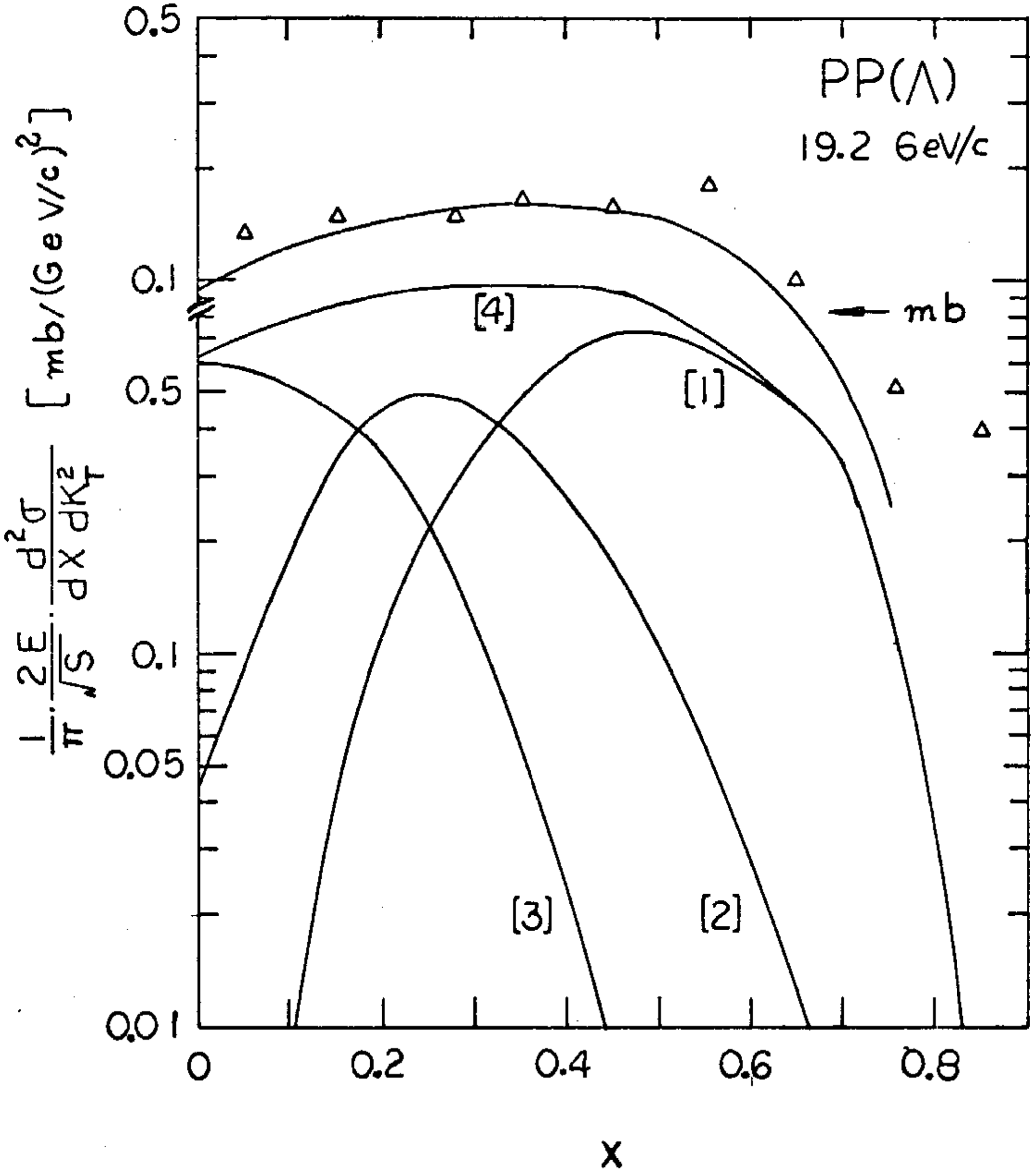
Fig. 1. Model calculation of the $k_T^2 = 0$ invariant distributions of pp(X). The experimental $\pi^\pm$ distributions are the 28 GeV/c data of Ref. 5; the remaining data points, for 19.2 GeV/c, are taken from Ref. 3. The calculations are an evaluation of Eq. (2), using $n_X(M)$ of Table 1, and are explained in detail in Sec. II.

Fig. 2. Invariant distribution of pp(Λ) and pp(K^+) at $k_T^2=0$. Curves [1], [2], and [3] are the contributions from novas of mass, 1.6 to 2.6, 2.6 to 3.6, and 3.6 to 5.2 GeV, respectively. Curve [4] is the sum of these contributions. The top curve in Fig. 2a is the distribution integrated over k_T^2 for pp(Λ), and the experimental data are taken from Ref. 6, where the data are averaged over 13 to 28 GeV/c in incident momentum.



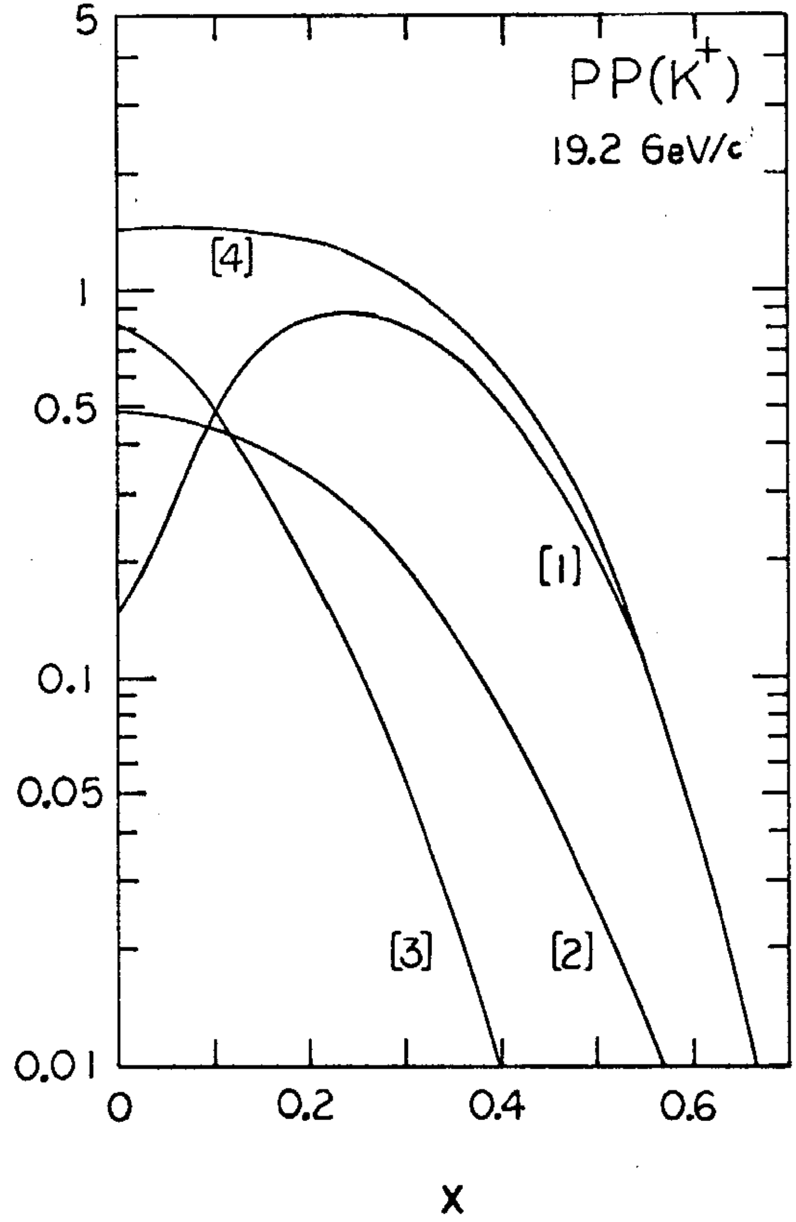
100
10
1
0.1
0.01
0.001
$\frac{1}{\pi} \frac{2E}{\sqrt{s}} \frac{d^2\sigma}{dx\, dK_T^2}$ [mb/(GeV/c)^2]
P
n
π^+
π^-
K^+
Λ
K^-
$\frac{1}{10} \times \bar{P}$
0
0.5
1.0
x

Fig. 1



0.5
0.1
0.5
0.1
0.05
0.01
$\frac{1}{\pi} \cdot \frac{2E}{\sqrt{S}} \cdot \frac{d^2\sigma}{dX\,dK_T^2}$ [mb/(GeV/c)^2]
PP(Λ)
19.2 GeV/c
mb
[4]
[1]
[3]
[2]
0
0.2
0.4
0.6
0.8
X

Fig. 2a



5
1
0.5
0.1
0.05
0.01
PP(K^+)
19.2 GeV/c
[4]
[1]
[3]
[2]
0
0.2
0.4
0.6
X

Fig. 2b