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THEORIES OF THE FINE STRUCTURE CONSTANT α

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Although the fine structure constant is one of the best determined numbers in physics, the reason why nature selects the particular value $\alpha = 1/137.03602 \pm 0.00021$ for the electromagnetic coupling strength is still a mystery, and has provoked much interesting theoretical speculation. For purposes of discussion, the speculations may be divided roughly into four general types: (a) Theories in which α is cosmologically determined; (b) Theories in which α is a constant which is determined microscopically through the interplay of the electromagnetic interaction with interactions of other types, either gravitational, weak or strong; (c) Theories in which α is microscopically determined through properties of the electromagnetic interaction alone, considered in isolation from other interactions; and (d) Numerological speculations.

(a) COSMOLOGICAL THEORIES

A cosmological idea which has received prominent attention recently is the suggestion that α may vary with the time t which has elapsed since the beginning of the universe. In one version[1] of this hypothesis α varies linearly with cosmic time,

$$\alpha \sim t \, , \tag{1a}$$

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while in another version[2] (suggested by a possible connection between electromagnetism and gravitation which we discuss below) the variation is logarithmic,

$$\alpha \sim (\ell n\ t)^{-1}. \tag{1b}$$

Assuming that the present age of the universe is $2 \cdot 10^{10}$ years, the rate at which α charges is

$$R = \dot{\alpha}/\alpha = \begin{array}{ll} 5 \cdot 10^{-11}\ \mathrm{year}^{-1} & \text{hypothesis (1a)} \\ -5 \cdot 10^{-13}\ \mathrm{year}^{-1} & \text{hypothesis (1b)}\ . \end{array} \tag{2}$$

Although these rates of variation are very small, it is remarkable that there is now strong experimental evidence ruling out both hypothesis (1a) and (1b), as summarized[3] in the following table:

Source	Limit on R
Fine structure of spectra of distant radio-galaxies	$\lvert R \rvert \leq 2 \cdot 10^{-12}\ \mathrm{year}^{-1}$
Nuclear α-decay: geophysical constancy of decay rate of U^{238}	$\lvert R \rvert \leq 2 \cdot 10^{-13}\ \mathrm{year}^{-1}$
Spontaneous fission: geophysical constancy of fission decay rate of U^{238}	$\lvert R \rvert \leq 5 \cdot 10^{-13}\ \mathrm{year}^{-1}$
Beta decay: agreement of laboratory and geophysical half-lives for decay Rhenium 187→Osmium 187 ($T \sim 5 \cdot 10^{10}$ years)	$\lvert R \rvert \leq 5 \cdot 10^{-15}\ \mathrm{year}^{-1}$

The only simple hypothesis which is compatible with these limits is R=0, which means a fine structure constant which is strictly a constant over the lifetime of the universe. Such a constancy could result if α were some sort of cosmological boundary condition, fixed, perhaps, by the detailed structure of the universe at the beginning of the present expansion phase. Clearly, if α is determined in such a fashion we could not, with our present knowledge, hope to calculate it. A more appealing explanation for the constancy of α is that α is microscopically determined by the basic particle interaction laws, independent of cosmological considerations.

(b) MICROSCOPIC THEORIES RELATING α TO THE GRAVITATIONAL OR WEAK INTERACTIONS

A basic issue in making a microscopic theory of α is deciding which of the four fundamental interaction types--strong, electromagnetic, weak and gravitational--must be included, and which can be neglected. Because of our very limited theoretical understanding of particle interactions, no systematic discussion of this problem can be given. The best we can do is to review various options which have been seriously studied in the past few years, with the caution that there is at present no proof that any of these theories actually includes the correct combination of interactions.

(bl) Theories Combining Electromagnetism and Gravity

A number of authors[2,4] have speculated that the length

$$a = (G\hbar/c^3)^{1/2} = 1.6 \cdot 10^{-33}\,\mathrm{cm} \tag{3}$$

characterizing quantized theories of gravitation may provide a natural cutoff which eliminates the logarithmic divergences of quantum electrodynamics. Apart from details (which in some versions are complicated), the basic consequence of this idea is obtained by requiring that the physical mass of the electron be equal to its electromagnetic self-mass, as calculated with a momentum cutoff Λ of order a^{-1}. Setting $\hbar = c = 1$, this gives

$$m_e = \delta m_e = m_e\, \kappa\, \alpha \ln\left(\frac{\Lambda^2}{m_e^2}\right) = m_e\, \kappa\, \alpha \ln\left(\frac{\kappa'}{m_e^2 G}\right), \tag{4}$$

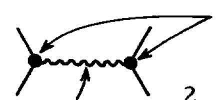
with κ and κ' numerical constants. Thus, Eq. (4) gives us the following relation between the gravitational coupling G and the fine structure constant α,

$$\alpha = \left[\kappa \ln\left(\frac{\kappa'}{m_e^2 G}\right)\right]^{-1}. \tag{5}$$

Assuming κ' is of order unity, Eq. (5) is satisfied with $\kappa \sim 137/104 \sim 1$, also near unity, making the relation plausible. The test of a detailed theory of this type would be whether it gives the correct value of κ, not an easy task since lowest order electrodynamic perturbation theory gives $\kappa = 3/4\pi$, which is substantially too small. An experimental test of Eq. (5) may be provided by radar ranging measurements[3] of the rate of change of G. Should Dirac's hypothesis[3] $G \sim t^{-1}$ prove correct, then Eq. (5), which would imply $\alpha \sim (\ln t)^{-1}$, would be ruled out by the stringent limits on the time rate of change of α. A constant G would, of course, be perfectly compatible with Eq. (5).

(b2) Theories Combining Electromagnetism and Weak Interactions

The fact that the electromagnetic and weak interactions both utilize vector type couplings suggests that they may have a common origin. Suppose, for example, that weak interactions are mediated by an intermediate vector boson of mass M_W which couples to leptons and hadrons with the electromagnetic coupling strength e.[5] The effective weak coupling coming from the W-exchange diagram



(6a)

would be

$$G_F \sim \frac{\alpha}{M_W^2} \; ; \tag{6b}$$

to agree with the experimental value of the Fermi constant $G_F \sim 10^{-5}/M_{proton}^2$ one would need an intermediate boson of mass

$$M_W \sim 30 \; M_{proton}, \tag{7}$$

well beyond the present experimental lower limit on M_W of a few proton masses. A particularly appealing version of this type of theory has been proposed by Weinberg,[6] who constructs a unified, renormalizable theory of weak and electromagnetic interactions. (Unlike the situation in the gravitational cutoff scheme discussed above, the renormalization constants in Weinberg's theory are themselves still infinite.) The basic test of models of this type will of course be the search for heavy intermediate bosons. While the models do not calculate α <u>a priori</u>, if they are proved correct there will be a strong indication that to calculate α one must take the weak interactions, as well as the electromagnetic interactions, into account.

(c) THEORIES IN WHICH α IS DETERMINED BY ELECTROMAGNETISM ALONE

Finally, let us discuss the possibility that α may be determined microscopically by properties of the electromagnetic interaction alone, with the neglect of gravitational, weak and strong interactions. To justify the neglect of gravity we can argue that so far there is no experimental evidence for quantum gravitational effects, and weak interactions may be negligible if they really are weak, rather than being of electromagnetic strength. An argument which may justify the neglect of strong interaction effects will be given later on. The basic requirement which we impose, in an attempt to get an eigenvalue condition for α, is that the renormalization constants of quantum electrodynamics should all be finite. These constants are

$$\begin{aligned} m_0 &= \text{electron bare mass,} \\ Z_2 &= \text{electron wave function renormalization,} \\ Z_3 &= \text{photon wave function renormalization;} \end{aligned} \tag{8}$$

we require that as the cutoff Λ used to calculate them becomes infinite, m_0, Z_2 and Z_3 should have finite limits. The condition on Z_3 can be stated in the alternative form that the renormalized photon propagator $d_c(-q^2/m^2, \alpha)$ [which is normalized to unity at $q^2=0$] should approach the finite constant $Z_3^{-1} = \alpha_0/\alpha$ as $-q^2/m^2 \to \infty$.

A systematic, non-perturbative attack on the problem of whether Z_3 can be finite was made by Gell-Mann and Low in their classic 1954 paper on the renormalization group.[7] They showed that there is indeed an eigenvalue condition imposed by requiring that Z_3 be finite, but that the condition takes the form

$$\psi(\alpha_0) = 0 \tag{9}$$

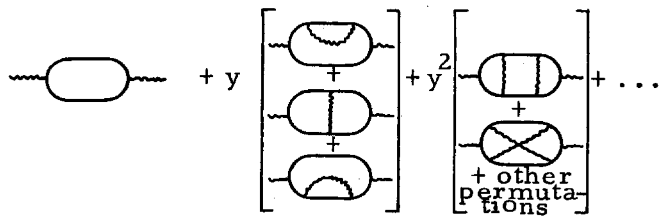
and determines the asymptotic coupling α_0 rather than the physical coupling α. Their analysis leaves α a free parameter of the theory, restricted only by the condition $\alpha < \alpha_0$ coming from spectral-function positivity. This essential conclusion was retained in the subsequent work of Johnson, Baker and Willey (JBW),[8] who made two important advances over the work of Gell-Mann and Low. First, they showed that if Z_3 is finite, then the renormalization constants Z_2 and m_0 can also be finite: The electron wave function renormalization Z_2, which is gauge-dependent, can be made finite by an appropriate choice of gauge (the Landau gauge), while the electron bare mass m_0 takes the simple scaling form

$$m_0 = \text{const} \times m\left(\frac{\Lambda^2}{m^2}\right)^{-\varepsilon}, \quad \varepsilon = \frac{3}{2}\,\frac{\alpha_0}{2\pi} + \frac{3}{8}\left(\frac{\alpha_0}{2\pi}\right)^2 + \ldots \tag{10}$$

and therefore vanishes in the limit of infinite Λ provided that $\varepsilon > 0$. (A vanishing bare mass means that the physical mass of the electron arises entirely from its self-interaction.) Second, Baker and Johnson[9] showed that the Gell-Mann Low eigenvalue condition $\psi(\alpha_0)=0$ implies the much simpler condition $F^{[1]}(\alpha_0) = 0$, where $F^{[1]}(y)$ is a function of coupling y defined as follows. Let us define the photon renormalized proper self-energy $\pi_c(-q^2/m^2, y)$ by

$$d_c(-q^2/m^2, y) = [\,1 + y\,\pi_c(-q^2/m^2, y)\,]^{-1}, \tag{11}$$

and let $\pi_c^{[1]}(-q^2/m^2, y)$ denote its <u>single-fermion-loop part,</u>

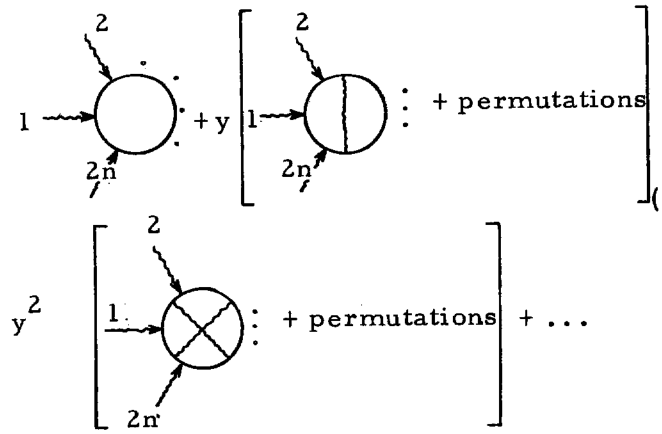
$\pi_c^{[1]}(-q^2/m^2, y) =$  (12)

In the limit of asymptotic $-q^2/m^2$ it can be shown that $\pi_c^{[1]}(-q^2/m^2, y)$ grows at worst as a <u>single power</u> of $\ln(-q^2/m^2)$ [higher powers of $\ln(-q^2/m^2)$ can only come from multiple-fermion-loop diagrams where vacuum polarization insertions appear inside fermion loops] ,

$$\pi_c^{[1]}(-q^2/m^2, y) = G^{[1]}(y) + F^{[1]}(y)\,\ln(-q^2/m^2) + \text{vanishing terms}. \tag{13}$$

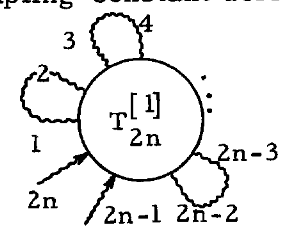
The coefficient of the logarithm in Eq. (13) is the function which gives the simplified eigenvalue condition; unlike the Gell-Mann Low function ψ, which involves <u>all</u> vacuum polarization diagrams, the function $F^{[1]}$ involves only a very special subclass of these diagrams.

In addition to showing that $\psi(\alpha_0) = 0$ implies $F^{[1]}(\alpha_0)=0$, the Baker-Johnson analysis also shows that $\psi(\alpha_0)=0$ implies $T_{2n}^{[1]}(m=0, y=\alpha_0) = 0$ for $n \geq 2$, where $T_{2n}^{[1]}(m, y)$ is the sum of single-fermion-loop $2n$-point functions

$T^{[1]}_{2n}(m, y) =$ 

(14)

Let us now use[10] this powerful result in the following way: We take a single-fermion-loop 2n-point function and contract n-1 pairs of external photon lines with n-1 photon propagators, leaving only two free external photons. The resulting object has the same Lorentz structure as the single-fermion-loop proper self energy $\pi_c^{[1]}$, but some simple combinatorics shows that it is not $\pi_c^{[1]}$ itself, but rather the coupling-constant derivative $(d/dy)^{n-1}\pi_c^{[1]}$. That is, we have

$(d/dy)^{n-1}\pi_c^{[1]} \propto$ 

(15)

But since $T^{[1]}_{2n}(m=0, y=\alpha_0)$ vanishes, we learn from Eq. (15) that

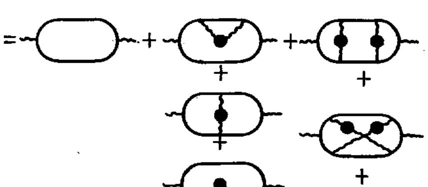
$$\frac{d^{n-1}}{dy^{n-1}} F^{[1]}(y)\Big|_{y=\alpha_0} = 0 \qquad n \geq 2, \tag{16}$$

that is, $F^{[1]}$ vanishes with an infinite order zero at $y = \alpha_0$. A similar argument shows that $T^{[1]}_{2n}(m=0, y)$ also vanishes with an infinite order zero at $y = \alpha_0$, and this in turn implies that the Gell-Mann Low function has a zero of infinite order. Hence, if the Gell-Mann Low function ψ has a zero for non-vanishing coupling, it must be a zero of infinite order-- we see that electrodynamics must satisfy an extraordinarily strong condition in order for Z_3 to be finite.

Whether $F^{[1]}(y)$ and $T^{[1]}_{2n}(m=0, y)$ have the required infinite order zero is an open calculational question. There are two

possibilities:
(A) $F^{[1]}(y)$ and $T_{2n}^{[1]}(m=0,y)$ do not have the required infinite order zero. Then the renormalization constants of electrodynamics cannot all be finite. [The only way to avoid this conclusion would be if a key technical assumption needed for the renormalization group analysis breaks down. The assumption states that terms which vanish asymptotically in each order of perturbation theory do not sum to give an asymptotically dominant result.]
(B) $F^{[1]}(y)$ and $T_{2n}^{[1]}(m=0,y)$ have an infinite order zero at $y = y_0 > 0$. As we have seen, this allows a class of solutions with finite Z_3, in which α_0 is fixed to be y_0 and $\alpha < y_0$ is undetermined. We will now show[10] that the presence of an infinite order zero allows one additional solution, in which the physical fine structure constant α is fixed to be y_0.

The possibility of an additional solution arises because when an infinite order zero (an essential singularity) is present, different orders of summing perturbation theory lead to inequivalent theories. One natural way of summing perturbation theory is to sum "vacuum-polarization-insertion-wise": One first sums all internal photon self-energy parts, and then inserts the resulting full photon propagators in the vacuum polarization skeleton graphs. This order of summation is the one used by JBW, and leads to their form of the eigenvalue condition $F^{[1]}(\alpha_0) = 0$. To see this we apply "vacuum-polarization-insertion-wise" summation to the single-fermion loop skeleton graphs for the photon proper self-energy, giving

$\pi_c^{[1]}[-q^2/m^2, \alpha d_c] =$  $+ \ldots$ (17)

$+$ permutations.

where each shaded blob denotes a full renormalized propagator insertion $\alpha d_c/q^2$. Let us now assume that in letting $-q^2/m^2 \to \infty$, we can take the limit inside the infinite sum over insertions represented by Eq. (17), and therefore we replace each blob by its asymptotic limit $\alpha d_c(\infty, \alpha)/q^2 = \alpha Z_3^{-1}/q^2 = \alpha_0/q^2$. This gives

$$\pi_c^{[1]}[-q^2/m^2, \alpha d_c] \sim \pi_c^{[1]}(-q^2/m^2, \alpha_0) \sim G^{[1]}(\alpha_0) + F^{[1]}(\alpha_0)\ln(-q^2/m^2) + \text{vanishing terms}, \tag{18}$$

so asymptotic finiteness requires the condition $F^{[1]}(\alpha_0)=0$, i.e. $\alpha_0=y_0$. Similar arguments apply to the multiloop skeleton diagrams, when summed "vacuum-polarization-insertion-wise", and again give the condition $\alpha_0=y_0$.

There is, however, another natural summation order, which is to proceed "loopwise": One first sums all single-fermion-loop vacuum polarization graphs, then one sums all two-fermion-loop vacuum polarization graphs, and so forth. The sum of all single-fermion-loop graphs is just

$$\pi_c^{[1]}(-q^2/m^2,\alpha) \underset{-q^2/m^2\to\infty}{\sim} G^{[1]}(\alpha)+F^{[1]}(\alpha)\ln(-q^2/m^2) + \text{vanishing terms}, \tag{19}$$

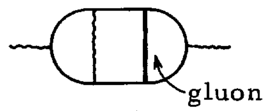
so asymptotic finiteness now requires $F^{[1]}(\alpha)=0$, i.e., $\alpha=y_0$. It is easily seen that the same condition $\alpha=y_0$ guarantees asymptotic finiteness of the multiloop vacuum polarization graphs. So we have found an additional, discrete solution in which α is fixed to have the value y_0. Since $\alpha_0>\alpha$, for this solution α_0 will be outside the region of analyticity of $F^{[1]}$ and so the interchange of limit with sum used in the "vacuum-polarization-insertion-wise" summation procedure is invalid. Hence the condition $F^{[1]}(\alpha_0)=0$ derived by JBW <u>does not</u> apply to the discrete solution in which α is fixed. (If it did, one would have the contradictory equations $\alpha=\alpha_0=y_0$, $\alpha<\alpha_0$.)

We conclude, then, that requiring the renormalization constants of electrodynamics to be finite, combined with "loopwise" summation, leads to an eigenvalue condition for α. <u>We conjecture that this is the mechanism which fixes the value of the fine structure constant.</u> The eigenvalue condition has the appealing property that it is <u>independent of the number of elementary charged fermion species which are present.</u> To see this, we note that when j species are present, the coefficient of the logarithmic divergence in the single-fermion-loop photon proper self-energy is

$$\sum_{\ell=1}^{j} F^{[1]}(\alpha_\ell), \tag{20}$$

which vanishes if all $\alpha_\ell = y_0$. The same condition guarantees vanishing of the multiloop vacuum polarization diagrams. So the value of α which is determined is the same as in the one species case, and the j species are all required to have the same basic electromagnetic coupling $\pm\sqrt{y_0}$. Hence charge quantization appears in a natural way.

Let us now give a possible argument for the neglect of the strong interactions. Suppose that elementary charged fermions are present which have strong interactions mediated by neutral boson exchange (the gluon model). Although the bosons do not themselves contribute vacuum polarization loops, they could modify the fermion vacuum polarization loops when they appear as internal radiative corrections, e.g.



(21)

However, let us now invoke the experimental observation of scaling in deep inelastic electron scattering, one explanation for which[11] is that the exchanges which mediate the strong interactions are actually much more strongly damped at high four-momentum transfer than is the free boson propagator $(q^2+\mu^2)^{-1}$. If this explanation proves correct, then vacuum polarization diagrams with gluon radiative corrections will by themselves be asymptotically finite, and therefore will not contribute to $F^{[1]}$. This means, in turn, that the presence of strong interactions will not alter the eigenvalue condition for α.

What can be said about the prospects of calculating $F^{[1]}(y)$? All that is known at present is the expansion through 6th order in perturbation theory,[12]

$$-y\, F^{[1]}(y) = \frac{2}{3}\left(\frac{y}{2\pi}\right) + \left(\frac{y}{2\pi}\right)^2 - \frac{1}{4}\left(\frac{y}{2\pi}\right)^3 + \dots \quad . \qquad (22)$$

Even though the perturbation theory calculations leading to Eq. (22) are quite horrendous, the resulting coefficients are remarkably simple. A possible clue to the origin of this simplicity may be the fact that $F^{[1]}$ is a property of electrodynamics in the zero fermion mass limit, in which limit the invariance group is the full conformal group, a much larger group than the usual inhomogeneous Lorentz group.[13] Perhaps this fact can be used to develop means for calculating $F^{[1]}$, or at least for approximating it well enough to determine the location of its singularities.

(d) NUMEROLOGICAL SPECULATIONS: WYLER'S FORMULA

So far we have discussed what might be termed[14] "theories in search of number". But no discussion of α would be complete without mentioning a much publicized "number in search of a theory", the formula for α proposed by Wyler,[15]

$$\alpha = \frac{9}{8\pi^4}\left(\frac{\pi^5}{2^4 5!}\right)^{1/4} = 1/137.03608. \tag{23}$$

Whether the agreement of Eq. (23) with experiment has a basis in physics, or is purely fortuitous, remains at present a completely open question.

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