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THE PROCESS $\nu_\mu + p \rightarrow \nu_\mu + p + \pi^0$ IN WEINBERG'S MODEL OF WEAK INTERACTIONS

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ABSTRACT

The ratio $R = \sigma(\nu_\mu + p \rightarrow \nu_\mu + p + \pi^0) / \sigma(\nu_\mu + n \rightarrow \mu + p + \pi^0)$ in Weinberg's model of weak interactions is estimated on the basis of a static model for the neutrino production of the 3,3 resonance. For the Weinberg mixing angle α given by $\sin^2\alpha \leq 0.35$ as deduced from the upper bound on $\sigma(\bar{\nu}_e + e \rightarrow \bar{\nu}_e + e)$, it is concluded that $R \gtrsim 0.4$ which is not consistent with the experimental bound $R \lesssim 0.14$ reported in the preceding paper.

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In view of the experimental results reported in the preceding paper,[1] it is of interest to see what the model of Weinberg[2] predicts for the cross-section of the process $\nu_\mu + p \rightarrow \nu_\mu + p + \pi^0$. This model is particularly attractive in unifying electromagnetic and weak interactions and in being renormalizable,[3] and appears to be typical of a class of models which accomplish the above two goals by utilizing a neutral current.

The effective Lagrangian for strangeness-conserving semi-leptonic processes is given in Weinberg's model by[2]

$$\mathcal{L} = \frac{G_F \cos\theta_c}{\sqrt{2}} \Big\{ \bar{\mu}\gamma_\alpha(1-\gamma_5)\mu \left(J_1^\alpha + i J_2^\alpha\right) + \bar{\nu}_\mu \gamma_\alpha (1-\gamma_5)\nu \left(J_3^\alpha - 2\sin^2\alpha \, J_{em}^\alpha\right) + h.c. \ldots \Big\} \tag{1}$$

where J_i^α are the three components of the V-A isospin currents, J_{em}^α is the electromagnetic current and α is the mixing angle of the Weinberg model.[4]

In estimating the ratio

$$R = \frac{\sigma(\nu_\mu + p \rightarrow \nu_\mu + p + \pi^0) + \sigma(\nu_\mu + n \rightarrow \nu_\mu + n + \pi^0)}{2\sigma(\nu_\mu + n \rightarrow \mu^- + p + \pi^0)} \tag{2}$$

we shall assume that the nucleon and π^0 in the final state are the decay products of the 3, 3 resonance Δ. Further, we shall assume that the Δ is excited by the magnetic dipole, axial electric dipole and axial longitudinal dipole. Under these assumptions

$$R = \frac{\sigma(\nu_\mu + p \to \nu_\mu + p + \pi^0)}{\sigma(\nu_\mu + n \to \mu + p + \pi^0)}$$

and the differential cross section for the process $\nu_\mu + p \to \mu + p + \pi^0$ is given by

$$\frac{d^2\sigma}{dk^2 dW} = \frac{(G_F \cos\theta_C)^2}{2(2\pi)^3} \frac{|\vec{q}|}{(k_{10}^L)} \Big\{ 2k^2 \Big(1 + \frac{2k_{10}k_{20}\cos^2(\theta/2)}{|\vec{k}|^2}\Big)$$

$$\times \big(|M_{1+}|^2 + |\mathcal{E}_{1+}|^2\big) + \frac{2k_{10}k_{20}\cos^2(\theta/2)}{|\vec{k}|^2} 8k_0^2 |\mathcal{L}_{1+}|^2$$

$$+ \frac{(k_{10}+k_{20})k^2}{2|\vec{k}|} (-8) \,\mathrm{Re}\, \big(M_{1+}\mathcal{E}_{1+}^*\big) \Big\} \tag{3}$$

Here we have used the notations of Adler,[5] and have neglected the muon mass. We have the same formula for the process $\nu_\mu + p \to \nu_\mu + p + \pi^0$ as above except that the magnetic dipole amplitude M_{1+} is to be multiplied by the factor $(1-2\sin^2\alpha)$.

For the multipole amplitudes M_{1+}, $\mathcal{E}_{1+}$ and $\mathcal{L}_{1+}$ we will take the static model expressions [Adler[5] (4E.1) - (4E.5)]

$$[M_{1+}]_{static} = \frac{1}{3} g_r (F_1^V + 2M_N F_2^V) \frac{|\vec{q}||\vec{k}|}{\omega M_N^2} h ,$$

$$
\begin{aligned}
[\mathcal{E}_{1+}]_{static} &= -2[\mathcal{L}_{1+}]_{static} \\
&\simeq -\frac{2}{3} g_r g_A \frac{|\vec{q}|}{\omega M_N} r , \\
r &= e^{i\delta_{3,3}} \sin\delta_{3,3} \left(|\vec{q}|^3 \frac{4}{3} f_r^2 / (\omega M_\pi^2) \right)^{-1} \\
f_r^2 &= \frac{1}{4\pi} \left(\frac{M_\pi g_r}{2 M_N} \right)
\end{aligned}
\tag{4}
$$

multiplied by the form factor

$$
F(k^2) = \left[1 + k^2 / 0.54\,(\mathrm{GeV})^2 \right]^{-1} \tag{5}
$$

as suggested by the work of Duffner and Tsai[6] [see their Table II. Here we assume that all three multiple amplitudes have a similar k^2 dependence]. The use of the static model [modified by the form factor (5)] for the vector current contribution is a reasonable approximation.[6] The static model for the weak pion production, however, underestimates the total cross-section perhaps by a factor of order of 2. To remedy this situation, we follow the dictum of Bijtebier[7] and Llewellyn-Smith[8] and add the isobar factor C_A^4. This amounts to modifying the axial multipole amplitudes by

$$
\begin{aligned}
\mathcal{E}_{1+} &= [\mathcal{E}_{1+}]_{static} F(k^2) \left[1 - \frac{W k_0}{M_\pi^2} \left(C_A^4 / C_A^5 \right) \right] , \\
\mathcal{L}_{1+} &= [\mathcal{L}_{1+}]_{static} F(k^2) \left[1 + \frac{W k^2}{M_\pi^2 k_0} \left(C_A^4 / C_A^5 \right) \right] ,
\end{aligned}
\tag{6}
$$

where C_A^4 and C_A^5 are defined in Bijtebier's paper.[7] Bijtebier requires[7]

$$C_A^4 / (M_\pi^2 C_A^5) \sim -(2.6 \sim 3.3)\,(\mathrm{GeV})^{-2}.$$

It is convenient to define the parameter η by

$$\eta = \frac{C_A^4}{M_\pi^2 C_A^5} W_r M_N \sim -3 \tag{7}$$

where $W_r \tilde{=} 1.3\, M_N$.

In the static limit, the differential cross-section may be written as

$$\begin{aligned}\frac{d^2\sigma}{dk^2 dW} &= \frac{(G_F\cos\theta_c)^2}{4\pi^3}\frac{(k_{10}-\omega)^2}{(\omega^2-M_\pi^2)^{1/2}}\left(\frac{2M_N}{g_r}\right)^2 \sigma_{33}(W)\,F^2(k^2)\\
&\Big\{ g_A^2\left(1+\sin^2(\theta/2)\right) + (1+\mu_V)^2 \sin^2(\theta/2)\frac{1}{M^2}\left(\frac{k_{10}^2+k_{20}^2}{2} + k_{10}k_{20}\sin^2(\theta/2)\right)\\
&- g_A^2\,\eta\,\frac{W k_0}{W_r M_N}\left(4\sin^2(\theta/2)\right) + 4g_A^2\eta^2\left(\frac{W}{W_r}\right)^2 \sin^2\left(\frac{\theta}{2}\right)\frac{1}{M_N^2}\left(\frac{k_{10}^2+k_{20}^2}{2}\right.\\
&\left.- k_{10}k_{20}\sin^2(\theta/2)\right) + 2g_A(1+\mu_V)\left(1-\eta\frac{W}{W_r}\frac{k_0}{M_N}\right)\frac{k_{10}+k_{20}}{2}\\
&\times \sin^2(\theta/2)\Big\}\end{aligned} \tag{8}$$

where $g_A \simeq 1.2$ and $\mu_V \simeq 3.7$. The ratio R is given by

$$R = \frac{\left[\begin{array}{c}(0.202-0.0212\eta+0.0431\eta^2)+0.263(1-2\sin^2\alpha)^2\\ +0.235(1-0.319\eta)(1-2\sin^2\alpha)\end{array}\right]}{\left[\begin{array}{c}(0.202-0.0212\eta+0.0431\eta^2)+0.263\\ +0.235(1-0.319\eta)\end{array}\right]} \tag{9}$$

for the incident neutrino energy of 1 GeV. To obtain Eq. (9) we integrate the static model differential cross-section (8) in the narrow width approximation $\sigma_{33}(W) \sim \delta(W-W_r)$.

The bound[9]

$$\sin^2\alpha \leq 0.35$$

deduced from the Reines' status report[10] on the experiment on $\bar{\nu}_e + e \rightarrow \bar{\nu}_e + e$ implies

$$R \geq 0.4 \text{ for } \eta = 0$$

and

$$R \geq 0.6 \text{ for } \eta = -3$$

Thus, the bound $R \leq 0.14$ (90% confidence) reported in the preceding paper, when taken together with the upper bound on the total cross-section for $\bar{\nu}_e + e \rightarrow \bar{\nu}_e + e$, rules out the existence of the neutral current predicted by Weinberg's model of weak interactions.

It must be emphasized that the concl usion reached here does not rule out the correctness of a unified electromagnetic and weak interaction theory based on a spontaneously broken gauge symmetry. It may be that the theory of leptons of Weinberg is substantially correct but that the hadronic neutral current consists of a $\Delta S=0$, isoscalar piece only. Or, it may be that the nature chooses a scheme which makes use of heavy leptons[11] and no neutral current that can show up in the process discussed in this paper.[12]

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