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A Model of Weak and Electromagnetic Interactions

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ABSTRACT

We present here a model of weak and electromagnetic interactions of leptons and hadrons based on the spontaneously broken gauge symmetry $0(3) \times 0(2)$. The advantages of the model are that:

(1) The universality of the β- and μ- decays emerges naturally;

(2) there appears only positively charged heavy leptons, and no neutral heavy leptons which might affect the muon (g-2) factor adversely;

(3) a neutral current shows up only as a short range weak parity violation in electromagnetism, and nowhere else.

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We present here a model of weak and electromagnetic interactions[1] of leptons and hadrons based on the spontaneously broken gauge symmetry[2] $0(3) \times 0(2)$. The model is in some sense intermediate between those of Weinberg,[1] and of Glashow and Georgi.[3] The universality of the β- and μ- decays emerges naturally. The model contains positively charged heavy leptons, but no neutral heavy leptons [A neutral heavy lepton in the Glashow-Georgi model may affect adversely the agreement between theory and experiment of the (g-2) factor of the muon if the mass of the neutral lepton is too large and/or the mass of the weak vector boson is too small[4].] . The model contains a neutral current, but it shows up only as a minute short-range parity violation in electromagnetism, and nowhere else. The model can be embedded in a bigger group $0(3) \times 0(3)$ (or even bigger ones) more or less naturally, but we shall not discuss it here. The model is anomaly-free[5] and renormalizable.[6]

We shall describe the model in terms of the electron first. We form a triplet

$$L_{\ell} = \frac{1-\gamma_5}{2} \begin{pmatrix} e^- \\ \nu^0 \\ E^+ \end{pmatrix}$$

with zero r-charge, and two singlets e^-_r, E^+_r :

$$e^-_r = \frac{1+\gamma_5}{2} e^- \quad , \quad E^+_r = \frac{1+\gamma_5}{2} E^+$$

with r-charge +1 and -1 respectively. Let W_μ, $W^\dagger_\mu$ and A^ℓ_μ be the $0(3)$ gauge bosons and A^r_μ be the r-charge $0(2)$ one. Their couplings to the currents are given by

$$g \left\{ W_\mu \left[\bar{e}\gamma^\mu \left(\frac{1-\gamma_5}{2}\right)\nu + \bar{\nu}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)E^+ \right] + \text{h.c.} \right.$$
$$\left. + A_\mu^\ell \left[\bar{e}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)e - \bar{E}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)E \right] \right\}$$
$$+ g' A_\mu^r \left[\bar{e}\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)e - \bar{E}\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)E \right]$$

To induce the Higgs phenomenon,[2] we postulate a real scalar triplet ϕ with r-charge 0, and a complex scalar triplet ξ with r-charge +1 and its complex conjugate $\xi^\dagger$. We arrange their mutual interactions in such a way that their vacuum expectation values are given by[7]

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix},$$

$$\langle \xi \rangle_0 = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}, \qquad \langle \tilde{\xi} \rangle_0 = \begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix}$$

where v and u are real numbers. Of the nine scalar mesons, three can be transformed away by a gauge transformation. There remain two neutral, two singly charged, two doubly charged scalar particles.

The charged vector bosons $W^\pm$ become massive with the mass given by

$$m_W^2 = g^2(u^2 + v^2).$$

One combination of two neutral vector bosons:

$$Z_\mu = A_\mu^\ell \cos\theta - A_\mu^r \sin\theta \quad , \quad \tan\theta = g'/g \; .$$

becomes massive:

$$m_Z^2 = 2(g^2 + g'^2)\, u^2$$

and couples to the parity violating neutral current

$$\sqrt{g^2+g'^2}\; Z_\mu \Big[\frac{\cos 2\theta}{2} \left(\bar{e}\gamma^\mu e - \bar{E}\gamma^\mu E \right) + \frac{1}{2} \left(\bar{e}\gamma^\mu\gamma^5 e - \bar{E}\gamma^\mu\gamma^5 E \right) \Big] .$$

The other linear combination

$$A_\mu = A_\mu^\ell \sin\theta + A_\mu^r \cos\theta$$

is the photon. The electric charge e is given by

$$e = \frac{g g'}{\sqrt{g^2+g'^2}}$$

and the Fermi constant $G_F/\sqrt{2}$ by

$$G_F/\sqrt{2} = \left[4 \left(u^2 + v^2 \right) \right]^{-1}$$

From these it follows that

$$m_W^2 = \frac{\sqrt{2}\, e^2}{4 G_F \sin^2\theta} \gtrsim (53 \text{ GeV})^2$$

The lepton masses are generated by the couplings

$$-\frac{m_E}{2u}\left[(\bar{L}_\ell \cdot \xi) E_r^+ + h.c. \right]$$

$$-\frac{m_e}{2u}\left[(\bar{L}_\ell \cdot \tilde{\xi}) e_r^- + h.c. \right]$$

To accommodate the muon, we need merely substitute μ^-, ν_μ, M^+ for e^-, ν, E^+ in the above discussions.

Integrally charged quarks p^+, n^o, λ^o may be incorporated into the scheme in the following way. We form two triplets of zero r-charge:

$$\begin{pmatrix} p \\ n' \\ q^- \end{pmatrix}_\ell , \qquad \begin{pmatrix} p' \\ \lambda' \\ q'^- \end{pmatrix}_\ell$$

where q^-, p'^+ and q'^- are the fourth, fifth and sixth quarks, and

$$n' = n \cos\theta_c + \lambda \sin\theta_c$$

$$\lambda' = -n \sin\theta_c + \lambda \cos\theta_c$$

with the Cabibbo angle θ_c. We treat the right handed quarks as singlets of appropriate r-charges. The quark masses, as well as the Cabibbo mixing can be generated by the couplings of ϕ and ξ to the quarks. In this scheme, the magnitude of the amplitude $K_L \to \mu^+\mu^-$ is proportional to $G_F^2(\Delta m)^2$ where Δm is a typical quark mass difference, rather than to $G_F(G_F M_W^2) \sim G_F \alpha$.

To estimate the size of the parity violating effects due to the Z meson, consider the μ-e force due to the Z meson exchange. For small momentum transfer it is given by

$$- \frac{1}{8u^2}\left[\bar{e}\gamma_\mu(\cos 2\theta + \gamma_5)e\right]\left[\bar{\mu}\gamma^\mu(\cos 2\theta + \gamma_5)\mu\right]$$

Since u is arbitrary, subject only to the constraint

$$\frac{G_F}{\sqrt{2}} \leq \frac{1}{4u^2}$$

the parity violating effect can be made easily to be of the order of the typical weak effects. The contribution of the above interaction to the hyperfine splitting of the muonium is, assuming $u^2 \simeq v^2$, about 10^2 cps, far below the present experimental detectability.

REFERENCES

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[7] We write

$$\xi = \begin{pmatrix} \xi^{--} \\ \xi^{-} \\ \xi^{0} \end{pmatrix} \quad \text{and} \quad \tilde{\xi} = \begin{pmatrix} \bar{\xi}^{0} \\ \xi^{+} \\ \xi^{++} \end{pmatrix} = \begin{pmatrix} (\xi^{0})^{*} \\ (\xi^{-})^{*} \\ (\xi^{--})^{*} \end{pmatrix}$$