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Bjorken Scaling and Structure Function Relations in Perturbation Theory

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ABSTRACT

We calculate in the leading logarithm approximation to perturbation theory the complete set of structure functions for inelastic electron, neutrino, and antineutrino scattering on proton and neutron targets. This extends our previous calculations of the νW_2 function of inelastic electron-proton scattering. The Lagrangian studied is that of the neutral vector gluon model-equivalently massive quantum electrodynamics. We find that, in spite of the logarithmic breakdown of Bjorken scaling which generally occurs in renormalizable field theories, all the structure function relations and sum rules of the parton model and formal light cone algebra are satisfied to leading logarithmic accuracy in the theory we study.

I. INTRODUCTION

In a previous paper[1] (hereafter called I) we examined in a renormalizable perturbation theory the structure function νW_2 of inelastic electron-proton scattering. The field theory studied was that of a spin 1/2 proton coupled to a neutral, massive vector meson field (neutral vector gluon model). Our calculations were carried out in the Bjorken limit[2] to leading logarithmic accuracy. Strict Bjorken scaling is violated by the presence of logarithms of the asymptotic variables. These logarithms were shown to exponentiate when summed to all orders in perturbation theory.

In this paper we continue our work by studying the second structure function W_1 of inelastic e-p scattering. In addition we introduce a neutron field and study inelastic e-n as well as inelastic ν and $\bar{\nu}$ scattering on p and n targets. The neutral vector field is taken to have equal (isoscalar) coupling to p and n.

We then examine the various relations among the structure functions and the sum rule constraints which are predicted by the parton model[3,4,5] and light cone[6] approach to Bjorken scaling. Such relations and sum rules are of central importance since they reveal information about the fundamental fields from which the electromagnetic and weak currents are constructed.[7] We find that they continue to hold in the vector gluon model in the leading logarithmic approximation in spite of the breakdown of Bjorken scaling. In a sense therefore the structure function relations and sum rules have a greater generality than might have been expected from work with models which possess strict Bjorken scaling. This possible greater generality remains circumscribed, however, by the fact that all perturbation theory evidence indicates that the relations and sum rules do not hold when one sums next-to-leading logarithms.

The plan of the paper is as follows. In Section II we fix our notation by recalling the standard definitions of the structure functions of inelastic lepton-nucleon scattering. We also state the parton model light cone relations and sum rules for the structure functions after appropriate modification to match the quantum numbers of the fields which we use. In Section III we start by reminding the reader of the philosophy of the leading logarithm approximation which is central to our work. We then give the perturbation theory results for the structure functions and compare to the relations and sum rules of Section II. Algebraic details are reserved for Appendix A. In Section IV we briefly discuss our results. In Appendix B the physically less interesting field theory of charged spin 0 particles coupled to a neutral vector particle is considered. We also correct a minor error involving this scalar case which was made in Appendix B of I.

II. KINEMATICS AND STRUCTURE FUNCTIONS

The cross section for inelastic electron nucleon scattering $e(\ell) + N(p) \to e(\ell') +$ anything, is proportional to the imaginary part of the spin-averaged amplitude for forward virtual photon-nucleon scattering (Fig. 1)

$$\frac{1}{\pi} \operatorname{Im} T_{\mu\nu}(Q^2,\nu)\Big|_{eN} = \frac{1}{2\pi}\int d^4x\, e^{iq\cdot x} \langle N(p)|[J_\mu^\gamma(x), J_\nu^\gamma(0)]|N(p)\rangle_{\substack{\text{spin}\\ \text{avg.}}}$$

$$= \frac{1}{m^2} W_2^{eN}(Q^2,\nu)\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right)$$

$$+ W_1^{eN}(Q^2,\nu)\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \qquad (2.1)$$

where $q = \ell - \ell'$ is the virtual photon four momentum, $-Q^2 = +q^2$ its mass, and $\nu = p\cdot(\ell - \ell')/m$ its lab energy. The nucleon mass is denoted by m.

Similarly the cross section for neutrino (antineutrino) scattering on a nucleon target, $\nu(\ell) + N(p) \to \mu(\ell') +$ anything, is proportional to[7]

$$\frac{1}{\pi} \operatorname{Im} T_{\mu\nu}(Q^2,\nu)\Big|_{\nu N(\bar\nu N)} = \frac{1}{2\pi}\int d^4x\, e^{iq\cdot x} \langle N(p)|[J_\mu^{-(+)}(x), J_\nu^{+(-)}(0)]|N(p)\rangle_{\substack{\text{spin}\\ \text{avg.}}}$$

$$= \frac{1}{m^2} W_2^{\nu N(\bar\nu N)}(Q^2,\nu)\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right)$$

$$+ W_1^{\nu N(\bar\nu N)}(Q^2,\nu)\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)$$

$$- \frac{i}{2m^2} W_3^{\nu N(\bar\nu N)}(Q^2,\nu)\, \epsilon_{\mu\nu\alpha\beta}\, p^\alpha q^\beta . \qquad (2.2)$$

The omitted terms in Eq. (2.2) contribute terms to the cross section which are proportional to lepton masses and thus may be ignored. In

Eqs. (2.1) and (2.2) J_μ^γ, J_μ^+, J_μ^- are the electromagnetic, weak charge raising, and weak charge lowering currents respectively. The latter two contain both vector and axial vector pieces.

The predictions[2] of Bjorken scaling are that in the limit $Q^2 \to \infty$, $\nu \to \infty$, $x = \omega^{-1} = Q^2/(2m\nu)$ fixed, one has

$$m W_1(Q^2, \nu) \to F_1(x) + O(1/\nu) \qquad (2.3a)$$

$$\nu W_2(Q^2, \nu) \to F_2(x) + O(1/\nu) \qquad (2.3b)$$

$$\nu W_3(Q^2, \nu) \to F_3(x) + O(1/\nu) \qquad (2.3c)$$

A. Spin Structure

To proceed further one must know something about the structure of the currents J_μ^γ, $J_\mu^\pm$. If these currents are bilinear forms in a spin 1/2 field the Callan-Gross relation[8] holds in the scaling limit

$$F_2(x) = 2x F_1(x) \qquad (2.4)$$

for ep, en, νp, $\bar{\nu}$p, νn, and $\bar{\nu}$n scattering. Two equivalent statements of Eq. (2.4) are

$$\lim_{\substack{\nu \to \infty \\ x \text{ fixed}}} \left[m W_1(Q^2, \nu) - 2x \nu W_2(Q^2, \nu) \right] = O(1/\nu) \qquad (2.4')$$

and

$$\lim_{\substack{\nu \to \infty \\ x \text{ fixed}}} R(Q^2, \nu) = O(1/\nu) \qquad (2.4'')$$

where $R = \sigma_S(Q^2, \nu)/\sigma_T(Q^2, \nu)$ is the ratio of the scalar and transverse cross sections of Hand.[9] For this reason the small experimental value[10] of R is taken as evidence that the electromagnetic current is predominantly constructed out of spin 1/2 fields.

For a current bilinear in spin 0 fields one has in place of Eq. (2.4)[8]

$$F_1(x) = 0. \tag{2.5}$$

Equivalently

$$\lim_{\substack{\nu\to\infty \\ x \text{ fixed}}} \left[m W_1(Q^2, \nu) \right] = O(1/\nu) \tag{2.5'}$$

or

$$\lim_{\substack{\nu\to\infty \\ x \text{ fixed}}} R^{-1}(Q^2, \nu) = O(1/\nu) . \tag{2.5''}$$

B. Internal Symmetry Relations

For simplicity we set the Cabbibo angle equal to zero. In this limit J_μ^+ and J_μ^- are related by an isotopic spin rotation and one has

$$W_i^{\nu p(\bar{\nu} p)}(Q^2, \nu) = W_i^{\bar{\nu} n(\nu n)}(Q^2, \nu) \; ; \; i = 1, 2, 3 . \tag{2.6}$$

In most parton model and light cone discussions the basic spin 1/2 fields are taken to be an SU(3) triplet with quark quantum numbers. In such a case one has the Llewellyn Smith relation[5]

$$F_1^{ep} - F_1^{en} = \frac{1}{12}\left(F_3^{\nu p} - F_3^{\bar{\nu} p} \right) \tag{2.7}$$

as well as the inequality[5]

$$F_1^{\nu p} + F_1^{\bar{\nu} p} \leq \frac{18}{5}\left(F_1^{ep} + F_1^{en}\right). \qquad (2.8)$$

The inequality in (2. 8) becomes an equality in the limit that none of the momentum of the proton and neutron is carried by strange quark constituents.

An additional relationship

$$F_2^{ep} - F_2^{en} = \frac{1}{6}\left(F_2^{\bar{\nu} p} - F_2^{\nu p}\right) \qquad (2.9)$$

has been suggested[11] which requires assumptions beyond the usual parton model or light cone hypotheses. For the status of this relation in the perturbation theory under discussion see Ref. (11).

If instead of an SU(3) triplet of quarks one constructs the currents out of an SU(2) doublet of nucleons it is trivial to see that the above relations become

$$F_1^{ep} - F_1^{en} = \frac{1}{4}\left(F_3^{\nu p} - F_3^{\bar{\nu} p}\right), \qquad (2.7')$$

$$F_1^{ep} + F_1^{en} = \frac{1}{2}\left(F_1^{\nu p} + F_1^{\bar{\nu} p}\right), \qquad (2.8')$$

and

$$F_2^{ep} - F_2^{en} = \frac{1}{2}\left(F_2^{\bar{\nu} p} - F_2^{\nu p}\right). \qquad (2.9')$$

C. Sum Rules

The primordial sum rule of inelastic lepton-nucleon scattering is the Adler sum rule[12]

$$\int_0^1 \frac{dx}{x}\left[\nu W_2^{\bar{\nu}p}(Q^2,\nu) - \nu W_2^{\nu p}(Q^2,\nu)\right] = 2. \qquad (2.10)$$

As the form of Eq. (2.10) indicates this sum rule holds without reference to Bjorken scaling. It requires for its existence only the SU(2) x SU(2) algebra of the weak charge densities along with a convergence hypothesis.

Using a model for the commutation relations of the space components of the currents Bjorken has derived the "backward" sum rule[13]

$$\int_0^1 dx\left[m W_1^{\bar{\nu}p}(Q^2,\nu) - m W_1^{\nu p}(Q^2,\nu)\right] = 1. \qquad (2.11)$$

This holds for currents constructed from spin 1/2 fields and is independent of their charges. Given scaling, Eq. (2. 3), and the Callan-Gross relation (2. 4) it is clear that the Bjorken sum rule (2. 11) is a consequence of the Adler sum rule (2. 10).

Consider next those sum rules which are sensitive to the quantum numbers of the constituent fields. First is the Gross-Llewellyn Smith[4] sum rule

$$\int_0^1 dx\left(F_3^{\bar{\nu}p} + F_3^{\nu p}\right) = -6. \qquad (2.12)$$

For the SU(2) doublet case this reads instead

$$\int_0^1 dx\left(F_3^{\bar{\nu}p} + F_3^{\nu p}\right) = -2. \qquad (2.12')$$

Finally one has the sum rule[14]

$$\epsilon = 1 + \int_0^1 dx\left[\frac{3}{4}\left(F_2^{\bar{\nu}p} + F_2^{\nu p}\right) - \frac{9}{2}\left(F_2^{ep} + F_2^{en}\right)\right] \qquad (2.13)$$

where the left hand side may be interpreted the fraction of the proton's momentum which is carried by neutral "gluon" constituents. The value of ϵ depends on the internal dynamics of the proton as well as the structure of the currents and is therefore a priori unknown. Eq. (2.13) is to be regarded as a way to measure ϵ, the only constraint being the inequality $0 \leq \epsilon \leq 1$. The sum rule (2.13) holds in the parton model and is independent of the form of the interaction since, of course, the usual parton model manipulations assume that the interactions among constituents may be neglected on the time scale which is important in the deep inelastic region. Eq. (2.13) has been derived by formal field theory-light cone techniques[14] in the neutral scalar and pseudoscalar gluon models only. For our case of a SU(2) doublet of spin 1/2 fields Eq. (2.13) takes the form

$$\epsilon = 1 + \int_0^1 dx \left[\left(F_2^{\bar{\nu} p} + F_2^{\nu p} \right) - \left(F_2^{ep} + F_2^{en} \right) \right] . \quad (2.13')$$

III. PERTURBATION THEORY RESULTS

Before stating the results of our perturbation theory calculations let us first recall the spirit behind such calculations. One wishes to calculate the imaginary part of the forward virtual Compton amplitude as illustrated in Fig. 2. The intermediate states, which are ultimately summed over, may be characterized by a multiplicity n_i as well as other labels. (The multiplicity n_i could be further broken down into its fermion and boson components, but we need not detail this here.) The leading logarithm approximation which we[1] use is defined by the following procedure: (i) first fix n_i ($n_i = 1, 2, \ldots$), (ii) then identify in each order of perturbation theory those diagrams which are asymptotically leading in the Bjorken limit, (iii) calculate the leading (logarithmic) term of these diagrams, (iv) sum the results to all orders of perturbation theory, and (v) finally perform the sum over the intermediate state multiplicity n_i.

It is clearly possible that in a given order of perturbation theory cancellations occur between terms corresponding to different n_i. Indeed this <u>partially</u> happens as one sees in Eq. (4.19) of I. (We discuss the physics behind this cancellation below Eq. (3.2)). Therefore, if instead of our procedure one asks in a given, fixed order of perturbation theory for the leading contribution <u>after</u> the sum over n_i has been carried out, and then sums this to all orders of perturbation theory, the results and list of dominant diagrams need not coincide with the results we find. This latter procedure is in fact the one adopted by Gribov and Lipatov[15] in their work on the vector gluon model (as well as the pseudoscalar model) and accounts for the difference when their results are compared to ours.[16] One may verify that in the region $x \approx 1$, $Q^2(x-1) >> m^2$ where the two procedures are essentially equivalent (see the discussion of "scale" below) the expression for νW_2 given in Ref. (15) coincides with our result in I.

It should be emphasized that the issue here is not the question of whether the summations over multiplicity and orders of perturbation theory commute. Rather one is discussing the option of identifying leading terms before or after carrying out the summation over n_i.[17] Obviously if one calculated and summed all subleading logarithms in addition to the leading logarithms the above distinctions would be irrelevant. However, since such a complete calculation is hopelessly difficult given the currently available calculation techniques, we are confronted with the task of gleaning some insight from less complete perturbation calculations if we are to use field theory at all. Given this situation it is not obvious which particular approximation procedure provides the best physical insight. We feel that our choice as outlined above and used in I is physically sensible and a good one. It allows us to examine in detail all properties of the final inelastic states and to see how these pieces fit together to give the final expression for the inelastic scattering. In particular, as discussed in I, we reveal the interplay between the asymptotic falloff of the elastic form factor and the buildup of the inelastic structure functions. Related to this as shown in I is a multiplicity growth ~ $\ln^2 Q^2$.[18] Finally our results are fully consistent with the general properties that the complete answer should have; there are no ghost cuts or singularities, and Lorentz invariance and gauge invariance are maintained throughout.

The result for νW_2 obtained in I is

$$\lim_{\substack{Q^2\to\infty \\ x \text{ fixed}}} \left(\nu W_2^{ep}(Q^2,\nu)\right) \equiv F_2^{ep}(x;Q^2)$$
$$= 4\lambda \frac{x^2}{1-x} \ln\left[\frac{Q^2(1-x)}{\mu^2}\right] \exp\left\{2\lambda \ln^2\left[\frac{Q^2(1-x)}{\mu^2}\right] - 2\lambda \ln^2\left[\frac{Q^2}{\mu^2}\right]\right\} \tag{3.1}$$

where μ is the mass of the vector particle, e is the coupling of the vector meson to the nucleon, and $\lambda = e^2/(16\pi^2)$. The intermediate states which contribute to Eq. (3.1) consist of a single fermion carrying all but an infinitesimal fraction of the momentum brought in by the external current and n_i-1 vector mesons which are relativistic but nevertheless "soft" compared to the fermion. Typical diagrams are shown in Figs. 3 and 4. For a given value of n_i states having fermion-antifermion pairs, whether from closed loops or from Z graphs, are smaller by at least a logarithm in each order of perturbation theory than the states included in Eq. (3.1). (Diffractive processes correspond to diagrams with closed fermion loops and warrant separate attention in any case.)

The result (from I) <u>before</u> carrying out the sum over n_i but <u>after</u> summing over all orders of perturbation theory is

$$F_2^{n_i}(x;Q^2) = \frac{4\lambda x^2}{1-x} \ln\left[\frac{Q^2(1-x)}{\mu^2}\right] \exp\left\{-2\lambda \ln^2\left[\frac{Q^2}{\mu^2}\right]\right\}$$
$$\times \frac{1}{(n_i-2)!} \left\{2\lambda \ln^2\left[\frac{Q^2(1-x)}{\mu^2}\right]\right\}^{n_i-2} . \qquad (3.2)$$

By inspection of the argument of the exponent in Eq. (3.1) we see that the $\ln^2(Q/\mu^2)$ terms cancel leaving $\nu W_2 \sim \exp\left\{4\lambda \ln\left(\frac{Q^2}{\mu^2}\right)\ln(1-x) + 2\lambda\ln^2(1-x)\right\}$. This is the partial cancellation alleged above and brings us to the important notion of "scale".

In any leading logarithm calculation there is a fundamental ambiguity in the scale factor (call it σ) used to make the argument of the logarithm dimensionless. That is, if σ_1 and σ_2 are two different scale factors then for $Q^2 \to \infty$

$$\ln\left(\frac{Q^2}{\sigma_1}\right) = \ln\left(\frac{Q^2}{\sigma_2}\right) + \ln\left(\frac{\sigma_2}{\sigma_1}\right) \approx \ln\left(\frac{Q^2}{\sigma_2}\right)$$

to leading logarithmic accuracy. The correct scale factor can be determined only if next-to-leading logarithms are kept. In our work we have chosen $\sigma \sim \mu^2$ in order to give it the proper dimensions. Since we treat $m^2/\mu^2 = O(1)$, $\sigma \sim m^2$ would work equally well. In the inelastic structure functions one has an additional feature. In general the scale σ will have x dependence. As we discussed in I the region $x \approx 1$ plays a special role in our work so we are careful to determing properly the x dependence of σ in this region. That is,[1] we treat $\ln^2(Q^2/\mu^2)$, $\ln(Q^2/\mu^2)\ln(1-x)$ and $\ln^2(1-x)$ on the same footing.[19] This is the origin of the 1-x factors in

Eqs. (3. 1) and (3. 2) and the reason we do not get a zero answer after the cancellation in Eq. (3. 1).

The Poisson form of Eq. (3. 2) is reminiscent of the infrared structure of radiative corrections in ordinary QED with massless photons and indeed has a very similar origin. The same applies to the partial cancellation in Eq. (3. 1). It is, in disguise, nothing more than the well known cancellation of the infrared divergences between real and virtual photon processes. Let us turn to our new results.

A. Spin Structure

In I W_2 was calculated by computing the $\mu = \nu = (+)$ component of $\mathrm{Im}\, T^{\mu\nu}_{(ep)}$. To select out W, we compute the $\mu = \nu = 1$ component. In each order of perturbation theory, as we show in Appendix A, one finds that the numerators in the $\mathrm{Im}\, T^{11}_{(ep)}$ calculation are universally proportional to the numerators of the $\mathrm{Im}\, T^{++}_{(ep)}$ case. The Feynman denominators are obviously the same, so all the work of I may be carried over unchanged. One finds

$$F_1^{ep}(x; Q^2) = \frac{1}{2x} F_2^{ep}(x; Q^2)\left[1 + O(1/\nu)\right] \qquad (3.3)$$

where

$$F_1(x; Q^2) \equiv \lim_{\substack{Q^2 \to \infty \\ x \text{ fixed}}} \left[m W_1(Q^2, \nu)\right]. \qquad (3.4)$$

Since our calculations are carried out only to leading logarithmic accuracy, the relation (3. 3) cannot be trusted beyond this accuracy. It is natural to ask if the relation (3. 3) might in fact hold to $O(1/\nu)$ if subleading logarithms are summed as well. The answer is no and follows from explicit calculation of all $O(\lambda)$ diagrams.[20] Namely, in $O(\lambda)$ F_1 and F_2 separately behave ~ $\ln Q^2$ but the appropriate difference is non vanishing

$$\left[2x\,F_1^{ep}(x;Q^2) - F_2^{ep}(x;Q^2)\right] = \text{const.} \neq 0 . \tag{3.5}$$

There is no reason to hope for a better result in higher order. Thus we see in perturbation theory, without cutoffs, the Callan-Gross relation (2. 4) is satisfied but in the weakest possible way.

Because the neutron has no electric charge, and since pair production does not contribute in the leading logarithm approximation as explained above, we have to the same accuracy as Eq. (3. 1).

$$F_2^{en}(x;Q^2) = F_1^{en}(x;Q^2) = 0. \tag{3.6}$$

Similarly since the weak currents are

$$J_\mu^{+} = \bar{p}(x)\,\gamma_\mu(1-\gamma_5)\,n(x) \tag{3.7}$$

and

$$J_\mu^{-} = \bar{n}(x)\,\gamma_\mu(1-\gamma_5)\,p(x) \tag{3.8}$$

we have by the same arguments which lead to Eq. (3. 6)

$$F_i^{\nu p} = F_i^{\bar{\nu} n} = 0 \,, \qquad i = 1, 2, 3 . \tag{3.9}$$

To calculate $F_2^{\bar{\nu}p}$ and $F_1^{\bar{\nu}p}$ we look at the $\mu = \nu = +$ and $\mu = \nu = 1$ components respectively of Im $T^{\mu\nu}_{(\bar{\nu}p)}$. The calculation is identical to that in I so one knows immediately

$$F_2^{\bar{\nu}p}(x;Q^2) = F_2^{\nu n}(x;Q^2) = 2\, F_2^{ep}(x;Q^2) \tag{3.10}$$

In Eq. (3.10) the factor of 2 arises because the square of the vector current and the square of the axial vector current contribute equally. (See Appendix A.) In direct parallel to Eq. (3.5) one has

$$2x\, F_1^{\rho}(x;Q^2) = F_2^{\rho}(x;Q^2)\left[1 + O(1/\nu)\right] \tag{3.11}$$

where $\rho = \bar{\nu}p, \nu n$.

Finally the vector-axial vector interference structure function F_3 may be picked out by evaluating Im $T^{12}_{(\bar{\nu}p)}$; we find (see Appendix A)

$$F_3^{\bar{\nu}p}(x;Q^2) = F_3^{\nu n}(x;Q^2) = -\frac{2}{x}\, F_2^{ep}(x;Q^2) . \tag{3.12}$$

B. Internal Symmetry Relations

Combining Eqs. (3.3), (3.6), (3.9), and (3.12) one sees that the Llewellyn Smith equality (2.7′) is satisfied in the leading logarithmic approximation. So also is his relation (2.8′) (now an equality) once Eqs. (3.3), (3.6), (3.9), and (3.11) are used. Just as with the Callan-Gross relation (see Eq. (3.3)), the Llewellyn Smith relations (2.7′) and

(2.8′) are satisfied to leading logarithmic accuracy only.

C. Sum Rules

With the exception of the Adler sum rule, the sum rules discussed in Sec. II put stringent demands on perturbation theory. Namely certain integrals of the structure functions over x are required to be independent of Q^2 in spite of the fact that the structure functions themselves do not scale.

In I we evaluated the two integrals

$$\Sigma_1(Q^2) = \int_0^1 dx \; F_2(x;Q^2) \qquad (3.13)$$

and

$$\Sigma_2(Q^2) = \int_0^1 \frac{dx}{x} \; F_2(x;Q^2) \qquad (3.14)$$

using Eq. (3.1) for F_2. In both integrals the region $x \approx 1$ is dominant and we have to leading logarithmic accuracy

$$\Sigma_1(Q^2) = 1 \qquad (3.15)$$

$$\Sigma_2(Q^2) = 1 \,. \qquad (3.16)$$

We note that Σ_1 and Σ_2 are independent of Q^2 to the accuracy of our calculation.

Using Eqs. (3.9), (3.10) and (3.16) the Adler sum rule (2.10) reads

$$2 \sum_2 (Q^2) = 2 \tag{3.17}$$

and is therefore satisfied to leading logarithmic accuracy. Similarly the "backward" Bjorken sum rule (2.11) is satisfied. With the help of Eqs. (3.9), (3.12), and (3.15) the Gross-Llewellyn Smith sum rule (2.12′) reduces to

$$-2 \sum_2 (Q^2) = -2. \tag{3.18}$$

and is therefore satisfied.

Lastly we have the sum rule (2.13′). Using Eqs. (3.1), (3.6), (3.9) and (3.10) we have

$$\epsilon = 1 - \left[2 \sum_1 (Q^2) - \sum_1 (Q^2)\right] = 0. \tag{3.19}$$

Thus from Eq. (3.15) we learn that the gluons in the neutral vector meson theory carry zero fraction of the nucleon's momentum in the leading logarithmic approximation. This is in complete accord with our direct calculation of the gluon momentum, Eq. (5.6) of I. A formal field theory derivation of Eq. (2.13′) (or equivalently Eq. (2.13)) has not yet been achieved for the vector gluon theory. This difficulty may be related to backward travelling partons in the parton model language.[21] We see in the leading logarithm approximation, at least, there are no problems. The gluons carry a vanishingly small fraction of the nucleon's momentum and Eq. (2.13′) is satisfied in the same way that all the other sum rules are satisfied.

IV. COMMENTS

Sec. III already contains extensive discussions of our methods and results. Rather than repeating this discussion here let us instead list a few directions in which our work may be extended. It is natural to ask what can be done about the violation of Bjorken scaling which comes from $\ln Q^2$ factors in each order of perturbation theory. Such logarithms seem characteristic of all renormalizable (as opposed to superrenormalizable) field theories.

One can, in the spirit of Drell, Levy, and Yan[22] impose an ad hoc transverse momentum cutoff and investigate the resulting model. Arbitrary modifications of field theory are always hazardous, but we will nevertheless report on the cutoff vector gluon model in a subsequent paper.

A physically more satisfactory way to generate the damping needed to obtain Bjorken scaling is to realize the target nucleons as bound states of some set of elementary particles, say an SU(3) triplet of quarks. (A heuristic treatment of nucleons as a three quark system can be found in Ref. (11).) This would remove the perhaps objectionable feature that the nucleons in our work so far are bare particles clothed in a cloud of neutral vector mesons and are not truly composite. Important problems would still remain, nevertheless. Composite nucleons made of point-like particles, as for example in the Bethe-Salpeter model of Drell and Lee,[23] are still likely to violate scaling when meson emission from the point like constituents is included. We will comment further on this point in our coming paper on the cutoff field theory model. Perhaps an infinitely composite picture, analogous to the one used by Stack[24] to discuss elastic form factors, is required to achieve Bjorken scaling.

An alternative and exciting possibility is to return to the unadorned neutral vector gluon model and to adopt a point of view analogous to that of Johnson, Baker and Willey.[25] These authors have investigated the question of whether or not ordinary quantum electrodynamics can be a self-consistent, finite field theory. In particular one cannot help but wonder if the eigenvalue condition which they find necessary for finite QED might simultaneously secure Bjorken scaling.

APPENDIX A

In this appendix we present the algebraic details behind the results quoted in Sec. III. All calculations are carried out in the notation and reference frame of I. Namely we use the +, - notation

$$a^\mu = (a^+, a^1, a^2, a^-) = (a^0+a^3, a^1, a^2, a^0-a^3) = (a^+, \vec{a}, a^-) \tag{A.1}$$

where a is any four vector. The frame is fixed by

$$p^\mu = (1, \vec{0}, m^2) \tag{A.2}$$

$$q^\mu = (O(s_{eN}^{-1}), 0, Q, 2m\nu) \approx (0, 0, Q, 2m\nu) \tag{A.3}$$

where p is the momentum of the target nucleon and $q = \ell - \ell'$ is the momentum transfer from the lepton system; Fig. 1. The quantity s_{eN} is the square of the center of mass energy of the initial lepton-nucleon pair. The momentum of any produced particle may be written

$$k_i^\mu = \left(x_i, \vec{k}_i, \frac{m_i^2 + \vec{k}_i^{\,2}}{x_i}\right). \tag{A.4}$$

(Note that in writing Eqs. (A.2), (A.3) and (A.4) we have scaled the + and - components to make the former dimensionless and to give the latter dimensions of mass squared.)

In terms of this notation it is clear from Eqs. (2.1) and (2.2) that

$$\frac{1}{\pi} \operatorname{Im} T^{++}\Big|_T = \frac{1}{m^2} W_2^T \tag{A.5}$$

$$\frac{1}{\pi} \operatorname{Im} T^{11}\Big|_T = W_1^T \tag{A.6}$$

for $\quad T = eN, \nu N$, and $\bar{\nu}N$

and $\quad \frac{1}{\pi}\, \mathrm{Im}\, T^{12}\Big|_{\sigma} = \frac{-i\nu}{2m} W_3^{\sigma} \qquad$ (A.7)

for $\quad \sigma = \nu N$ and $\bar{\nu}N$.

For the Born term (Fig. 5) Eqs. (3.3), (3.10), (3.11) and (3.12) are trivial to verify. Consider Fig. 3(a) which is one of the diagrams which makes a leading contribution in $0(\lambda)$,

$$\frac{1}{\pi}\, \mathrm{Im}\, T^{\mu\nu}_{(ep)} = \frac{-e^2}{16\pi^2}\left(\frac{1}{4\pi m}\right)\int_0^1 \frac{dx_1}{x_1}\int_0^1 \frac{dy}{y}\int d^2k_1 \int d^2p'\, \delta(1-x_1-y)$$

$$\times\, \delta(\vec{Q}-\vec{k}_1-\vec{p}')\, \delta\left(\frac{\vec{p}'^2+m^2}{y} + \frac{\vec{k}_1^2+\mu^2}{x_1} - m^2 - 2m\nu\right)\frac{N^{\mu\nu}}{D} \qquad \text{(A.8)}$$

where

$$D = \left[(p-k_1)^2 - m^2\right]\left[(p+q)^2 - m^2\right] \qquad \text{(A.9)}$$

and

$$N^{\mu\nu}_{ep} = \mathrm{Tr}\left\{(\not{p}+m)\gamma^{\mu}(\not{p}+\not{q}+m)\gamma^{\alpha}(\not{p}'+m)\gamma^{\nu}(\not{p}-\not{k}_1+m)\gamma_{\alpha}\right\}. \qquad \text{(A.10)}$$

In I we showed that in computing the leading term of N^{++}_{ep} one may make the approximations

$$\not{p}+m \rightarrow \not{p} \rightarrow \tfrac{1}{2}\gamma^{-}$$

$$\not{p}+\not{q}+m \rightarrow \not{q} \rightarrow -Q\gamma^{2}$$

$$\gamma^{\alpha}\cdots\gamma_{\alpha} \rightarrow \tfrac{1}{2}(\gamma^{-}\cdots\gamma^{+})$$

$$\not{p}'+m \rightarrow \not{p}+\not{q}-\not{k}_1 \rightarrow -Q\gamma^{2}$$

$$\not{p}-\not{k}_1+m \rightarrow \not{p}-\not{k}_1 \rightarrow \tfrac{1}{2}(1-x_1)\gamma^{-}.$$

Thus to leading order

$$N_{ep}^{++} = Q^2\frac{(1-x_1)}{8}\,\mathrm{Tr}\left\{\gamma^-\gamma^+\gamma^2\gamma^-\gamma^2\gamma^+\gamma^-\gamma^+\right\}$$

$$= 16\,Q^2(1-x_1) \qquad (A.11)$$

For N_{ep}^{11} the leading term comes from

$$\not p + m \rightarrow \not p \rightarrow \tfrac{1}{2}\gamma^-$$

$$\not p + \not q + m \rightarrow \not q \rightarrow \tfrac{1}{2}(2m\nu)\gamma^+$$

$$\gamma^\alpha \cdots \gamma_\alpha \rightarrow \tfrac{1}{2}(\gamma^- \cdots \gamma^+)$$

$$\not p' + m \rightarrow \not q \rightarrow \tfrac{1}{2}(2m\nu)\gamma^+$$

$$\not p - \not k_1 + m \rightarrow \not p - \not k_1 \rightarrow \tfrac{1}{2}(1-x_1)\gamma^-_0$$

Hence to leading order

$$N_{ep}^{11} = \frac{(2m\nu)^2}{32}(1-x_1)\,\mathrm{Tr}\left\{\gamma^-\gamma^1\gamma^+\gamma^-\gamma^+\gamma^1\gamma^-\gamma^+\right\}$$

$$= 8m\nu(1-x_1)(2m\nu) = 8m\nu(1-x_1)(Q^2/x). \qquad (A.12)$$

Thus for Fig. 3(a)

$$2x\,(m N_{ep}^{11}) = (m^2\nu\, N_{ep}^{++}) \qquad (A.13)$$

which in turn implies Eq. (3.11).

For the general case, Fig. 4(a), the numerator trace is of the form (dropping terms ~ m)

$$N_{ep}^{\mu\nu} = \mathrm{Tr}\left\{ \cdots \gamma^{\mu} (\not{p} + \not{q} - \sum_{j=1}^{r} \not{k}_j') \cdots (\not{p} + \not{q} - \sum_{i=1}^{\ell} \not{k}_i) \gamma^{\nu} \cdots \right\}. \tag{A.14}$$

Hence the leading terms are

$$N_{ep}^{++} = \mathrm{Tr}\left\{ \cdots \gamma^{+} (-Q\gamma^{2}) \cdots (-Q\gamma^{2}) \gamma^{+} \cdots \right\} \tag{A.15}$$

and

$$N_{ep}^{11} = \mathrm{Tr}\left\{ \cdots \gamma^{1} (m\nu\gamma^{+}) \cdots (m\nu\gamma^{+}) \gamma^{1} \cdots \right\} \tag{A.16}$$

where all unwritten terms are identical in the two traces and consist of a series of γ^{+}'s interlaced with γ^{-}'s. After successive anticommutation to bring the γ^{2}'s and γ^{1}'s together in Eqs. (A.15) and (A.16) respectively one establishes Eq. (A.13) as a general result and hence the Callan-Gross relation.

Consider next the structure functions for neutrino scattering. We will illustrate the calculation for Fig. 3(a) only since the extension to the general case is obvious from the above. In place of (A.10) one has

$$N_{\bar{\nu}p}^{\mu\nu} = \mathrm{Tr}\left\{ (\not{p} + m) \gamma^{\mu} (1-\gamma_5) (\not{p} + \not{q} + m) \gamma^{\alpha} (\not{p}' + m) \gamma^{\nu} \times (1-\gamma_5)(\not{p} - \not{k}_1 + m) \gamma_{\alpha} \right\}. \tag{A.17}$$

As before we may drop all terms ~ m, and therefore after running the $1-\gamma_5$ factors together we have

$$N^{\mu\nu}_{\bar{\nu}p} = 2N^{\mu\nu}_{ep} - 2\,\mathrm{Tr}\left\{\gamma_5 \not{p}\, \gamma^{\mu} (\not{p} + \not{q}) \gamma^{\alpha} \not{k}\, \gamma^{\nu} (\not{p} - \not{k}_1) \gamma_{\alpha}\right\}. \tag{A.18}$$

For $N^{++}_{\bar{\nu}p}$ and $N^{11}_{\bar{\nu}p}$ the second term in Eq. (A. 18) gives a negligible contribution which moreover vanishes after the phase space integration over $\vec{k}_1$ is carried out in Eq. (A. 8). Thus the results quoted in Eqs. (3. 10) and (3. 11) are established.

For $N^{12}_{\bar{\nu}p}$ the first term in Eq. (A. 18) is negligible; the second may be simplified to

$$\begin{aligned} N^{12}_{\bar{\nu}p} &= -2\,\mathrm{Tr}\left\{\gamma_5 \left(\tfrac{1}{2}\gamma^{-}\right) \gamma^{1} (m\nu\gamma^{+}) \gamma^{\alpha} (m\nu\gamma^{+}) \gamma^{2} \left(\tfrac{1}{2}(1-x_1)\gamma^{+}\right) \gamma_{\alpha}\right\} \\ &= 16\, i\, m\nu Q^2 (1-x_1)/x. \end{aligned} \tag{A.19}$$

Therefore with the help of Eqs. (A. 5), (A. 7), and (A. 11) we establish the result quoted in Eq. (3. 12) of Sec. III.

APPENDIX B

Here we briefly examine the field theory of charged spin zero particles (say pions) interacting with massive neutral vector mesons. We consider an isotriplet of charged pions interacting with the isoscalar vector field. The theory is much simpler than the spinor theory since an axial current cannot be constructed which is bilinear in the spin zero field. In Appendix B of I we calculated in the leading logarithmic approximation $F_2^{e\pi^\pm}$. Here we correct a slight error in that calculation which was made when we neglected the "seagull" diagrams.

In lowest nontrivial order diagrams which make leading contributions to Im $T^{++}_{(e\pi^\pm)}$ are the ones shown in Fig. 6(a-d). Diagram 6(a) was considered in I and has a numerator

$$N^{++}(6a) = 8Q^2(2-x_1)(1-x_1)/x \,. \tag{B.1}$$

The denominators and all other factors are the same as in the spinor case. (See Eqs. (A.8-A.10).)

The "seagull" diagram 6(b) has a numerator

$$N^{++}(6b) = -8(1-x_1)(2-x_1). \tag{B.2}$$

Furthermore, except for lacking a denominator factor $[(p+q)^2-m^2] = Q^2(1-x)/x$ diagram 6(b) has a structure identical to that of 6(a). The sum of 6(a) and 6(b) is therefore found by replacing N^{++} of the spinor calculation by

$$N^{++} = N^{++}(6a) + \frac{Q^2(1-x)}{x} N^{++}(6b)$$
$$= 8Q^2 (1-x_1)(2-x_1) \,. \tag{B.3}$$

After carrying out the phase space integrations in Eq. (A.8) the value of x_1 is fixed to be $x_1 = 1-x$. Thus we find

$$F_2^{e\pi^\pm} = \left(\frac{1+x}{2}\right) F_2^{ep} . \tag{B.4}$$

The pattern generalizes in an obvious way; thus Eq. (B.4) holds in every order of perturbation theory. Eq. (B.4) differs from the result quoted in I by a factor of x. Since the sum rules are dominated by the region $x \approx 1$ they are unaffected.

One may alternatively identify νW_2 by computing Im T^{22}. If one does this one finds that the leading contributions come only from Fig. 6(a) and not from the "seagull" Fig. 6(b). Of course, the result is identical to that given in Eq. (B.4), as gauge invariance dictates.

To calculate $W_1^{e\pi^\pm}$ we consider Im $T^{11}_{(e\pi^\pm)}$. Diagram 6(a) gives a vanishing contribution and all seagull diagrams give contributions which are down by a power of Q^2. Thus

$$F_1^{e\pi^\pm} = 0 \tag{B.5}$$

in the leading logarithmic approximation. This is the Callan-Gross relation[8] for currents constructed from spin zero fields.

The neutrino scattering results are related to the electron scattering results by a simple Clebsch-Gordan coefficient. Namely

$$F_i^{\phi} = 2\, F_i^{e\pi^{\pm}} ; \qquad i = 1,2 \tag{B.6}$$

where $\phi = \bar{\nu}\pi^{+}$, $\bar{\nu}\pi^{o}$, $\nu\pi^{-}$, $\nu\pi^{o}$ and

$$F_i^{\rho} = 0 \tag{B.7}$$

for $\rho = \bar{\nu}\pi^{-}$, $\nu\pi^{+}$. Moreover, because of the absence of an axial vector contribution,

$$F_3 = 0 \tag{B.8}$$

in all cases.

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[17] Since there exists no non-perturbative way to solve field theory one is

forced always to identify leading terms before carrying out the sum over orders of perturbation theory.

[18] This multiplicity growth will not be altered by including subleading logarithms.

[19] That is to say, we sum also a selected class of non-leading logarithmic contributions. Gribov and Lipatov, Ref. 15, have by first summing over n_i covertly obtained the cancellation of the $\lambda \ln^2(Q^2/\mu^2)$ terms and have then gone on to sum [all] terms proportional to $\lambda \ln(Q^2/\mu^2)$. These latter [next-to-leading in our language] logarithms aside from the $\lambda \ln(Q^2/\mu^2) \ln(1-x)$ terms which we keep come from "ultraviolet" regions of integration and thus have a very different physical origin from the "infrared" logarithms which we consider. Summation of ultraviolet logarithms in both γ_μ and γ_5 field theories (for a discussion of this in the context of the elastic form factor see: T. Appelquist and J. Primack, Phys. Rev. D1, 1144 (1970).) seems to lead always to results which have a malevolent analytic behavior as in Ref. 15. On the contrary, infrared logarithms correspond to simple paths of momentum flow in diagrams and are well behaved when summed. See Ref. 1 for a discussion of momentum flow.

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FIGURE CAPTIONS

Fig. 1 Kinematics for lepton-nucleon inelastic scattering.

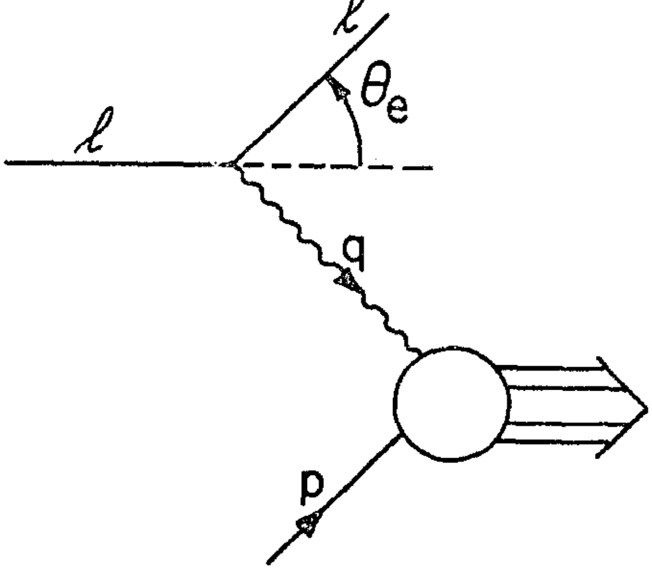
Fig. 2 Diagram representing the discontinuity of the forward virtual Compton amplitude. The intermediate states are on shell and have a multiplicity n_i.

Fig. 3 Diagrams of $0(\lambda)$ which make a leading contribution in the Bjorken limit. The vertical line represents the unitarity cut and crosses those lines which are on shell.

Fig. 4 Leading diagrams in the general case. We sum over all permutations of the emission of the ℓ photons on the left side and all permutations of absorption of the r photons on the right. (a) All vector mesons are real. (b) Diagrams which have some virtual vector mesons which build up the elastic form factor corrections to the external current vertices.

Fig. 5 The Born term for inelastic lepton-nucleon scattering.

Fig. 6 Leading diagrams in $0(\lambda)$ in the scalar-vector field theory. In addition to the ones shown one has virtual photon diagrams as in Fig. 3(b) and (d).



$$S_{ep} = (\ell + p)^2$$

$$-q^2 = Q^2 > 0$$

$$m\nu = p \cdot q$$

$$x = \frac{Q^2}{2m\nu}$$

Fig. 1

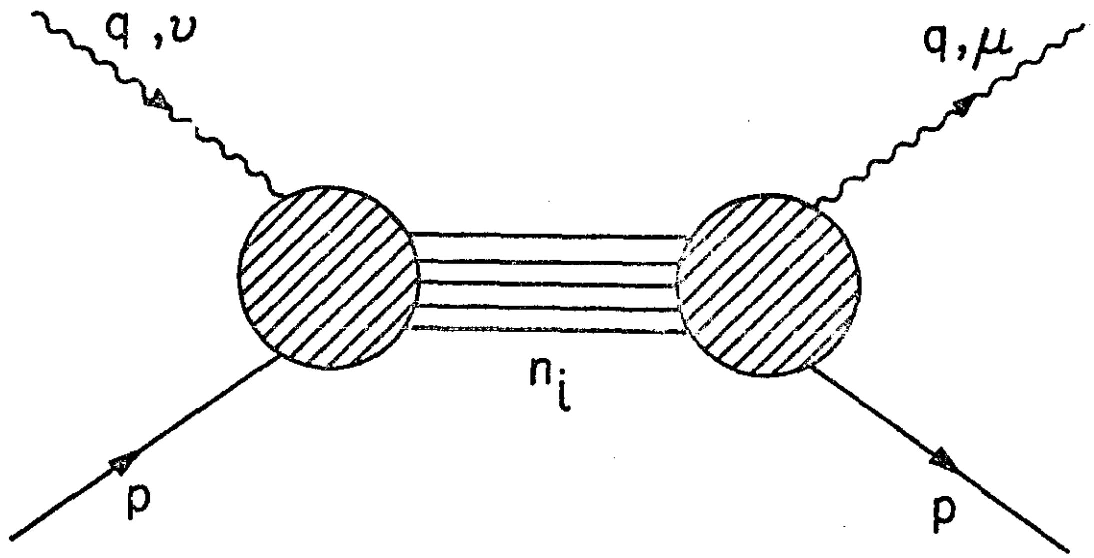


Fig. 2

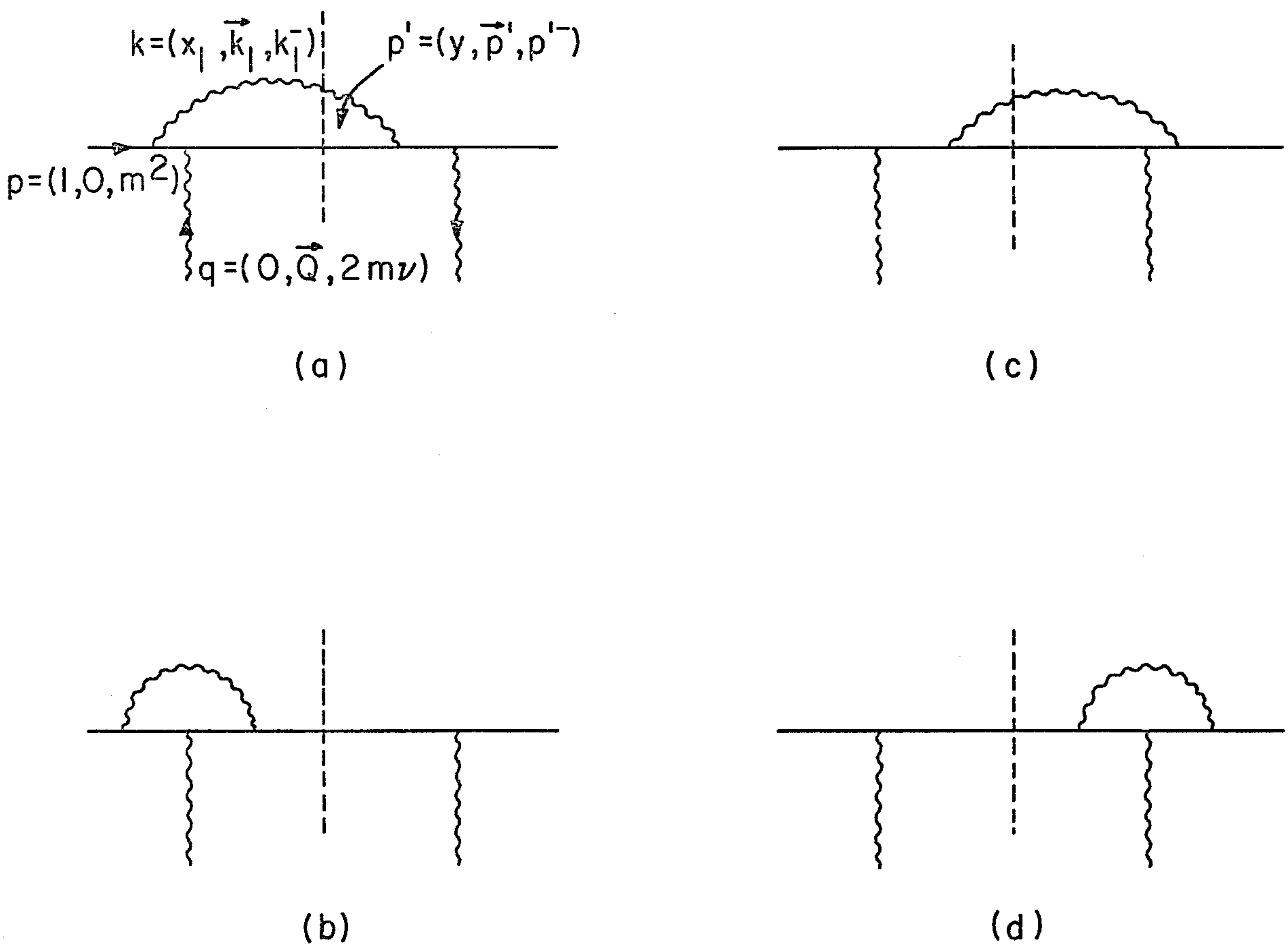


Fig. 3

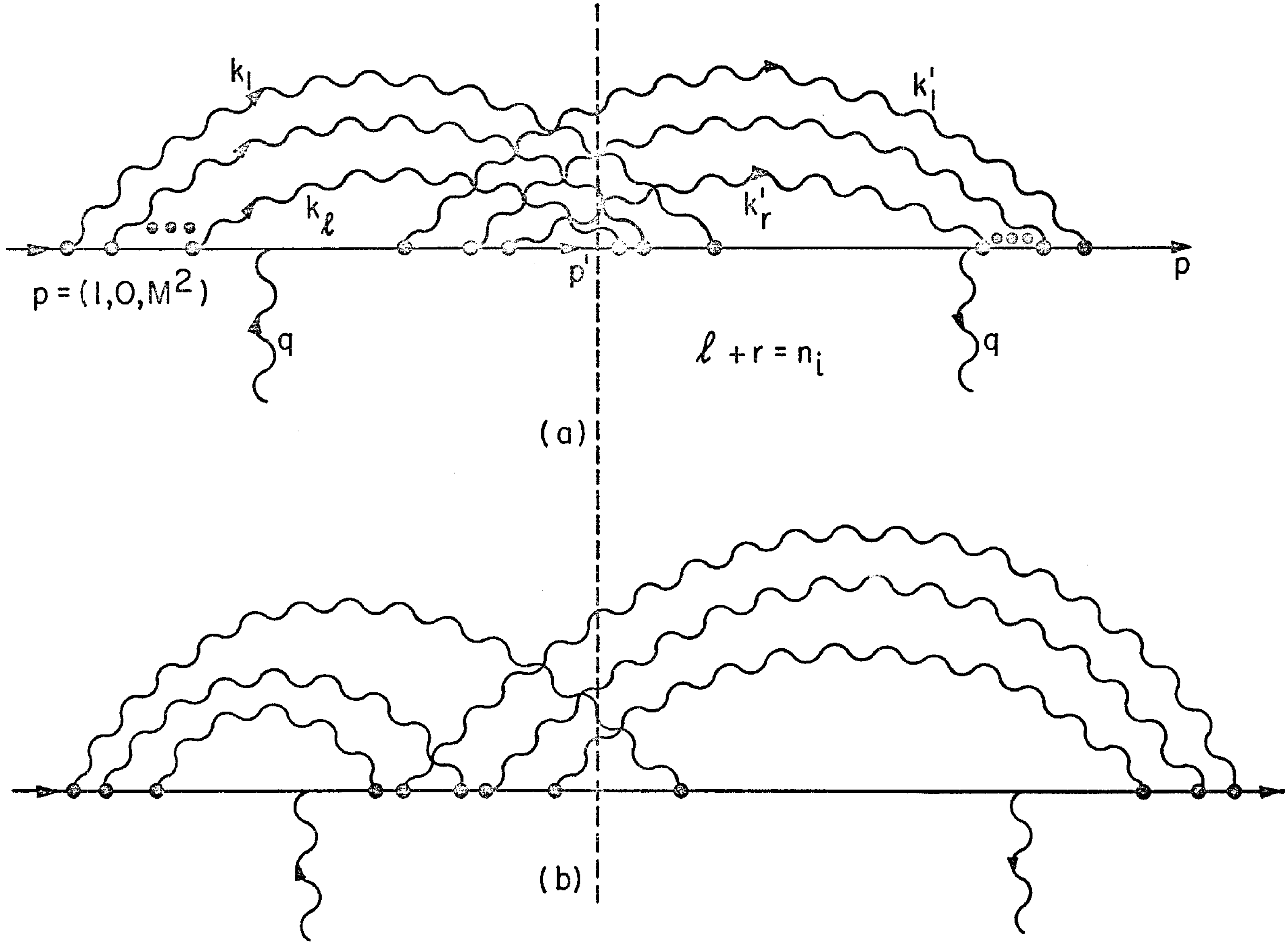


Fig. 4

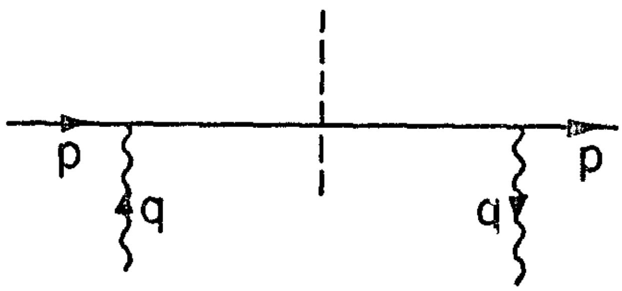


Fig. 5

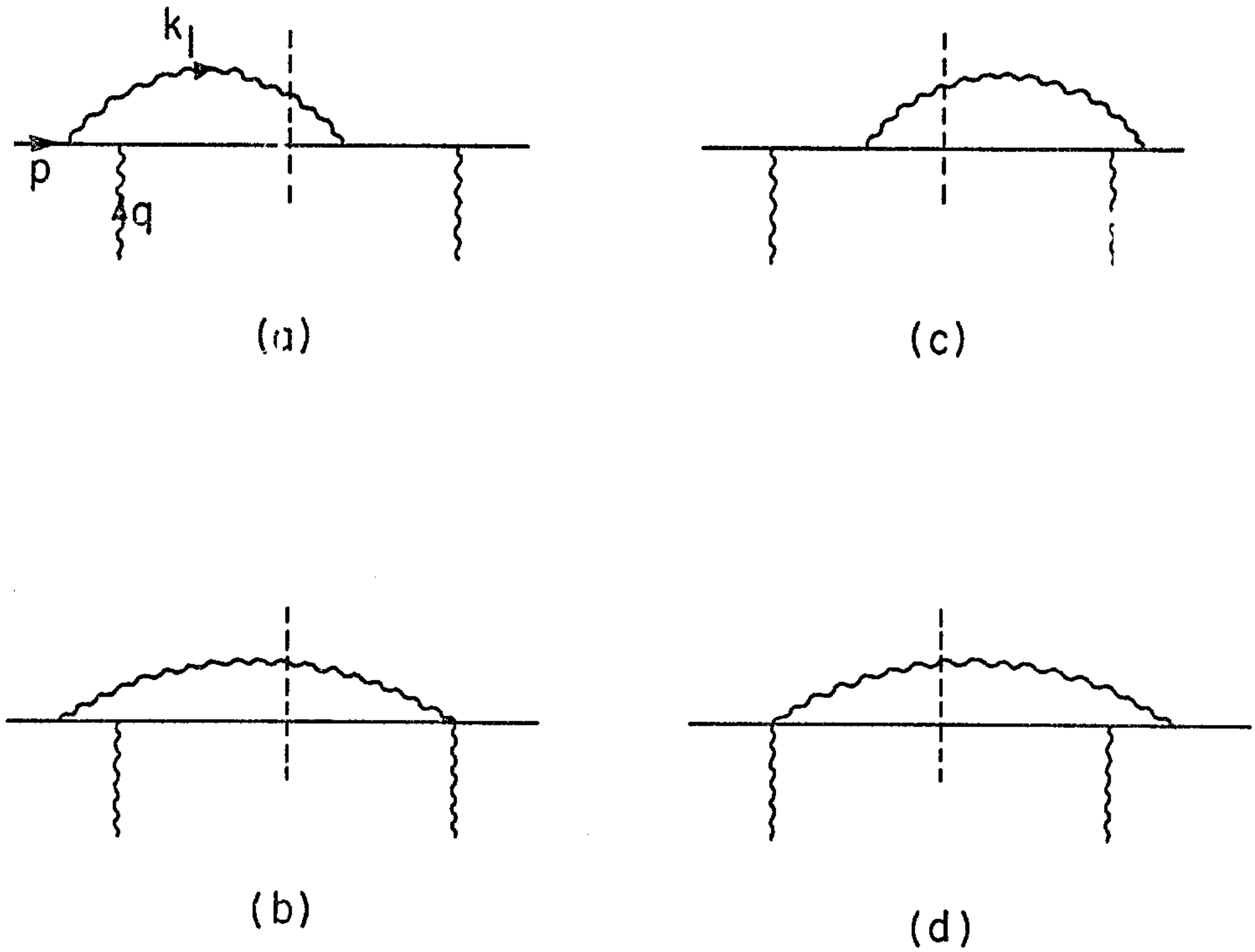


Fig. 6